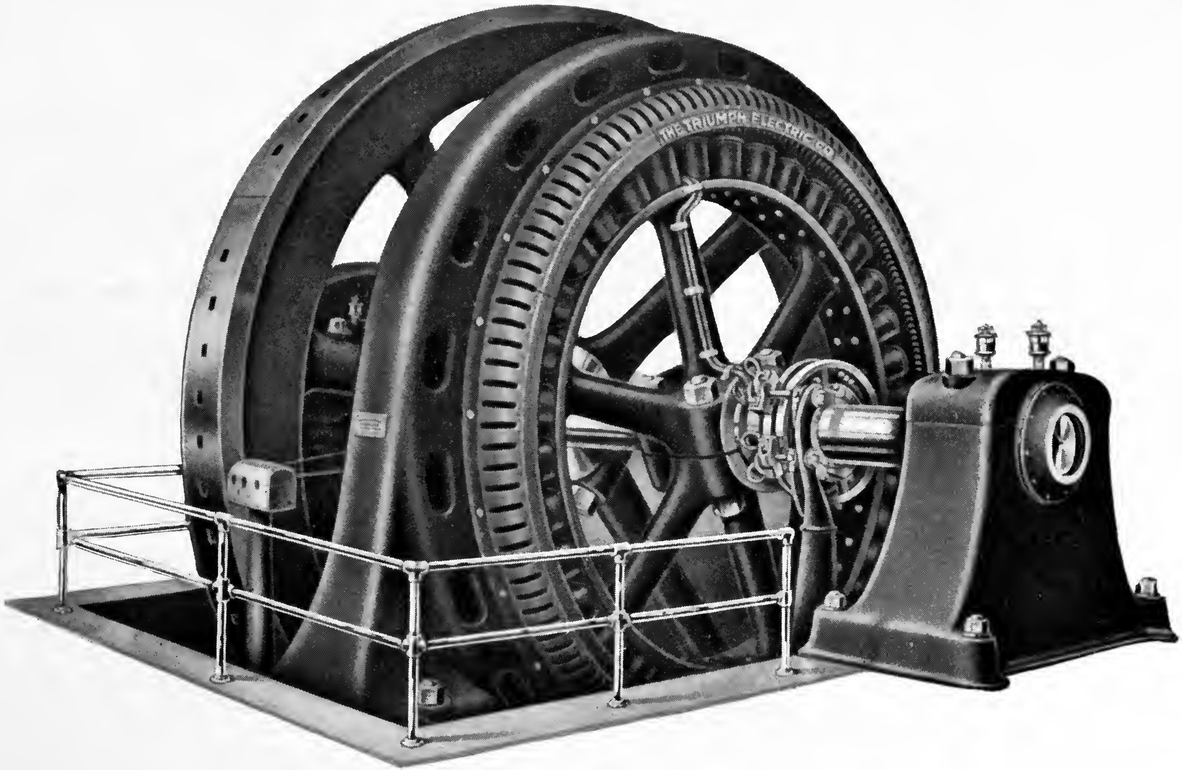

(*Frontispiece*)

Three-phase Alternating-current Generator.

ESSENTIALS OF
ELECTRICAL ENGINEERING

A TEXT BOOK FOR COLLEGES AND
TECHNICAL SCHOOLS

BY

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282 ILLUSTRATIONS

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PREFACE

The widely prevalent belief that continuous and alternating currents are *not* subject to the same general laws, is entirely erroneous. The principles and laws which relate to the flow of continuous currents also govern the flow of alternating currents.

This volume, which is offered as a text for the use of students pursuing either electrical or non-electrical engineering courses, is the result of the writer's class-room experience, and seeks to emphasize the fact that continuous and alternating currents are governed by the same laws. To this end the fundamental laws of the electric circuit are fully developed before any study of machines is attempted. With a thorough knowledge of the electric circuit as a foundation, the student should have little trouble in comprehending the physical phenomena taking place in the more common types of electrical apparatus.

The student is expected to be familiar with trigonometry, and a knowledge of calculus will be found advantageous but not indispensable. The mathematical developments of the formulæ for the calculation of inductance and capacitance have been placed in appendices at the back. These and other portions of the text may be omitted when, for lack of time or for any other reason, it is necessary to shorten the course.

The fact that the ideas advanced in this volume have developed with the science of Electrical Engineering, and may be regarded as the common property of the science, would make any attempt to give specific credit burdensome (and often impossible), but the writer wishes to specifically acknowledge his indebtedness to both standard and current literature, particularly to those works listed on page 333. Students desiring a more detailed discussion of particular subjects are referred to this list.

The writer also wishes to express his obligation to the following men, each of whom read all or part of the manuscript, and offered valuable suggestions for its improvement: Professor C. M. Jan-

sky, of the University of Wisconsin; Professor H. H. Higbie, Professor A. H. Lovell, Mr. A. H. Stang and Mr. W. L. Bice, of the University of Michigan.

Some idea of the practical construction of electrical machinery and instruments is given by means of a limited number of illustrations of actual apparatus. These illustrations are offered through the courtesy of the manufacturers.

J. F. W.

Ann Arbor, Mich.
June, 1915.

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NOTATION*

A = area.
B = magnetic flux density.
b = susceptance.
C = capacitance (electrostatic capacity).
D = electrostatic flux density.
e = electromotive force, instantaneous alternating electromotive force.
E_m = maximum alternating electromotive force.
E = continuous electromotive force, effective alternating electromotive force.
f = frequency, force.
$\mathcal{F}$ = magnetomotive force.
F = electrostatic field intensity.
g = conductance.
H = magnetic field intensity, magnetizing force.
i = instantaneous current.
I_m = maximum alternating current.
I = continuous current, effective alternating current.
K = dielectric constant.
kva = kilovolt-amperes.
kw = kilowatts.
L = inductance.
l = length.
m = unit magnet pole.
n = speed (revolutions per second).
N = number of armature conductors, number of turns in a winding.
P = power in continuous-current circuit, average power in alternating-current circuit.
p = number of poles on a dynamo, instantaneous power in alternating-current circuit.
p' = number of paths into which an armature winding is divided.
q, Q = quantity of electricity.
R = resistance.
$\mathcal{R}$ = reluctance.
s = slip of an induction motor.
t = time.
T = torque, temperature.
V = velocity, volume.
W = work.
X = reactance.
Y = admittance.
Z = impedance.
α = an angle.

* Based on the report of the Standardization Committee of the American Institute of Electrical Engineers (1915).

β = an angle.
ϕ = magnetic flux, an angle.
θ = an angle.
ψ = dielectric flux.
μ = permeability.
ω = angular velocity (radians per second).
ρ = resistivity.

Essentials of Electrical Engineering

CHAPTER I

THE ELECTRIC CIRCUIT

1. Introduction. — The ultimate nature of electricity has never been discovered but it is generally conceived to be a medium, without weight or form, by means of which the energy of heat or motion may be transferred from one point to another. By means of this medium the energy of motion, as developed by the electric generator, may be transferred and caused to reappear at some distant point in the form of motion (the electric motor), or as heat (the electric lamp). This conception of electricity is upheld by the similarity of an electric and a hydraulic system.

Contrary to a widely prevalent belief, electricity is not erratic in its action but is governed by simple and well-defined laws. In the following pages the fundamental laws of the electric circuit are developed, and the physical phenomena which take place in the more common types of electrical apparatus explained.

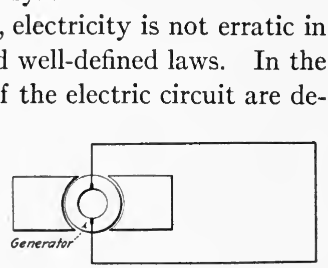


Fig. 1. The Electric Circuit.

2. The electric current. — The electric circuit (Fig. 1) is analogous, in many respects, to a hydraulic system, consisting of the pump and pipe connections shown in Fig. 2. The rotary pump indicated in Fig. 2a produces a continuous and uniform flow of water in the direction indicated by the arrows; the piston pump indicated in Fig. 2b produces a flow which is always in the direction indicated by the arrows but which is not uniform, *i.e.*, when the piston speed is reduced and its motion reversed at the end of the stroke, the flow of water slackens or ceases altogether; the valveless pump indicated in Fig. 2c produces a flow which is not constant, either in direction or in value, but surges first in one direction and then in the other through the system.

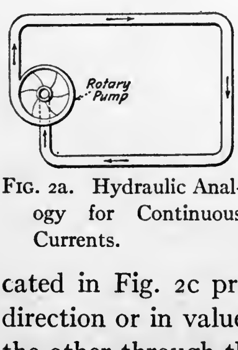


Fig. 2a. Hydraulic Analogy for Continuous Currents.

Similarly the construction of the generator determines the characteristics of the electric current flowing in the circuit. Electric currents may be divided into three classes: (*a*) continuous, (*b*) pulsating, (*c*) alternating.

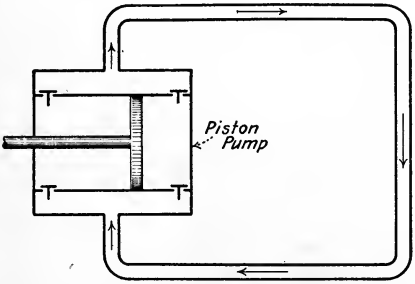


FIG. 2b. Hydraulic Analogy for Pulsating Currents.

(*a*) *Continuous currents.* — When the current in a given circuit flows continuously in one direction and the rate of flow is uniform, it is said to be a "continuous current" and is commonly called a "direct current." The voltaic cell and certain forms of the electric generator cause a unidirectional current to flow at an approximately constant rate and are, therefore, continuous-current apparatus.

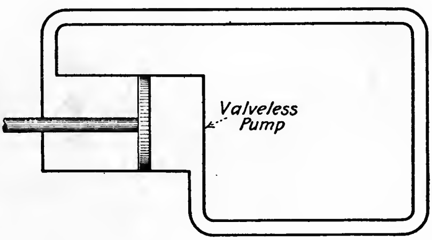


FIG. 2c. Hydraulic Analogy for Alternating Currents.

(*b*) *Pulsating currents.* — If the current in a given circuit flows in one direction but at a momentarily changing rate, it is a "pulsating current." Pulsating currents are largely used in telephone work and in telegraphy.

(*c*) *Alternating currents.* — When the current in a given circuit, starting at zero, increases to maximum, decreases to zero, increases to maximum in the opposite direction, and again decreases to zero (the process being repeated periodically), it is an "alternating current." The wave forms produced by commercial alternators vary greatly, but comparisons and mathematical calculations, unless specifically stated otherwise, are based on the harmonic or sine wave form.* While this wave form is seldom or never attained, its assumption is usually sufficiently accurate for practical purposes. Calculations for the actual form of the wave may become extremely complicated.

3. **Manifestations of the electric current.** — The presence of an electric current is made manifest in a number of ways. The prin-

* See Appendix A.

cipal effects of the electric current with which the electrical engineer is concerned are: (*a*) chemical, (*b*) magnetic and (*c*) heating.

(*a*) *Chemical effect.* — When water containing a small quantity of acid forms part of a circuit carrying a unidirectional current, bubbles may be seen to rise through the liquid. When the gases causing these bubbles are collected, they are found to be oxygen and hydrogen, the elements which enter into chemical combination to form water. The same action takes place in a circuit carrying alternating current, but each half cycle destroys the chemical effect of the preceding half cycle so that the net effect is zero. Hence, alternating currents are *not* used when chemical effects are desired.

(*b*) *Magnetic effect.* — A current-carrying conductor exhibits all the characteristics of a magnet in that it attracts pieces of iron, and is attracted to or repelled from a magnet or another current-carrying conductor. Attraction or repulsion takes place according to the polarity of the magnet, or the relative directions of the currents in the conductors. This magnetic effect is intensified if the conductor is in the form of a spiral. It is still further increased if the spiral is wound about an iron core.

(*c*) *Heating effect.* — When an electric current flows in a wire, heat is liberated. The heating may be so slight as to be unnoticed as in the ordinary electric bell circuit, or so great as to heat the conductor to incandescence as in the electric glow lamp. Light is not a direct manifestation of the electric current, but is due to the high temperature to which the current heats the conductor.

4. Series circuits. — A series circuit, represented in Fig. 3a, is one in which the entire current in the circuit flows successively through each piece of apparatus. The commercial application of the series circuit is somewhat limited, the most common use being in connection with street and other outdoor lighting.

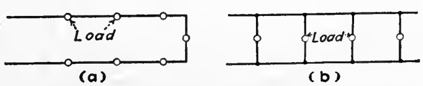


FIG. 3. Series and Parallel Circuits.

5. Parallel circuits. — A parallel or multiple circuit, represented in Fig. 3b, is one in which the current divides and flows through two or more branches. The current in each branch of a parallel circuit is independent of that in the other branches. Motors are always connected in parallel.

6. Electric units.* — The units of the electric system are: (*a*) the ohm, (*b*) the ampere, (*c*) the volt, (*d*) the coulomb, (*e*) the joule and (*f*) the watt.

(*a*) *The ohm.* — The opposition to the flow of an electric current is measured in ohms. The standard ohm is represented by the opposition offered to an unvarying electric current by a column of mercury at a temperature of melting ice, 14.4521 grams in mass, of a constant cross section, and of the length 106.3 centimeters.

(*b*) *The ampere.* — The rate at which electricity is transferred is measured in amperes, analogous to miner's inches or gallons per second, and the ampere is represented by the unvarying current which, when passed through a standard solution of nitrate of silver in water, deposits silver at the rate of 0.001118 gram per second.

(*c*) *The volt.* — The force which causes or tends to cause an electric current to flow is termed an "electromotive force," the unit of which, the volt, is the electromotive force that, steadily applied to a conductor whose resistance is one ohm, produces a current of one ampere.

(*d*) *The coulomb.* — Quantity of electricity is expressed in coulombs, analogous to gallons or cubic feet, and one coulomb is the quantity of electricity transferred in one second by a current of one ampere.

(*e*) *The joule.* — The unit of energy or work in the electric system is the joule, and is the energy expended in one second when a current of one ampere flows in a circuit having a resistance of one ohm. The energy equivalent of one joule has been experimentally determined to be

= 0.24 calorie.
= 0.00095 B.T.U.
= 0.737 foot-pound.
= 10,000,000 ergs.

(*f*) *The watt.*—The watt, which is the unit of electrical power, is represented by the expenditure of one joule in one second. Since

* There is a definite relation between the C.G.S. units (the so-called absolute units) and the ohm, the ampere and the volt, but these units are to be regarded as arbitrary units, similar to the foot and the pound, which have been adopted by international agreement, and legalized by statutory enactment.

the energy equivalent of one joule is 0.737 foot-pound, and 550 foot-pounds per second equal one horse power,

$$\text{h.p.} = \frac{550}{0.737}$$
$$= 746 \text{ watts.}$$

The kilowatt (kw.) is a commonly used unit of power and is equal to 1000 watts.

7. Resistance. — Resistance is the inherent property of a *material* which opposes the flow of an electric current, and by virtue of which electrical energy is converted into heat. It is analogous, in many respects, to mechanical friction, and has been experimentally determined to be directly proportional to length, and inversely proportional to cross-sectional area.

$$R = \rho \frac{\text{length}}{\text{area}}. \qquad (1)$$

The unit of length commonly used is the foot; that of area, the circular mil. The constant ρ is, then, the resistance in ohms of a section of the material one foot long and having a cross-sectional area of one circular mil. The average value of ρ for commercial copper may be taken as 10.8 at ordinary temperatures (25° C.).

Materials whose resistivity is low, such as copper, silver, aluminum, etc., are known as conductors; those whose resistivity is high, such as glass, rubber, mica, porcelain, etc., are known as insulators.

8. Circular mil. — The circular mil is an arbitrary unit of area much used in electrical calculations. Since electrical conductors are largely circular in cross section, it is convenient to have a unit of area that bears a simple relation to the diameter of a circle. The area of a circular cross section, in circular mils, is obtained by squaring the diameter in thousandths of an inch (mils). Hence, the area in circular mils is to the area in square measure as 4 is to π.

$$\frac{\text{area in circular mils}}{\text{area in millionths of a square inch}} = \frac{4}{\pi}. \qquad (2)$$

9. Temperature coefficient. — An increase in the temperature of a copper wire causes its resistance to increase. The ratio of the change in the resistance of a conductor, per degree change in

its temperature, to its resistance at its initial temperature, is its temperature coefficient. The temperature coefficient of most materials is not constant, but changes with the temperature. By reference to Table I it will be found that the temperature coefficient of copper decreases as the temperature increases.

TABLE I

TEMPERATURE COEFFICIENTS FOR COPPER

$$a = \frac{1}{234.5 + T}$$

T	a	T	a	T	a	T	a
0	0.00427	13	0.00404	26	0.00383	39	0.00366
1	0.00425	14	0.00403	27	0.00381	40	0.00364
2	0.00424	15	0.00401	28	0.00380	41	0.00362
3	0.00422	16	0.00399	29	0.00379	42	0.00361
4	0.00420	17	0.00397	30	0.00378	43	0.00360
5	0.00418	18	0.00396	31	0.00377	44	0.00360
6	0.00416	19	0.00394	32	0.00375	45	0.00358
7	0.00414	20	0.00393	33	0.00374	46	0.00356
8	0.00412	21	0.00391	34	0.00373	47	0.00355
9	0.00411	22	0.00390	35	0.00371	48	0.00354
10	0.00409	23	0.00388	36	0.00370	49	0.00352
11	0.00408	24	0.00387	37	0.00369	50	0.00351
12	0.00406	25	0.00385	38	0.00367	.....	....

$$R_t = R(1 \pm at).$$

R = the resistance at temperature T °C.
R_t = the resistance at temperature $(T \pm t)$.
a = the temperature coefficient at temperature T °C.
t = the change in temperature.

The resistance of some materials, of which carbon is an example, decreases as the temperature increases. Glass, at ordinary temperatures, has a very high resistivity, but in the liquid state its resistivity is comparatively low. These materials are said to have negative temperature coefficients. Certain metallic alloys have a practically constant resistance over a wide range of temperature.

10. Ohm's Law. — *The electromotive force required to overcome the opposition due to the resistance of any circuit is proportional to the current flowing in the circuit.*

When the electromotive force (e) is expressed in volts and the current (i) in amperes, the proportionality factor (R) is in ohms.

$$e = Ri. \tag{3}$$

The experimental fact stated above is the first fundamental law of the electric circuit and was first demonstrated and enunciated by Dr. G. S. Ohm. It applies equally to continuous, pulsating and alternating currents.

11. **Inductance.** — That property of a body which tends to maintain any existing state or condition of motion of the body is termed its inertia. The force (f) required to neutralize the effect of inertia is proportional to the rate at which the speed or velocity of the body changes, and the proportionality factor (M) is the mass of the body. When the velocity of a body is changed from V' to V'' in t seconds

$$\text{av.}f = \frac{M\,(V' - V'')}{t}, \tag{4}$$

and $$f = M \times \text{rate at which } V \text{ changes.} \tag{5}$$

Suppose the speed of a steam engine is increased from 100 r.p.m. to 200 r.p.m. in 10 seconds. A force is required to overcome the inertia of the flywheel and other rotating parts, but the energy used in causing the speed of the engine to increase is stored as kinetic energy in the rotating parts, and is returned to the system when the engine speed decreases to its original value. Therefore, the net expenditure of energy due to the inertia of the moving parts of a machine is zero when the final state of motion is the same as the initial state.

An electric circuit has a property, similar to that of inertia, which tends to maintain any state or condition of current flowing in the circuit. The electromotive force (e) required to overcome the effect of this property and cause the current to increase or decrease, is proportional to the rate at which the current changes, and the proportionality factor (L) is the *inductance* of the circuit. The unit of inductance is the henry. When the current in a circuit is changed from i' to i'' in t seconds

change

$$\text{av.}\,e = L\frac{i' - i''}{t}, \tag{6}$$

and $$e = L \times \text{rate at which } i \text{ changes.} \tag{7}$$

It is evident that the energy expended in causing a current to increase is equal and opposite to that expended in causing it to decrease to its original value. Therefore, the net expenditure of energy due to the inductance of an electric circuit is zero when the final current is equal to the initial current.

12. Capacitance. — In the system shown in Fig. 4, consisting of a pump, pipe connections, and a cylinder across which is stretched an elastic membrane, the pressure set up by the pump stresses the membrane until the reaction due to the stress is equal to the pressure of the pump, and the quantity of water displaced or stored is proportional to the pressure.

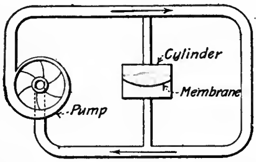


FIG. 4. Mechanical Analogy of Capacitance.

$$\text{gallons} = \text{constant} \times \text{pressure}. \qquad (8)$$

If the pressure due to the pump decreases, the stress in the membrane becomes less, and energy which tends to maintain the original pressure set up by the pump, is returned to the system.

An electric condenser consists, essentially, of two conductors separated by an insulating material, or dielectric. In the system shown in Fig. 5, the generator sets up a pressure which stresses the dielectric separating the plates PP until the reaction equals the electromotive force of the generator, and the quantity of electricity displaced or stored is proportional to the electromotive force (e) of the generator. The proportionality factor (C) is termed the *capacitance* of the condenser. The unit of capacitance is the farad.

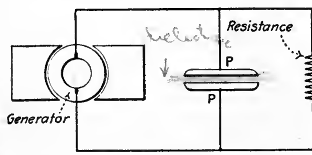


FIG. 5. Electric Capacitance.

$$q = Ce. \qquad (9)$$

When the applied electromotive force is changed from e' to e'' in t seconds

$$\text{av. } i = C\frac{e' - e''}{t} \qquad (10)$$

and

$$i = C \times \text{rate at which } e \text{ changes}. \qquad (11)$$

If the electromotive force of the generator decreases, energy is returned to the system and tends to maintain the electromotive force of the system at its former value, the stress in the dielectric being reduced proportionately. The energy returned to the system when the condenser is completely discharged is equal to the energy stored, the condenser being so constructed that the losses are negligible.

13. Alternating-current circuits containing resistance only. — From equation (3)

$$e = Ri \tag{12}$$

$$= RI_m \sin \omega t, \tag{13}$$

i.e., the current and the electromotive force in an alternating-current circuit containing resistance only, are in phase as shown in Fig. 6a.

14. Alternating-current circuits containing inductance only. — Since the electromotive force required to overcome the effects of inductance is proportional to the rate at which the current changes, the electromotive force required to overcome the inductance of an alternating-current circuit in which the current varies harmonically is

$$e = \omega LI_m \sin (\omega t + 90°),* \tag{14}$$

i.e., the current and the electromotive force in an alternating-current circuit containing inductance only, are 90 degrees out of phase, and the current lags behind the applied electromotive force as shown in Fig. 6b.

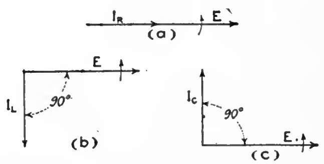


FIG. 6a. Vectors of Current and Electromotive Force in a Non-inductive Circuit.

FIG. 6b. Vectors of Current and Electromotive Force in an Inductive Circuit.

FIG. 6c. Vectors of Current and Electromotive Force in a Capacitive Circuit.

The maximum value of $\sin (\omega t + 90°)$ equals 1, and

$$E_m = \omega LI_m. \tag{15}$$

The quantity ωL is termed the inductive reactance of an alternating-current circuit and is expressed in ohms. (Symbol X_L.)

15. Alternating-current circuits containing capacitance only. — Since the quantity of electricity displaced in a condenser circuit is proportional to the applied electromotive force, the current (rate of displacement) in the circuit is proportional to the rate of change in the electromotive force. When the electromotive force varies harmonically

$$i = \omega CE_m \sin (\omega t + 90°),* \tag{16}$$

i.e., the current and the electromotive force in an alternating-current circuit containing capacitance only, are 90 degrees out of phase, and the current leads the applied electromotive force as shown in Fig. 6c.

* See Appendix A, Section 7.

The maximum value of $\sin(\omega t + 90°)$ equals 1,

$$I_m = \omega C E_m \tag{17}$$

and

$$E_m = \frac{I_m}{\omega C}. \tag{18}$$

The quantity $\frac{1}{\omega C}$ is termed the capacitive reactance * of an alternating-current circuit and is expressed in ohms. (Symbol X_C.*)

16. Reactance of alternating-current circuits. — The reactance of an alternating-current circuit is, from equations (15) and (18), the ratio of the current flowing in the circuit and the electromotive force required to overcome the combined effects of inductance and capacitance.

17. Alternating-current circuits containing resistance and inductance in series. — In a series circuit containing resistance and inductance, the applied voltage required to cause a given current to flow in the circuit must be equal to that required to overcome the combined effects of resistance and inductance.

$$e = e_R + e_L \tag{19}$$

$$= Ri + \omega L i \tag{20}$$

$$= RI_m \sin \omega t + \omega L I_m \sin(\omega t + 90°). \tag{21}$$

The right-hand member of equation (21) is the sum of two harmonic electromotive forces 90 degrees out of phase. The applied electromotive force is, therefore, a harmonic quantity, the maximum value of which is the geometric sum of the maximum values of the quadrature electromotive forces.

$$E_m = \sqrt{(RI_m)^2 + (\omega L I_m)^2} \tag{22}$$

$$= I_m \sqrt{R^2 + (\omega L)^2} \tag{23}$$

and the current lags behind the applied electromotive force by the angle

$$\phi = \tan^{-1} \frac{\omega L}{R}. \tag{24}$$

* Capacitive reactance is to be considered a negative quantity.

18. Alternating-current circuits containing resistance and capacitance in series. — In a series circuit containing resistance and capacitance,

$$e = e_R + e_C \tag{25}$$

$$= Ri + \frac{i}{\omega C} \tag{26}$$

$$= RI_m \sin \omega t + \frac{I_m \sin (\omega t - 90^\circ)}{\omega C}; \tag{27}$$

therefore,

$$E_m = \sqrt{(RI_m)^2 + \left(\frac{I_m}{\omega C}\right)^2} \tag{28}$$

$$= I_m \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}, \tag{29}$$

and the current leads the applied electromotive force by the angle

$$\phi = \tan^{-1} \frac{1}{\omega CR}. \tag{30}$$

19. Alternating-current circuits containing resistance, inductance and capacitance in series. — In a series circuit containing resistance, inductance and capacitance,

$$e = e_R + e_L + e_C \tag{31}$$

$$= Ri + \omega Li + \frac{i}{\omega C} \tag{32}$$

$$= RI_m \sin \omega t + \omega LI_m \sin (\omega t + 90^\circ) + \frac{I_m \sin (\omega t - 90^\circ)}{\omega C}. \tag{33}$$

From equation (33) $E_m = \sqrt{R^2 I_m{}^2 + \left(\omega LI_m + \frac{I_m}{\omega C}\right)^2}$ (34)

$$= I_m \sqrt{R^2 + \left(\omega L + \frac{1}{\omega C}\right)^2} \tag{35}$$

and the current leads or lags behind the applied electromotive force, as the quantity $\left(\omega L + \frac{1}{\omega C}\right)$ is negative or positive, the angle of lead or lag being

$$\phi = \tan^{-1} \frac{\omega L + \frac{1}{\omega C}}{R}. \tag{36}$$

20. Voltage triangle. — When an alternating current flows in a circuit containing resistance and reactance in series, the applied electromotive force is the resultant of quadrature electromotive forces, and the voltages of the circuit may be represented graphically by means of a right triangle. Fig. 7a. Because of the impossibility of eliminating resistance from a reactance, just as it is impossible to eliminate friction from a machine having moving parts, the more common form of the voltage triangle is an obtuse triangle, the hypotenuse and the sides of which are, respectively, the applied electromotive force, the voltage between the terminals of the resistor, and that between the terminals of the reactor. Fig. 7b.

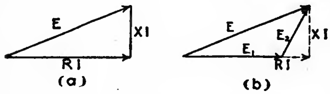


FIG. 7. Voltage Triangles.

21. Impedance and the impedance triangle. — Dividing equation (35) by I_m, and representing the quantity $\omega L + \frac{1}{\omega C}$ by X

$$\frac{E_m}{I_m} = \sqrt{R^2 + X^2}. \tag{37}$$

The quantity $\sqrt{R^2 + X^2}$ is termed the *impedance* of an alternating-current circuit, and is the ratio of the applied electromotive force and the current flowing in the circuit. The unit of impedance (Z) is the ohm.

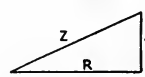


FIG. 8. Impedance Triangle.

The impedance, the resistance and the reactance of an alternating-current circuit form respectively, the hypotenuse, the base, and the altitude of a right triangle, and may be represented graphically as in Fig. 8.

Example. — A series circuit has the following:

$$E_m = 220 \text{ volts.}$$
$$f = 60 \text{ cycles.}$$
$$R = 30 \text{ ohms.}$$
$$L = 0.1 \text{ henry.}$$
$$C = 0.00013 \text{ farad.}$$

Find:

(a) The reactance; (b) the impedance; (c) the current.

Solution.

$$X_L = 377 \times 0.1 = 37.7 \text{ ohms.}$$

$$X_C = \frac{1}{377 \times 0.00013} = 20.4 \text{ ohms.}$$

$$X = X_L + X_C = 17.3 \text{ ohms.}$$

$$Z = \sqrt{(30)^2 + (17.3)^2} = 34.7 \text{ ohms.}$$

$$I_m = \frac{E_m}{Z} = \frac{220}{34.7} = 6.3 \text{ amperes.}$$

22. Alternating-current circuits containing resistance and inductance in parallel. — At any instant the total current supplied to a parallel system is the *algebraic* sum of the currents flowing in the branches.

$$i = i_R + i_L. \tag{38}$$

By definition

$$i_R = (I_m)_R \sin \omega t. \tag{39}$$

From equation (14)

$$i_L = (I_m)_L \sin (\omega t - 90^\circ). \tag{40}$$

Substituting in equation (38),

$$i = (I_m)_R \sin \omega t + (I_m)_L \sin (\omega t - 90^\circ). \tag{41}$$

The right-hand member of equation (41) is the sum of two harmonic currents 90 degrees out of phase. The total current supplied to the system is, therefore, a harmonic quantity, the maximum value of which is the *geometric* sum of the maximum values of the currents flowing in the branches.

$$I_m = \sqrt{(I_m)_R^2 + (I_m)_L^2} \tag{42}$$

and the total current lags behind the applied electromotive force by the angle

$$\phi = \tan^{-1} \frac{(I_m)_L}{(I_m)_R}. \tag{43}$$

23. Alternating-current circuits containing resistance and capacitance in parallel. — When resistance and capacitance are connected in parallel

$$i = i_R + i_C, \tag{44}$$

$$i_R = (I_m)_R \sin \omega t. \tag{45}$$

From equation (16)

$$i_C = (I_m)_C \sin (\omega t + 90^\circ). \tag{46}$$

Substituting in equation (44),

$$i = (I_m)_R \sin \omega t + (I_m)_C \sin (\omega t + 90^\circ), \tag{47}$$

$$I_m = \sqrt{(I_m)_R^2 + (I_m)_C^2} \tag{48}$$

and the total current leads the applied electromotive force by the angle

$$\phi = \tan^{-1} \frac{(I_m)_C}{(I_m)_R}. \tag{49}$$

24. Alternating-current circuits containing resistance, inductance and capacitance in parallel. — When resistance, inductance and capacitance are connected in parallel

$$i = i_R + i_L + i_C, \tag{50}$$

$$i_R = (I_m)_R \sin \omega t, \tag{51}$$

$$i_L = (I_m)_L \sin (\omega t - 90^\circ), \tag{52}$$

$$i_C = (I_m)_C \sin (\omega t + 90^\circ). \tag{53}$$

Substituting in equation (50),

$$i = (I_m)_R \sin \omega t + (I_m)_L \sin (\omega t - 90^\circ) + (I_m)_C \sin (\omega t + 90^\circ), \tag{54}$$

$$I_m = \sqrt{(I_m)_R^2 + [(I_m)_L - (I_m)_C]^2} \tag{55}$$

and the total current leads the applied electromotive force if $[(I_m)_L - (I_m)_C]$ is negative, and lags behind the applied electromotive force if $[(I_m)_L - (I_m)_C]$ is positive, the angle of lead or lag being

$$\phi = \tan^{-1} \frac{(I_m)_L - (I_m)_C}{(I_m)_R}. \tag{56}$$

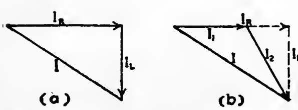


FIG. 9. Current Triangles.

25. Current triangle. — The current relations of a parallel circuit are shown graphically by means of a right triangle. Fig. 9a. Because of resistance in the inductive or capacitive branch the currents in parallel circuits are seldom 90 degrees out of phase, and Fig. 9b shows the usual relations of the component and the resultant currents.

26. Joule's Law. — *The work done in any electrical circuit is proportional to the square of the current flowing in the circuit and to the time the flow continues.*

When the electromotive force (e) is expressed in volts, the current (i) in amperes and the time (t) in seconds, the unit of work is the joule (A), and the proportionality factor is in ohms.

$$A \propto i^2t. \tag{57}$$

Joule's Law is the second fundamental law of the electric circuit and, like Ohm's Law, has been shown, experimentally, to be universal.

27. Work done in an electric circuit. — From equation (57)

$$A_R = Ri^2t \tag{58}$$

when current flows in a circuit containing resistance only;

$$A_L = \omega Li^2t \tag{59}$$

when current flows in a circuit containing inductance only;

$$A_C = \frac{i^2t}{\omega C} \tag{60}$$

when current flows in a circuit containing capacitance only.

28. Power in an electric circuit. — Power is, by definition, the rate of doing work. Therefore,

$$\text{Power} = \frac{\text{work}}{\text{time}} \tag{61}$$

and

$$p_R = Ri^2 \text{ watts} \tag{62}$$

when current flows in a circuit containing resistance only;

$$p_L = \omega Li^2 \text{ watts} \tag{63}$$

when current flows in a circuit containing inductance only;

$$p_C = \frac{i^2}{\omega C} \text{ watts} \tag{64}$$

when current flows in a circuit containing capacitance only.

From equation (3)

$$Ri = e_R. \tag{65}$$

From equation (14)

$$\omega Li = e_L. \tag{66}$$

From equation (16)

$$\frac{i}{\omega C} = e_C. \tag{67}$$

Therefore, the instantaneous power (watts) in any electrical circuit is equal to the product of the applied electromotive force (volts) and the current (amperes) flowing in the circuit.

$$p = ei. \tag{68}$$

In a continuous-current circuit, e and i are constant and

$$P = EI \text{ watts.} \tag{69}$$

In an alternating-current circuit containing resistance only

$$p_R = E_m \sin \omega t I_m \sin \omega t \tag{70}$$

$$= E_m I_m \sin^2 \omega t. \tag{71}$$

From Appendix A, Section 9, the average value of $\sin^2 \omega t$ during one complete cycle is $\frac{1}{2}$. Therefore,

$$\text{av. } p_R = \frac{E_m I_m}{2} \text{ watts.} \tag{72}$$

In an alternating-current circuit containing inductance only

$$p_L = E_m \sin \omega t I_m \sin (\omega t - 90^\circ) \tag{73}$$

$$= E_m I_m \sin \omega t \cos \omega t, \tag{74}$$

but the average value of $\sin \omega t$ or $\cos \omega t$ during one complete cycle is zero.* Therefore,

$$\text{av. } p_L = 0. \tag{75}$$

In an alternating-current circuit containing capacitance only

$$p_C = E_m \sin (\omega t + 90^\circ) \omega t I_m \sin \omega t \tag{76}$$

$$= E_m I_m \cos \omega t \sin \omega t, \tag{77}$$

which is identical with equation (74) and

$$\text{av. } p_C = 0. \tag{78}$$

In any alternating-current circuit

$$p = E_m \sin \omega t\, I_m (\sin \omega t \pm \phi) \tag{79}$$

$$= E_m I_m \sin \omega t (\sin \omega t \cos \phi \pm \cos \omega t \sin \phi) \tag{80}$$

$$= E_m I_m (\sin^2 \omega t \cos \phi \pm \sin \omega t \cos \omega t \sin \phi). \tag{81}$$

* See Appendix A, Section 8.

From Appendix A, Section 9,

$$\text{av. } \sin^2 \omega t = \tfrac{1}{2} \tag{82}$$

and

$$\text{av. } \sin \omega t \cos \omega t \sin \phi = 0. \tag{83}$$

Therefore,

$$\text{av. } p = \frac{E_m I_m \cos \phi}{2} \text{ watts,} \tag{84}$$

i.e., the average power * (watts) in any alternating-current circuit is equal to one-half the product of the maximum electromotive force (volts), the maximum current (amperes) and the cosine of the phase angle.

29. Effective current and electromotive force. — *The steady current (or electromotive force) which, acting in a circuit of constant resistance and zero reactance, transfers energy at the average rate of transfer when an alternating current (or electromotive force) acts in the same circuit, is the effective value of the alternating current (or electromotive force).*

From equation (62)

$$p = Ri^2 \tag{85}$$

$$= RI_m^2 \sin^2 \omega t. \tag{86}$$

Then

$$\text{av. } p = RI_m^2 \text{ av. } (\sin^2 \omega t)\dagger \tag{87}$$

$$= \frac{RI_m^2}{2}. \tag{88}$$

For continuous currents

$$P = RI^2. \tag{89}$$

Therefore,

$$RI^2 = \frac{RI_m^2}{2} \tag{90}$$

and

$$I = \frac{I_m}{\sqrt{2}} \tag{91}$$

$$= 0.707\, I_m. \tag{92}$$

Similarly,

$$\frac{E^2}{R} = \frac{E_m^2}{2R} \tag{93}$$

* A watt-meter indicates the average power in an alternating-current circuit.

† See Appendix A, Section 9.

and

$$E = \frac{E_m}{\sqrt{2}} \qquad (94)$$

$$= 0.707\, E_m. \qquad (95)$$

Therefore, the *effective* value of a harmonic alternating current (or electromotive force) is equal to its maximum value divided by the square root of 2, and effective values may be substituted in any of the above equations containing maximum values.

Effective values are indicated by ammeters and voltmeters, and are always to be assumed in alternating-current problems unless the values given or required are specifically stated to be maximum or instantaneous.

30. **Power factor.** — The cosine of the phase angle ($\cos \phi$) is termed the power factor of an alternating-current circuit. The power factor may also be expressed as the ratio of the resistance to the impedance,

$$\cos \phi = \frac{R}{Z}, \qquad (96)$$

or by the ratio of the watts to the product of volts and amperes,

$$\cos \phi = \frac{P}{EI}. \qquad (97)$$

31. **Resistances in series.** — The resistance of a series circuit is the *arithmetical* sum of the resistances of its parts.

$$E = E_1 + E_2 \qquad (98)$$

$$= I\,(R_1 + R_2). \qquad (99)$$

Dividing by I

$$R = R_1 + R_2. \qquad (100)$$

32. **Reactances in series.** — The reactance of a series circuit is the *algebraic* sum of the reactances of its parts.

$$E = E_1 + E_2 \qquad (101)$$

$$= I\,(X_1 + X_2). \qquad (102)$$

Dividing by I

$$X = X_1 + X_2. \qquad (103)$$

33. Impedances in series. — The impedance of a series circuit is the *geometric* sum of the impedances of its parts. From equation (35)

$$E = \sqrt{E_r^2 + E_x^2} \tag{104}$$

$$= I\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}. \tag{105}$$

Dividing by I

$$Z = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}. \tag{106}$$

34. Resistances in parallel. — In a parallel system the branches of which contain resistance only, the reciprocal of the resistance of the system is the *arithmetical* sum of the reciprocals of the resistances of the branches.

$$I = I_1 + I_2, \tag{107}$$

$$\frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2}. \tag{108}$$

Dividing by E

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{109}$$

35. Reactances in parallel. — In a parallel system the branches of which contain reactance only, the reciprocal of the reactance of the system is the *algebraic* sum of the reciprocals of the reactances of the branches.

$$I = I_1 + I_2, \tag{110}$$

$$\frac{E}{X} = \frac{E}{X_1} + \frac{E}{X_2}. \tag{111}$$

Dividing by E

$$\frac{1}{X} = \frac{1}{X_1} + \frac{1}{X_2}. \tag{112}$$

36. Resistance and reactance in parallel. — In a parallel system consisting of a branch containing resistance only and a branch containing reactance only, the reciprocal of the impedance of the system is the *geometric* sum of the reciprocal of the resistance and the reciprocal of the reactance of the branches.

From equation (55)

$$I = \sqrt{I_1^2 + I_2^2}, \tag{113}$$

$$\frac{E}{Z} = \sqrt{\left(\frac{E}{R}\right)^2 + \left(\frac{E}{X}\right)^2}. \tag{114}$$

Dividing by E

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X^2}}. \tag{115}$$

37. Impedances in parallel. — Any impedance containing both resistance and reactance (Fig. 10) may be replaced by a parallel

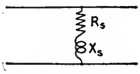


FIG. 10. Resistance and Inductance in Series.

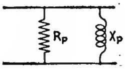


FIG. 11. Resistance and Inductance in Parallel.

system the branches of which contain only resistance or reactance (Fig. 11). Referring to Fig. 10 let

E = the applied electromotive force,
I = the current,
Z = the impedance,
$\cos\phi$ = the power factor.

Then $I_r = I\cos\phi$ (= power component of current),
$I_x = I\sin\phi$ (= wattless component of current),
$R_s = Z\cos\phi$,
$X_s = Z\sin\phi$.

Referring to Fig. 11, let

E = the applied electromotive force,
I = the line current,
I_r = the current in the resistance,
I_x = the current in the inductance,
R_p = the resistance,
X_p = the reactance,
Z = the impedance.

i.e., the applied electromotive force, the current, the impedance and the power factor of one system are made equal to those of the other system, and the systems are, therefore, equivalent.

$$R_p = \frac{E}{I_r} \tag{116}$$

$$= \frac{E}{I\cos\phi} \tag{117}$$

$$= \frac{Z}{\cos\phi} \tag{118}$$

$$= \frac{Z^2}{Z \cos \phi} \tag{119}$$

$$= \frac{Z^2}{R_s} \tag{120}$$

$$X_p = \frac{E}{I_x} \tag{121}$$

$$= \frac{E}{I \sin \phi} \tag{122}$$

$$= \frac{Z}{\sin \phi} \tag{123}$$

$$= \frac{Z^2}{Z \sin \phi} \tag{124}$$

$$= \frac{Z^2}{X_s}. \tag{125}$$

From equation (115)

$$\frac{1}{Z_p} = \sqrt{\frac{1}{R_p^2} + \frac{1}{X_p^2}} \tag{126}$$

$$= \sqrt{\left(\frac{R_s}{Z_s^2}\right)^2 + \left(\frac{X_s}{Z_s^2}\right)^2}. \tag{127}$$

Therefore, for a parallel system,

$$\frac{1}{Z} = \sqrt{\left(\frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2} + \frac{R_n}{Z_n^2}\right)^2 + \left(\frac{X_1}{Z_1^2} + \frac{X_2}{Z_2^2} + \frac{X_n}{Z_n^2}\right)^2}, \tag{128}$$

$$R = Z^2\left(\frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2} + \frac{R_n}{Z_n^2}\right), \tag{129}$$

$$X = Z^2\left(\frac{X_1}{Z_1^2} + \frac{X_2}{Z_2^2} + \frac{X_n}{Z_n^2}\right). \tag{130}$$

Example.—Let the circuit shown in Fig. 12 have: $R_1 = 6$ ohms, $R_2 = 8$ ohms, $R_3 = 10$ ohms, $X_1 = 5$ ohms, $X_2 = 4$ ohms, $X_3 = 3$ ohms. Then,

$$Z_1^2 = 6^2 + 5^2 = 61,$$

$$Z_2^2 = 8^2 + 4^2 = 80,$$

$$Z_3^2 = 10^2 + 3^2 = 109,$$

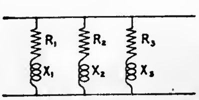


FIG. 12. Impedances in Parallel.

and

$$\frac{1}{Z} = \sqrt{\left(\frac{6}{61} + \frac{8}{80} + \frac{10}{109}\right)^2 \left(\frac{5}{61} + \frac{4}{80} + \frac{3}{109}\right)^2}$$

$$= \sqrt{(0.098 + 0.1 + 0.092)^2 + (0.082 + 0.05 + 0.028)^2}$$

$$= 0.331.$$

$$Z = \frac{1}{0.331} = 3.02 \text{ ohms.}$$

$$R = (3.02)^2 (0.098 + 0.1 + 0.092) = 2.65 \text{ ohms,}$$

$$X = (3.02)^2 (0.082 + 0.05 + 0.028) = 1.46 \text{ ohms.}$$

Also,

$$\tan \phi = \frac{0.082 + 0.05 + 0.028}{0.098 + 0.1 + 0.092} = \frac{0.159}{0.289} = 0.55.$$

$$\cos \phi = 0.876.$$

$$R = Z \cos \phi = 3.02 \times 0.876 = 2.65 \text{ ohms.}$$

$$\sin \phi = 0.482.$$

$$X = Z \sin \phi = 3.02 \times 0.482 = 1.46 \text{ ohms.}$$

38. Resonance of alternating-current circuits. — In a series circuit the electromotive force required to overcome the effect of inductance leads the current by 90 degrees; the electromotive force required to overcome the effect of capacitance lags behind the current by 90 degrees. These two harmonic forces are, then, 180 degrees out of phase, and tend to neutralize. When the inductive electromotive force is equal, numerically, to the capacitive electromotive force, the circuit is said to have *voltage* resonance.

$$\omega LI + \frac{I}{\omega C} = 0 \tag{131}$$

Example. — Find the capacitance required to neutralize an inductance of 0.1 henry in a 60-cycle alternating-current circuit, the capacitance to be connected in series with the inductance.

Solution. — From equation (131)

$$C = \frac{1}{\omega^2 L}$$

$$= \frac{1}{\pi^2 \times 4 \times 3600 \times 0.1}$$

$$= 0.007042 \text{ farad.}$$

In a parallel circuit, the wattless component of current in an inductive branch lags 90 degrees behind the applied electromotive

force; the wattless component of current in a capacitive branch is 90 degrees ahead of the applied electromotive force. These wattless components of current are, then, 180 degrees out of phase, and tend to neutralize. When the leading wattless component is equal, numerically, to the lagging wattless component, the circuit is said to have *current* resonance.

$$I_{\text{lag}} \sin \phi' = I_{\text{lead}} \sin \phi'' \qquad (132)$$

when I_{lag} = the current flowing in the inductive branch,
I_{lead} = the current flowing in the capacitive branch,
ϕ' = the angle between the vector of electromotive force and that of the current in the inductive branch,
ϕ'' = the angle between the vector of electromotive force and that of the current in the capacitive branch.

Example. — The impedance of a 60-cycle inductive circuit is 22 ohms, the current flowing in the circuit is 10 amperes, and lags 30 degrees behind the applied electromotive force. Find the capacitance to be connected in parallel with the impedance, the electromotive force and the resultant current to be in phase.

Solution. — The voltage across the condenser is

$$E_c = 22 \times 10 = 220 \text{ volts.}$$

and the wattless component of current is

$$I_w = 10 \times 0.5 = 5 \text{ amperes.}$$

From equation (18)

$$C = \frac{5}{377 \times 220}$$
$$= 0.00006 \text{ farad.}$$

39. Use of electrical measuring instruments. — A voltmeter is connected in parallel with that part of a circuit in which it is desired to measure the drop of potential. Fig. 13a.

An ammeter is connected in series with the circuit in which it is desired to measure the current, the entire current, or a constant proportion of it, going through the coils of the instrument. Fig. 13b.

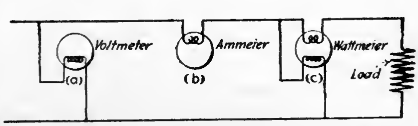


FIG. 13. Voltmeter, Ammeter and Wattmeter Connections.

A wattmeter consists of two current-carrying coils, a voltage

coil connected in parallel with the load and a current coil connected in series with the load. The reaction set up by the coils is proportional to the product of the currents flowing in the coils, *i.e.*, to the power in the circuit, and causes the deflection of a movable element.

Permanent-magnet instruments indicate on continuous-current circuits only; induction instruments on alternating-current circuits only; dynamometer instruments on either continuous- or alternating-current circuits.

CHAPTER I — PROBLEMS

1. Find the area in circular mils of: (*a*) a circle $\frac{1}{4}$ inch in diameter, (*b*) a rectangle $\frac{1}{4} \times \frac{1}{4}$ inch.

2. Find the diameter of a circle, the area of which is: (*a*) 211,600 circular mils, (*b*) 52,240 circular mils, (*c*) 10,400 circular mils, (*d*) 1000 circular mils.

3. Find the side of the square, the area of which is: (*a*) 211,600 circular mils, (*b*) 52,240 circular mils, (*c*) 10,400 circular mils, (*d*) 1000 circular mils.

4. Find the resistance, at 20° C., of 1000 feet of copper wire: (*a*) 0.1 inch in diameter, (*b*) 0.25 inch in diameter, (*c*) 0.6 inch in diameter, (*d*) 1 inch in diameter.

5. Find the resistances of the wires specified in Problem 4 at a temperature of 45° C.

6. Find the electromotive force required to cause a current of 10 amperes to flow in a circuit, the resistance of which is: (*a*) 1 ohm, (*b*) 5 ohms, (*c*) 18 ohms, (*d*) 25 ohms, (*e*) 40 ohms.

7. Find the average electromotive force of self-induction in a circuit, the inductance of which is 1 henry, when the current changes: (*a*) from 1 ampere to 5 amperes in 0.1 of a second, (*b*) from 10 amperes to zero in 0.01 of a second, (*c*) from 25 amperes to zero in 0.2 second.

8. Find the capacitance of a circuit when 5 coulombs of electricity are displaced by an electromotive force which changes: (*a*) from 25 volts to zero in 1 second, (*b*) from 50 volts to 100 volts in $\frac{1}{2}$ of a second, (*c*) from zero to 10 volts in 0.01 of a second.

9. The resistance of an electric circuit is 10 ohms. Find the maximum value of the current when the maximum value of the alternating electromotive force applied to the terminals of the circuit is: (*a*) 25 volts, (*b*) 40 volts, (*c*) 90 volts, (*d*) 150 volts, (*e*) 500 volts.

10. The inductance of a circuit is 0.1 henry. Find the electromotive force required to cause an alternating current of 100 amperes (maximum) to flow in the circuit when the frequency of the alternating current is: (*a*) 10, (*b*) 25, (*c*) 40, (*d*) 50, (*e*) 60, (*f*) 100.

11. Find the voltage required to charge a 60-microfarad condenser with 5 coulombs of electricity.

12. The charge on a condenser changes from 1 coulomb to 2 coulombs in

0.01 of a second. Find the average current flowing in the circuit during the period of change.

13. An electric circuit has an inductance of 0.1 henry. Find the capacitance required to cause resonance when the frequency of the alternating electromotive force is: (*a*) 25, (*b*) 40, (*c*) 60, (*d*) 100.

14. Find the impedance of an electric circuit having a resistance of 8 ohms and a reactance of 6 ohms connected in series.

15. Find the reactance of a circuit having an inductance of 0.15 henry when the frequency of the applied electromotive force is: (*a*) 15, (*b*) 25, (*c*) 40, (*d*) 60, (*e*) 100.

16. Find the reactance of a 10-microfarad condenser when the frequency of the applied electromotive force is: (*a*) 15, (*b*) 25, (*c*) 40, (*d*) 60, (*e*) 100.

17. The resistance of an electric circuit is 2 ohms. Find: (*a*) the total heat developed in the circuit when 50 amperes continuous current flows steadily in the circuit for 10 minutes, (*b*) the maximum value of the alternating-current required to develop the same quantity of heat in the same length of time, (*c*) the power (watts) in the circuit.

18. The continuous electromotive force applied to an electric circuit is 220 volts. This electromotive force produces a current of 20 amperes. Find: (*a*) the resistance of the circuit, (*b*) the power used in the circuit.

19. The effective value of an alternating current is 20 amperes, the applied electromotive force (effective) is 220 volts, and the power factor of the circuit is 0.85. Find: (*a*) the impedance of the circuit, (*b*) the resistance of the circuit, (*c*) the reactance of the circuit, (*d*) the power used in the circuit.

20. A rheostat and a condenser are connected in series to a 110-volt, 25-cycle alternating-current generator. The value of the resistance is 10 ohms and the capacitance of the condenser is 0.1 microfarad. Find: (*a*) the reactance of the circuit, (*b*) the impedance of the circuit, (*c*) the power factor of the circuit, (*d*) the current flowing in the circuit, (*e*) the power component of electromotive force, (*f*) the wattless component of electromotive force.

21. Three impedances, the values and power factors of which are indicated below, are connected in series. Find: (*a*) the impedance of the circuit, (*b*) the power factor of the circuit.

$Z_1 = 10$ ohms. p. f. = 0.9 (lagging).
$Z_2 = 15$ ohms. p. f. = 1.
$Z_3 = 20$ ohms. p. f. = 0.6 (lagging).

22. The impedances specified in Problem 21 are connected in parallel. Find: (*a*) the impedance of the circuit, (*b*) the resistance of the circuit, (*c*) the reactance of the circuit, (*d*) the power factor of the circuit.

23. Find the resistance of 1000 feet of copper wire, 102 mils in diameter and at a temperature of 40° C.

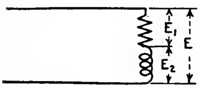
24. Find the length of copper wire, 204 mils in diameter, the resistance of which, at the same temperature, is equal to the resistance of the wire specified in Problem 23.

25. The resistance of the field windings of a shunt motor was found to be 70 ohms at 20° C. After the motor had been in operation for three hours, it was

found that the resistance of the windings was 10 per cent greater than when the first measurement was made. Find the temperature of the field windings.

26. Two loads A and B are connected in parallel to alternating-current mains. The power factor of A is 0.9 and $I_a = 100$ amperes; the power factor of B is 0.3 and $I_b = 50$ amperes. Find: (*a*) the current in the line, (*b*) the power factor of the system.

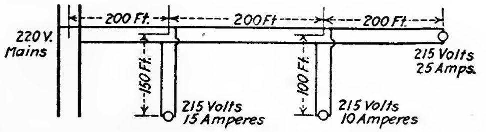
27. Measurements taken on the series system indicated in the accompanying sketch were as follows:



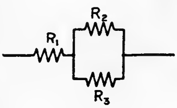
$I = 100$ amperes, $E = 2300$ volts, $E_1 = 1800$ volts, $E_2 = 900$ volts.

Find: (*a*) the power factor of the circuit, (*b*) the resistance of the circuit, (*c*) the reactance of the circuit, (*d*) the impedance of the circuit, (*e*) the power used in the circuit.

28. The continuous-current distributing system indicated in the accompanying sketch is connected to 220-volt supply mains. Determine: (*a*) the sizes of wires required for the different parts of the circuit so that 215 volts may be delivered at the terminals of each load, (*b*) the resistance of the distributing system (not including the load), (*c*) the power (watts) loss due to the resistance of the wires, (*d*) the total output of the generator.



29. Calculate the resistance of the circuit indicated in the accompanying sketch when:



$R_1 = 10$ ohms, $R_2 = 5$ ohms, $R_3 = 8$ ohms.

30. Find the impedance of the system specified in Problem 29 when a reactance of 2 ohms is connected in series with R_2.

31. The capacitance of each of three condensers is 10 microfarads. Find the capacitance of the circuit when: (*a*) the condensers are connected in series, (*b*) the condensers are connected in parallel.

32. Find the current flowing in a 25-microfarad condenser when connected to a 220-volt, 60-cycle, alternating-current system.

33. Three impedances A, B and C are connected in series to 60-cycle mains.

$$R_a = 10 \text{ ohms}, \; R_b = 5 \text{ ohms}, \; R_c = 12 \text{ ohms}, \; L_a = 0.1 \text{ henry},$$
$$L_b = 0.08 \text{ henry}, \; L_c = 0.2 \text{ henry}.$$

A current of 10 amperes (effective) flows in the circuit. Find the voltage between the terminals of: (*a*) impedance A, (*b*) impedance B, (*c*) impedance C, (*d*) the circuit (the applied voltage).

34. The impedances specified in Problem 33 are connected in parallel and to 110-volt, 25-cycle mains. Find: (*a*) the impedance of the system, (*b*) the resistance of the system, (*c*) the reactance of the system, (*d*) the current in each parallel branch, (*e*) the total current flowing in the circuit (line).

35. An induction motor is operated in parallel with 100 incandescent lamps. The motor takes 50 amperes and has a power factor of 0.8; the lamps each require 0.4 ampere and their power factor is unity. Find: (*a*) the current supplied to the combination, (*b*) the power factor of the circuit, (*c*) the power component of line current, (*d*) the wattless component of line current.

36. The current in a line supplying two induction motors is 100 amperes, and lags 45 degrees behind the electromotive force. One motor has a power factor of 0.85, and takes 60 amperes. Find the power factor of the other motor, and the current supplied to it.

37. The maximum value of an alternating current flowing in a circuit, the impedance of which is 25 ohms and the ratio $\frac{X}{R} = 0.75$, is 100 amperes. Find: (*a*) the effective value of the current, (*b*) the power in the circuit, (*c*) the power factor of the circuit.

38. Find the size wire required to supply 10 amperes to lamps 200 feet from the generator, when the allowable resistance of the wires is 0.2 ohm.

39. Find the current flowing in the circuit when the impedances of Problem 33 are connected to 25-cycle mains, the voltage of which is equal to the voltage found in part (*d*) of Problem 33.

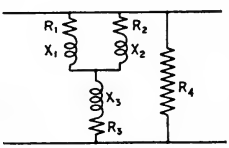
40. Three impedances are connected in parallel.

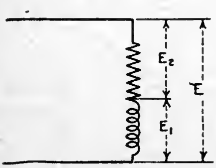
$I_a = 75$ and lags 30 degrees behind the electromotive force,
$I_b = 150$ and lags 45 degrees behind the electromotive force,
$I_c = 225$ and lags 15 degrees behind the electromotive force. Find: (*a*) the current in the supply line, (*b*) the power factor of the system.

41. The resistances and reactances indicated in the accompanying sketch have the following values:

$$R_1 = 5,\ R_2 = 3,\ R_3 = 6,\ R_4 = 10,\ X_1 = 8,\ X_2 = 4,\ X_3 = 3.$$

Find: (*a*) the impedance of the circuit, (*b*) the power factor of the circuit.



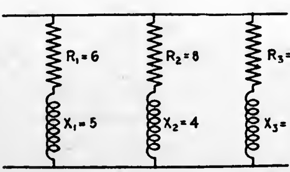


42. Referring to the accompanying sketch:

$E_1 = 100$, $E_2 = 150$, $I = 10$, E_1 and E_2 are 60 degrees out of phase.

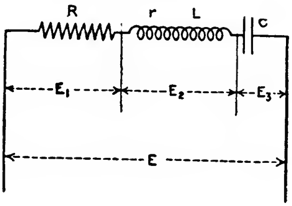
Find: (*a*) E, (*b*) Z, (*c*) R, (*d*) X, (*e*) power factor, (*f*) power in the circuit.

43. Find the impedance, the resistance, the reactance and the power factor of the following circuit:



44. Find the impedance, the resistance and the reactance of the circuit in Problem 43 when X_1 is a capacitance equivalent to 5 ohms.

45. Referring to the accompanying figure:



$$E = 220 \text{ volts}, \; R = 40 \text{ ohms}, \; r = 0.5 \text{ ohm}, \; L = 0.1 \text{ henry},$$
$$C = 0.00012 \text{ farad}, \; f = 60 \text{ cycles}.$$

Find: (*a*) the current, (*b*) E_1, (*c*) E_2, (*d*) E_3, (*e*) power in the circuit, (*f*) power factor.

Draw vector diagram representing the current and the voltages in the circuit.

46. A voltmeter, the resistance of which is 3,000 ohms (reactance negligible), is connected in series with a condenser between 250-volt, 60-cycle alternating-current mains. The indication of the meter is 150. Find the capacitance of the condenser, and the voltage between its terminals.

47. A current of 8 amperes flows in an impedance connected between 240-volt, 60-cycle alternating-current mains. The power absorbed in the impedance is 1280 watts. Find: (*a*) the power factor of the circuit, (*b*) the resistance of the circuit, (*c*) the reactance of the circuit, (*d*) the inductance of the circuit. Draw vector diagram of the voltages and the current.

48. When an impedance (resistance and inductance in series), the power factor of which is 0.707, is connected to 60-cycle, 250-volt alternating-current mains, 25 amperes flow in the circuit. Find: (*a*) the capacitance required to be connected in the circuit to cause voltage resonance, (*b*) the current in the resonant circuit, (*c*) the voltage across the resistance, (*d*) the voltage across the inductance, (*e*) the voltage across the capacitance.

49. The impedance in Problem 48 is connected to 60-cycle, 250-volt alternating-current mains in parallel with a condenser. The resistance of the condenser circuit is negligible. Find the capacitance when the power factor of the circuit is: (*a*) unity, (*b*) 0.90 lagging, (*c*) 0.90 leading.

50. Same as Problem 49 except the resistance of the condenser circuit is 5 ohms.

51. Show that when the applied voltage and the resistance of a series circuit are constant, and the reactance of the circuit varies from zero to infinity, the locus of the current vector is a semi-circle, the diameter of which makes zero angle with the vector of applied electromotive force.

52. Show that when the applied voltage and the reactance of a series circuit are constant, and the resistance of the circuit varies from zero to infinity, the locus of the current vector is a semi-circle, the diameter of which makes an angle of 90 degrees with the vector of applied electromotive force.

CHAPTER II

MAGNETISM AND MAGNETIC INDUCTION

1\. **Magnetism.** — A body which has the power to attract a piece of iron is termed a magnet, and the property by virtue of which attraction takes place is termed magnetism. The ultimate nature of magnetism, like that of electricity, has never been determined, but electricity and magnetism are intimately associated.

2\. **Magnetic field.** — The space in and around a magnet is a magnetic field. A magnetic field is represented graphically by means of lines (lines of magnetic force or magnetic induction) which pass from the north pole to the south pole of a magnet and form complete loops or circuits. Fig. 14. The total number of lines of magnetic induction leaving a north pole and entering a south pole is a magnetic flux.

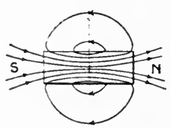


Fig. 14.

3\. **Production of a magnetic field.** — The only known means for the production of a magnetic field is the electric current (electricity in motion). It has been proved experimentally that the space surrounding a current-carrying conductor is a magnetic field, that the lines of magnetic induction are concentric circles,* and that their direction is clockwise or counter-clockwise as the current in the conductor flows away from or toward the observer. The direction of a magnetic flux is assumed to be that indicated by the north-seeking pole of the magnetic needle. The magnetic field around a current-carrying wire is represented, in direction and in intensity, in Fig. 15.

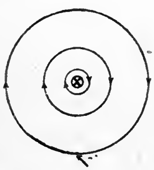


Fig. 15.

4\. **The solenoid.**—If a current-carrying conductor is wound into a helix, the lines of magnetic induction are no longer concentric circles but pass through the helix as indicated in Fig. 16. If the helix is long in

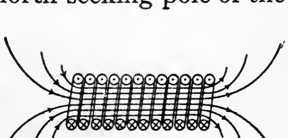


Fig. 16.

* When the conductor is isolated and uninfluenced by other magnetic fields.

comparison to its diameter, the magnetic flux is uniformly distributed over its cross-sectional area except near the ends where the lines begin to diverge.

5. The electromagnet. — The magnetic effect of a current-carrying wire is concentrated by winding it into a helix, and greatly increased if the helix is wound on an iron core. A current-carrying coil wound on an iron core is an electromagnet, and is used for the production of the intense magnetic fields required for modern electrical apparatus. When once magnetized, iron retains a part of the magnetic effect of the current-carrying coil. This permanent effect is termed residual magnetism, and varies with the quality of the iron.

The polarity of an electromagnet is easily determined from the direction of the current in its exciting coil. If the direction of current around the coil is that of the rotating motion of a right-handed screw, the direction of magnetic flux is that of the longitudinal movement of the screw.

6. Magnetomotive force. — Magnetomotive force is that property by virtue of which a magnetic flux is established or maintained.

7. Reluctance. — The opposition offered to the establishment or to the maintenance of a magnetic flux is termed the reluctance of the magnetic circuit. The reluctance of a magnetic circuit is directly proportional to its length and inversely proportional to its cross-sectional area.

8. Permeability. — Some materials, notably iron and many of its alloys, offer less opposition to the establishment or the maintenance of a magnetic flux than do others. The ratio of the flux established or maintained in a given length and cross section of a material by unit magnetomotive force, to the flux established or maintained by unit magnetomotive force in the same length and cross section of vacuum is termed the permeability of the material.

9. Magnetic units. — The magnetic units are:

(*a*) *Unit pole.* — Unit magnet pole is one which repels with a force of one dyne a similar and equal magnet pole placed at a distance of one centimeter in air. (Symbol m.)

(*b*) *Field intensity.* — A magnetic field has unit intensity (one line per square centimeter of cross-sectional area) when it reacts on a unit pole, placed in the field, with a force of one dyne. (Symbol H.)

(*c*) *Flux density.* — By flux density is meant the number of lines of magnetic force or induction per unit of cross-sectional area of a magnetized material. (Symbol B.)

Note. — When a magnetic flux is propagated in air, or other non-magnetic material, the flux density is equal to the field intensity.

(*d*) *Magnetic flux.* — The total flux in a magnetic circuit is equal to the product of the average flux density and the cross-sectional area of the circuit. The unit is the maxwell. (Symbol ϕ.)

(*e*) *Reluctance.* — Unit reluctance (the œrsted) is that opposition offered by a cubic centimeter of vacuum,* each face of which is one centimeter square, to the passage of a magnetic flux between its parallel faces. (Symbol $\mathfrak{R}$.)

(*f*) *Magnetomotive force.* — The magnetomotive force of a circuit is equal to the work in ergs done when a unit magnet pole is moved around the circuit against the force, and is measured in gilberts. (Symbol $\mathfrak{F}$.) Unit difference of potential exists between two points when one erg is required to transfer a unit pole from one point to the other.

(*g*) *Magnetizing force.* — Magnetizing force is the ratio of the magnetomotive force of the circuit to the length of the circuit. Magnetizing force is numerically equal to field intensity, and is indicated by the same symbol.

(*h*) *Permeability.* — The permeability of a material is the ratio of the flux density and the field intensity, or the magnetizing force. (Symbol μ.)

10. Law of the magnetic circuit. — *The flux in any magnetic circuit is directly proportional to the magnetomotive force and inversely proportional to the reluctance of the circuit.*

$$\phi = \frac{\mathfrak{F}}{\mathfrak{R}}, \tag{1}$$

when ϕ = the magnetic flux in maxwells,
$\mathfrak{F}$ = the magnetomotive force in gilberts,
$\mathfrak{R}$ = the reluctance in œrsteds.

11. Faraday's Law. — *An electromotive force is induced in any closed circuit when the magnetic flux linking with the circuit changes in value, and the magnitude of the electromotive force induced is propor-*

* Air has practically the same specific reluctance as vacuum.

tional to the rate at which the magnetic flux linking with the circuit changes.

12. Lenz' Law. — *The direction of an induced current is such that its reaction opposes any change in the value of the magnetic flux linking with the closed circuit in which the induced current flows.*

Lenz' Law is simply a special application of the general principle of mechanics that action and reaction are equal and opposite.

13. Induced currents. — From Faraday's Law it is evident that an electric conductor forming part of a closed circuit and lying partly or wholly within a magnetic field has an electromotive force induced in it when: (*a*) the relative position of the conductor and the magnetic field changes, (*b*) the intensity of the magnetic field varies.

(*a*) *Induction by motion.* — The following propositions are self-evident or are easily demonstrated by experiment:

(1) The relative position of the conductor and the magnetic field is changed by the movement of *either* the conductor or the field.

(2) The induction of an electromotive force occurs only during the period of motion.

(3) The induced electromotive force is proportional to the number of lines of magnetic induction across which the conductor cuts per unit of time.

(4) The direction of the induced electromotive force is reversed when the direction of motion is reversed.

Let Fig. 17 represent a cross section of a magnetic field, the flux in which flows toward the observer, and a copper conductor *A* which may be moved to the right or to the left, the circuit being completed through the rails *B*, *C* and *D*. When the conductor *A* is moved to the right an electromotive force is induced, the magnitude of which is proportional to the rate of motion, and the direction of the current in the circuit is that indicated by the arrows. If the wire is moved to the left at the same speed, the same value of current flows in the wire but its direction of flow is reversed.

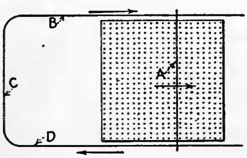


FIG. 17.

In the above case the induced electromotive force is proportional to the speed of the conductor, the length of the conductor and the density of the magnetic field being constant. In any construction,

the number of lines of magnetic induction cut per unit of time depends on:

(1′) The length of conductor lying within the magnetic field.

(2′) The rate at which the conductor moves across the magnetic field.

(3′) The density of the magnetic field (the number of lines of magnetic induction per unit of cross-sectional area).

The length of the conductor, the speed at which the conductor or the magnetic field moves, and the flux density may be expressed in any desired units, provided the length of the conductor and its speed are in the same unit, and the unit of area is the square of this linear unit. The unit on which the electromotive force of a circuit is based is that difference of potential found to exist when a conductor one centimeter long moves* at a uniform speed of one centimeter per second across a uniform magnetic field having a flux density of one maxwell (one line of magnetic force per square centimeter). This unit of electromotive force is inconveniently small and the commercial unit (the volt) is taken as 100,000,000 times the fundamental unit (the abvolt). Then

$$e = l \times V \times B \times 10^{-8}, \tag{2}$$

when e = the electromotive force in volts induced in the conductor,
l = the length of the conductor,
V = the relative speed (per second) of the conductor and the magnetic field,
B = the density of the magnetic field (maxwells per unit of cross-sectional area),

i.e., an electromotive force of one volt is induced in a closed circuit when the magnetic flux linking with the circuit changes at the rate of 100,000,000 maxwells per second.

As pointed out above, a reversal of the direction of motion causes a reversal of the direction of current flow. There is, then, a definite relation between the direction of motion, the direction of induced electromotive force and the direction of magnetic flux. By refer-

* The direction of motion and the axis of the conductor are here assumed to be at right angles to each other and to the direction of the lines of magnetic force. When this condition does not exist, the effective length of the conductor is equal to l times the sine of the acute angle between the axis of the conductor and that of the magnetic field, and the effective velocity is equal to V times the sine of the acute angle between the direction of motion and the axis of the magnetic field.

ence to Fig. 17 it will be observed that any one of these three quantities is at right angles to each of the other two. A simple rule for determining the relations of these quantities was deduced by Dr. J. A. Fleming and is known as Fleming's finger rule. If the thumb, index and middle fingers of the *right* hand are placed as indicated in Fig. 18, the thumb indicates the direction of motion across the magnetic field, the index finger the direction of flux, and the middle finger the direction of induced electromotive force.

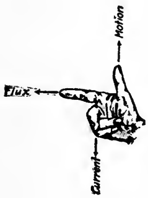


FIG. 18.

If a ring or closed loop is rotated in a magnetic field, one side of the loop cuts the flux in one direction and the opposite side cuts it in the other direction. Hence, a current flows around the loop but its direction is periodically reversed, and the induced electromotive force passes through successive values from zero to maximum and then from maximum to zero.

Let Fig. 19 represent a loop of wire and a uniform magnetic field, the loop being rotated at a constant angular velocity in the direction indicated by the arrow. Starting from the position shown, the electromotive force increases to maximum, decreases to zero, increases to maximum in the opposite direction, and again decreases to zero for each revolution of the loop. The flux passing through (linking with) the loop decreases from maximum to zero, or increases from zero to maximum, in one quarter of a revolution, and the electromotive force induced in the loop is proportional, at any instant, to the sine of the angle through which the loop has rotated, *i.e.*, the electromotive force is harmonic as indicated in the rectangular representation where the abscissas are angular distances (degrees) passed through by the rotating loop, and the ordinates are the instantaneous values of electromotive force induced in the loop. Since the electromotive force induced in one

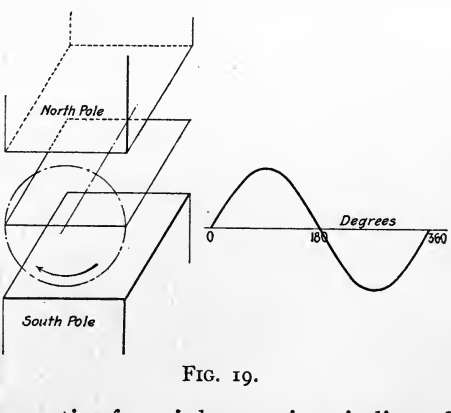


FIG. 19.

side of the loop tends to produce a current in the same direction around the loop as the electromotive force induced in the other side of the loop, the electromotive force of the circuit is twice as great as if only one side of the loop were in the magnetic field.

(*b*) *Induction by varying the flux density.* — If two coils of insulated wire are wound on an iron core as indicated in Fig. 20, and coil A is supplied with a current of constant value (continuous current) no current will flow in coil B. If the current in coil A is made variable, the following effects may be noted:

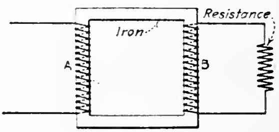


FIG. 20.

(1) A current flows in coil B when the current in coil A either increases or decreases.

(2) A current flows in coil B only when the value of the current in coil A is changing.

(3) The electromotive force induced between the terminals of coil B is proportional to the rate at which the flux in the iron ring changes. Within certain limits, the change in the flux is approximately proportional to the change in the current flowing in coil A.

(4) The direction of current in coil B is reversed by reversing the direction of current in coil A.

(5) The direction of current in coil B is always such that it opposes the change in flux which produced it.

From the above considerations it is evident that electrical energy may be transferred from one circuit to another, provided the current in the supply circuit is of varying value, *i.e.*, either pulsating or alternating.

14. Reaction between a magnetic field and a current-carrying conductor. — Let

f = the force required to move a conductor across a uniform magnetic field at a velocity of V centimeters per second,

B = the density of the magnetic field in maxwells,

l = the length of the conductor in centimeters lying in the magnetic field and perpendicular to both the direction of motion and the direction of the magnetic flux,

i = the current in amperes flowing in the conductor,

e = the electromotive force in volts induced in the conductor ($= e \times 10^8$ c.g.s. units),

ds = the distance through which the conductor moves,
dt = the time required for the conductor to move through the distance ds.

Then
$$f\,(ds) = ei\,(dt) \text{ joules} \tag{3}$$
$$= ei\,(dt) \times 10^7 \text{ ergs} \tag{4}$$
$$= \frac{lBVi\,(dt)}{10} \text{ ergs} \tag{5}$$
and
$$f\frac{ds}{dt} = \frac{lBVi}{10}. \tag{6}$$
But
$$\frac{ds}{dt} = V. \tag{7}$$
Therefore,
$$f = \frac{lBi}{10} \text{ dynes} \tag{8}$$
$$= \frac{lBi}{9810} \text{ grams} \tag{9}$$
$$= \frac{22\,lBi}{981 \times 10^4} \text{ pounds.} \tag{10}$$

From the above considerations it is evident that:

(*a*) A current-carrying conductor lying within a magnetic field tends to move across the field, the direction of motion being at right angles to the axis of the conductor and to the direction of the flux.

(*b*) The direction in which the conductor tends to move is reversed by reversing *either* the direction of the magnetic flux or the direction of the current flowing in the conductor.

(*c*) The force tending to move the conductor across the magnetic field is proportional to the product of the current flowing in the conductor, the density of the magnetic field, and the length of conductor lying in the magnetic field.

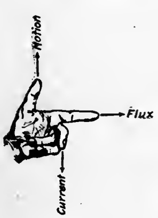


FIG. 21.

The relations between the direction of the magnetic flux, the direction of the current and the direction of the resultant force on the current-carrying conductor are shown in Fig. 21 by the index finger, the middle finger and the thumb of the *left* hand.

15. Magnetic flux and field intensity due to unit pole.—From the definitions of unit pole and field intensity given above, it

follows that the field intensity due to unit pole is unity at any point on the surface of a sphere one centimeter in radius, and of which the unit pole is the center. But the area of such a sphere is 4π square centimeters. Therefore, the total flux leaving or entering a unit pole equals 4π maxwells, and the field intensity due to an isolated magnet pole is inversely proportional to the square of the distance x of a surface from the pole.

$$H = \frac{m}{x^2} \tag{11}$$

16. Field intensity produced by an electric current. — As stated in Section 3, the space surrounding a current-carrying conductor is a magnetic field. The intensity of the magnetic field at the center of a circular loop, and that surrounding a long straight wire, are of particular interest.

(*a*) *Field intensity at the center of a loop of r centimeters radius, in which flows a current of i amperes.* — From equation (11), the intensity of the field set up by a magnet pole m placed at the center of the loop is

$$H' = \frac{m}{r^2}; \tag{12}$$

and the force with which the current-carrying conductor and the magnetic field react is, from equation (8),

$$f = \frac{m}{r^2} \times \frac{2\pi r i}{10} \tag{13}$$

$$= \frac{2\pi i m}{10 r} \text{ dynes.} \tag{14}$$

If H'' is the field intensity at the center of the loop due to the current in the loop,

$$f = mH'', \tag{15}$$

$$mH'' = \frac{2\pi i m}{10 r}, \tag{16}$$

and

$$H'' = \frac{0.2\pi i}{r}. \tag{17}$$

(*b*) *Field intensity x centimeters distant from the axis of a long straight wire in which flows a current of i amperes.* — Let m magnet poles per centimeter length of the wire be uniformly distributed along a line parallel to and x centimeters distant from the axis of

the current-carrying wire. The 4π lines which emanate from each unit pole radiate like the spokes of a wheel, and are in a plane perpendicular to the axis of the current-carrying wire. Therefore, the intensity of the field at the wire is

$$H' = \frac{4\pi m}{2\pi x} \tag{18}$$

$$= \frac{2m}{10x} \tag{19}$$

and the force exerted on each centimeter length of the conductor is

$$f = \frac{2mi}{10x} \text{ dynes.} \tag{20}$$

If H'' is the field intensity at the poles due to the current-carrying wire,

$$f = mH'' \text{ dynes,} \tag{21}$$

$$mH'' = \frac{0.2\,mi}{x} \tag{22}$$

and

$$H'' = \frac{0.2\,i}{x}. \tag{23}$$

17. Relation of the gilbert to the ampere-turn. — Let a current of i amperes flow in a coil of n turns. When the coil is rotated about a unit pole, or a unit pole is moved around a path threading the coil, the work done is

$$W = \frac{\phi ni}{10} \tag{24}$$

$$= \frac{4\pi ni}{10} \text{ ergs.} \tag{25}$$

The magnetomotive force due to the current-carrying coil is, by definition,

$$f = \frac{4\pi ni}{10} \tag{26}$$

$$= 1.257\, ni \text{ gilberts.} \tag{27}$$

The quantity ni is termed "ampere-turns," and the ampere-turn is commonly used as a unit of magnetomotive force.

18. The C. G. S. unit of electric current. — The C. G. S. unit of current is that current which, when flowing in a wire in and at right angles to a magnetic field having a density of one maxwell

per square centimeter, causes a mechanical force of one dyne to be exerted on each centimeter length of the wire. From equation (8), the C. G. S. unit is equal to ten amperes.

19. Force of magnetic traction. — Lines of magnetic force act like rubber threads under tension and tend to shorten, and the tractive force or pull between two surfaces in contact is proportional to the square of the flux density of the magnetic field within which the surfaces lie.

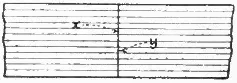


FIG. 22.

Let the flux passing from surface X (Fig. 22) be divided into equal parts, and enter poles m' and m'' on surface Y. Pole m' lies in the magnetic field of pole m'' and pole m'' lies in the magnetic field of pole m', and there is a reaction between each pole and one-half the total flux. By definition,

$$m' = \frac{\phi}{2 \times 4\pi}. \tag{28}$$

Therefore,
$$f' = \frac{\phi}{2 \times 4\pi} \times \frac{B}{2} \tag{29}$$

$$= \frac{\phi B}{16\pi} \text{ dynes.} \tag{30}$$

Similarly,
$$f'' = \frac{\phi B}{16\pi} \text{ dynes.} \tag{31}$$

Therefore,
$$f = f' + f'' \tag{32}$$

$$= \frac{\phi B}{8\pi} \text{ dynes.} \tag{33}$$

But in a uniform field
$$\phi = AB \tag{34}$$

and
$$f = \frac{AB^2}{8\pi} \text{ dynes} \tag{35}$$

$$= \frac{AB^2}{24{,}655} \text{ grams} \tag{36}$$

$$= \frac{AB^2}{11{,}183{,}000} \text{ pounds,} \tag{37}$$

when A = the area of the surface in square centimeters,
B = the flux density in maxwells per square centimeter.

$$f = \frac{AB^2}{72{,}134{,}000} \text{ pounds,} \tag{38}$$

when A = the area of the surface in square inches,
B = the flux density in lines per square inch.

TABLE II

PERMEABILITY OF CAST IRON, CAST STEEL AND SHEET STEEL

H	Cast iron		Cast steel		Sheet steel	
	B	μ	B	μ	B	μ
10	5000	500	9,800	980	12,500	1250
20	6600	330	13,400	670	16,700	835
30	7200	240	14,400	460	18,000	600
40	7750	194	15,200	380	19,000	480
50	8150	163	15,700	314	19,800	396
60	8500	142	16,150	269	20,500	342
70	8800	126	16,550	236	21,100	301
80	9075	114	16,925	211	21,600	270
90	9325	104	17,275	192	22,000	244
100	9600	96	17,600	176	22,300	223

The quality of iron varies greatly, and the above are to be considered simply as representative values.

20. Counter-electromotive force. — When the reaction between a current-carrying conductor and a magnetic field causes the conductor to move across the field, an electromotive force which opposes the flow of the current is induced in the conductor, and the current flowing in the conductor is proportional to the geometrical difference of the applied and the induced (counter) electromotive forces.

21. Magnetic leakage. — Unlike the electric current, a magnetic flux cannot be confined to a definite path, except under con-

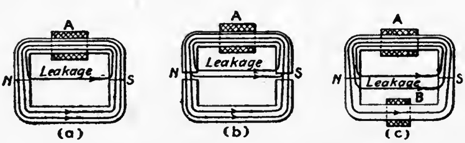


FIG. 23. Magnetic Leakage.

ditions which are not applicable to commercial electrical apparatus. The total flux in any magnetic circuit may be regarded as flowing in two or more parallel branches, the flux in each branch being inversely proportional to the reluctance of that branch. In Fig. 23 that part of the flux which flows through the air is termed the leakage flux. The ratio of the total flux set up by a magneto-

motive force to the useful flux is the leakage coefficient. The leakage coefficient of a dynamo depends on its size and design, and varies from 1.1 to 1.5.

In any magnetic circuit containing iron, the leakage flux is increased by:

(*a*) Increasing the total flux in the circuit. Since the permeability of iron is a function of the flux density, the reluctance of the path

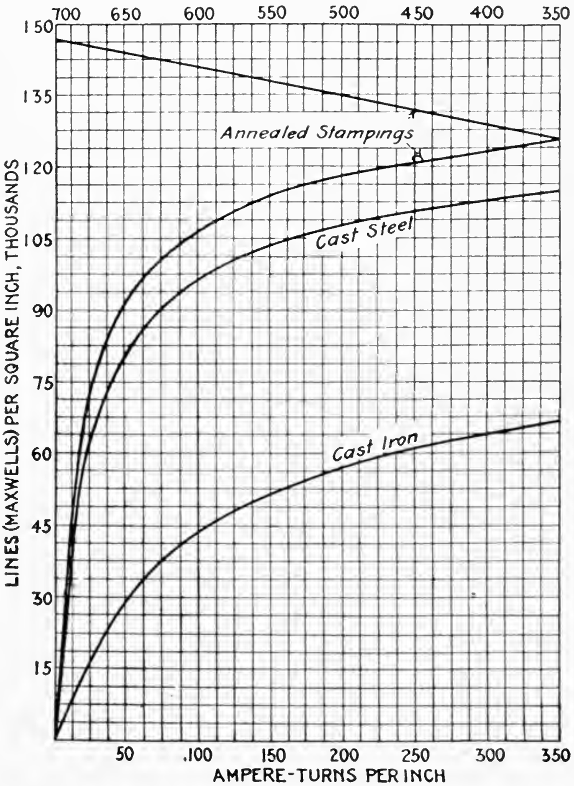


FIG. 24. Magnetization Curves.

through the iron increases as the total flux set up by coil *A* increases, the reluctance of the path through the air remains constant, and a larger part of the total flux is diverted through the air. Fig. 23a.

(*b*) Increasing the length of the air gap between the metallic parts of the magnetic circuit. Increasing the air gap increases the reluctance of one branch of the magnetic circuit without affecting that

of the other, and diverts a larger proportion of the flux through the leakage path. Fig. 23b.

(*c*) An auxiliary coil which sets up an opposing magnetomotive force. The counter-magnetomotive force set up by coil *B* acts as an additional opposition to the flow of the magnetic flux through that part of the circuit, and increases the relative quantity of flux diverted through the air. Fig. 23c.

22. Magnetization curves. — Typical magnetization curves are shown in Fig. 24 for cast iron, cast steel and annealed stampings.

It will be observed that for low flux densities the magnetization curve is an approximately straight line, that it bends sharply to the right when the magnetizing force is increased beyond a certain definite value, and again becomes an approximately straight line if the magnetizing force is sufficiently increased. From these curves it is evident that the reluctance of a magnetic circuit in iron is not constant, but increases with increasing flux density.

23. The elementary dynamo. — A simple generator producing an alternating electromotive force may be constructed by rotating

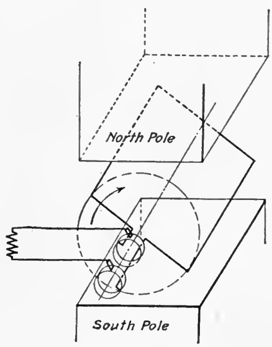


FIG. 25.

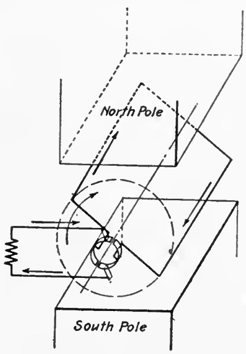


FIG. 26.

an open loop of copper wire or other conducting material between the poles of a magnet, and providing means for connecting the terminals of the loop to an outside circuit. This arrangement is illustrated in Fig. 25 where the terminals of the loop are connected to insulated copper rings mounted on the shaft. Stationary conductors (brushes) press against these rings and through them connection is made to an outside circuit.

Unidirectional currents are obtained in the outside circuit by the arrangement shown in Fig. 26. Instead of connecting the terminals

of the loop to separate rings, connection is made to a single ring which is divided, the segments being insulated from each other. By placing the brush mid-way of the pole face, it passes from one segment to the other when the loop is in the position where it is generating zero electromotive force, and the current in the outside circuit always flows in the direction indicated by the arrows. The magnitude of the electromotive force varies with the position of the loop, the values during one revolution of the loop being plotted in Fig. 27. If, instead of a single loop, a number of symmetrically spaced loops are connected in series the electromotive force of the system is the sum of the instantaneous electromotive forces induced in the individual loops, and the potential difference between the stationary brushes is approximately constant. Fig. 28 shows the electromotive forces induced in each of three symmetrically spaced loops, and the resultant electromotive force due to the combined action of the three loops. The greater the number of loops in the series, the more nearly the resultant approximates a straight line.

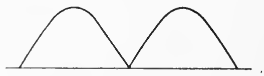


FIG. 27.

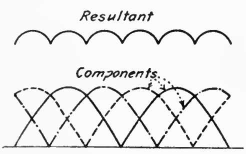


FIG. 28.

Structurally the electric generator and the electric motor are identical, and the same machine may be used for the production of electrical energy or for its conversion into mechanical energy.

CHAPTER II — PROBLEMS

1. Find the magnetomotive force set up by a winding of 100 turns when a current of 5 amperes flows in the coil.

2. Determine the permeability of the iron around which the coil in Problem 1 is wound if the length of the coil is 10 inches, and the flux density in the iron is 10,000 lines per square centimeter.

3. The reluctance of a magnetic circuit is 10 oersteds. Find the flux set up by a magnetomotive force of 150 gilberts.

4. Find the flux density when the force (pull) between an electromagnet and its armature is 60 pounds per square inch.

5. A copper wire is moved across a uniform magnetic field at the rate of 500 feet per minute. Find the electromotive force, per unit length, induced in the wire if the density of the magnetic field is 75,000 lines per square inch.

6. A closed loop rotates in a uniform magnetic field at the rate of 200 r.p.m. and the maximum flux linking with the loop is 1,000,000 lines. Find: (*a*) the

effective value of the electromotive force induced in the loop, (*b*) the current flowing around the loop if the resistance of the loop is 0.01 ohm. Neglect any inductance.

7. Find the force acting on a current-carrying wire 10 inches long, lying wholly within a uniform magnetic field ($B = 45{,}000$ lines per square inch), and carrying a continuous current of 1000 amperes.

8. Find the average force acting on the wire in Problem 7 when an alternating current of 1000 amperes (effective) flows in the wire.

9. A series magnetic circuit is composed of: 6 inches of cast iron, density 35,000; 8 inches of cast steel, density 75,000; $\frac{1}{4}$-inch air, density 50,000. Find the ampere-turns required in the winding. Use curves in Fig. 24 for the iron and the steel.

10. A magnetic circuit 10 inches in length is made up of sheet-iron punchings (laminations), and excited by 1000 turns. Find the exciting current when the average flux density is 40,000, and 90% of the cross-sectional area is iron.

11. A conductor 15 inches long moves across a magnetic field having a uniform density of 48,000 lines per square inch. Find the electromotive force induced in the conductor if the conductor moves at the rate of 4500 feet per minute.

12. If the magnetic field in Problem 11 consists of poles 9 inches long and separated by a distance of 3 inches, find the average electromotive force induced in the conductor.

13. A continuous-current armature is 18 inches long and 30 inches in diameter. The electromotive force induced in each conductor as it passes under an interpole 15 inches long is 1 volt when the armature rotates at a speed of 600 r.p.m. Find the flux density in the air gap under the interpole.

14. Show that the field intensity in a long solenoid is

$$H = \frac{0.4\,\pi Ni}{l}$$

15. Show that the ampere-turns in the exciting coil of an electromagnet are

$$NI = \frac{0.8\,Bl}{\mu}$$

when $B =$ the maxwells per square centimeter,
$l =$ the length of the magnetic circuit in centimeters.

16. Show that the ampere-turns in the exciting coil of an electromagnet are

$$NI = \frac{0.3133\,Bl}{\mu}$$

when $B =$ the maxwells per square inch,
$l =$ the length of the magnetic circuit in inches.

17. An electromagnet lifts 5 tons (10,000 pounds). The mean length of the magnetic circuit = 60 inches; cross-sectional area = 20 square inches; $\mu =$ 1000. Find: (*a*) the total flux, (*b*) the flux density, (*c*) the ampere-turns in

the exciting coil, (*d*) the magnetomotive force (gilberts), (*e*) the field intensity, (*f*) the reluctance of the circuit.

18. A hollow wrought-iron cylinder (outside diameter = 12 inches, inside diameter = 6 inches) 6 inches long forms part of a magnetic circuit, the flux in which is parallel to the axis of the cylinder. Average density = 61,000 lines per square inch. $\mu = 20$. Find: (*a*) ampere-turns required, (*b*) flux density in the iron, (*c*) field intensity, (*d*) reluctance of the cylinder.

19. A point lies on the axis of a circular loop of wire in which flows a current of i amperes. The radius of the loop is r centimeters, and the point is x centimeters distant from the plane of the loop. Show that the field intensity at the point is

$$H = \frac{0.2\,\pi r^2 i}{(r^2 + x^2)^{\frac{3}{2}}}.$$

20. The currents in two long parallel wires are equal but flow in opposite directions. Show that the field intensity at any point on a line joining the centers of the wires is

$$H = \frac{0.2\,i}{x} + \frac{0.2\,i}{D - x}$$

when D = the distance in centimeters between the axes of the wires,
x = the distance in centimeters of the point from the axis of one wire.

21. A long solenoid has n turns per centimeter of its total length. Show that the field intensity at any point inside the solenoid, except near the ends, is

$$H = 0.4\,\pi n i$$

when i amperes flow in the windings.

CHAPTER III

PRACTICAL CONSTRUCTION OF THE DYNAMO

1. **Parts.** — The principal parts of a commercial dynamo are: (*a*) frame, (*b*) field poles, (*c*) field windings, (*d*) armature core, (*e*) armature winding, (*f*) yoke, (*g*) commutator, (*h*) collector rings, (*i*) brushes, (*j*) brush holders.

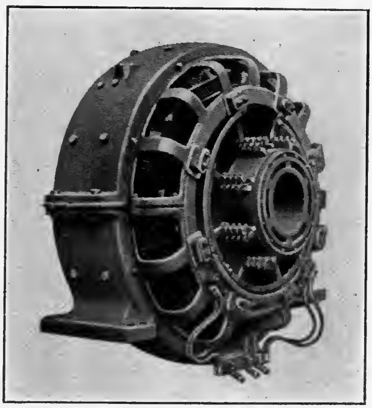


Fig. 29. Typical Continuous Current Dynamo. General Electric Co.

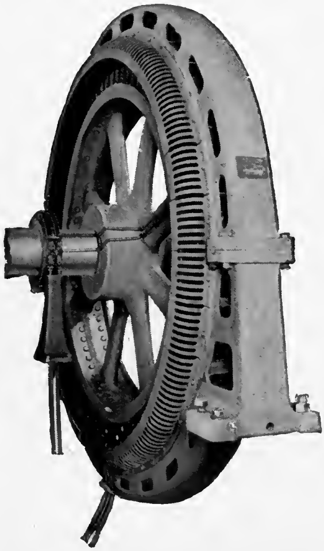


Fig. 30. Alternating-current Dynamo. General Electric Co.

(*a*) *Frame.* — The frame of a dynamo is the supporting structure, and includes the base and the supports for the bearings in which the armature shaft rests, as well as the yoke. It is usually made of cast iron or cast steel.

(*b*) *Field poles.* — The field core is a body of iron around which the field winding is placed. Its function is to reduce the reluctance of the magnetic circuit (increase the flux for a given field winding and current), and to supply a mechanical support for the field winding. It may be made of cast iron, of cast steel, or be built up of stampings bolted or riveted together. Fig. 32 shows a typical form

of field pole. The part next the armature is usually spread out to give a larger cross-sectional area as it is desirable to have a smaller flux density (flux per unit of cross-sectional area) in the air gap than in the body of the pole. This enlarged part is termed the pole shoe, and is sometimes made as a separate piece and bolted to the core.

Fig. 31. Frame and Field Structure of Triumph Continuous Current Dynamo.

The field poles and the yoke of small machines are sometimes cast in a single piece. When made separately, they are bolted together, the joint being made as close as possible in order to reduce the reluctance of the magnetic circuit.

(*c*) *Field windings.* — The field winding of a dynamo is that part which produces the magnetic field or flux across which the electrical conductors move, or which moves relatively to the conductors. It consists (Fig. 33) of a coil of insulated copper wire, the exciting current being supplied by the dynamo itself (self-excited) or from some outside source (separately excited).

Fig. 32. Typical Field Core for Continuous Current Dynamos. Western Electric Co.

(*d*) *Armature core.* — The armature core is that part of the dynamo over or around which the armature conductors are placed. It serves as a mechanical support for these conductors and as a path through which the magnetic circuit is completed. Armature cores are of two classes: (1) ring, (2) drum.

(1) *Ring cores.* — The armature cores of early dynamos were iron rings through which the conductors were threaded, as shown in Fig. 34a. Because of mechanical and electrical deficiencies, the ring construction is seldom used in present day machines.

(2) *Drum cores.* — The drum core consists of an iron cylinder (Fig. 34b) on the surface of which the armature conductors are placed.

Armature cores are built up of thin stampings (laminations) of

soft iron or steel. The purpose of this construction is to reduce the eddy currents which flow in the body of the core and cause it to heat. Laminations vary in thickness from 0.01 inch to 0.04 inch.

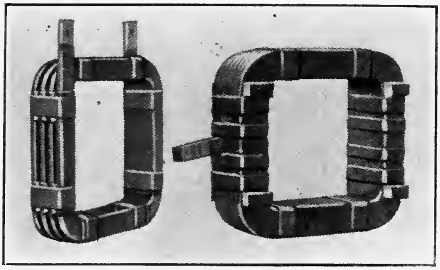


Fig. 33. Typical Field Coils (Westinghouse).

Early dynamos were built with smooth cores, the armature conductors on their surface being held in place by binding wires. In later types, the armature conductors are placed in slots as shown in Fig. 37, and retained by means of wooden or fiber wedges. Different slot forms are used by different manufacturers, from an entirely closed slot, through which the armature conductors are threaded, to the rectangular (open) slot.

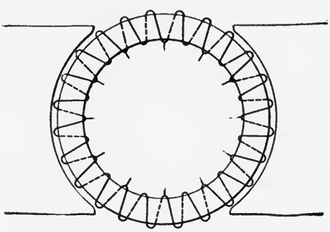


Fig. 34a. Ring Armature.

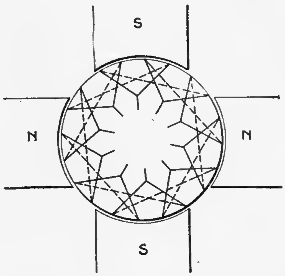
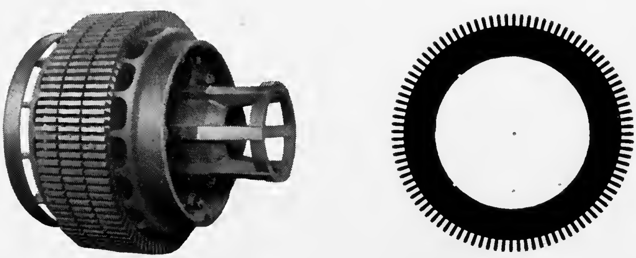


Fig. 34b. Drum Armature.



(a) Core. (b) Lamination.

Fig. 35. Armature Core and Lamination for Crocker-Wheeler Continuous Current Dynamo.

(*e*) *Armature windings.* — The armature winding consists of electrical conductors which move relatively to the magnetic field.

For machines of the usual commercial voltages, the winding consists of several hundred insulated copper wires or bars, divided into groups, the wires of each group being connected in series and the groups in parallel. The number of parallel groups is never greater than the number of poles on the dynamo (simplex windings), and there may be only two parallel groups on a continuous-current armature, regardless of the number of poles. The armature conductors of an alternator are usually all connected in series (single-phase alternator). A typical armature coil having one turn is shown in Fig. 38.

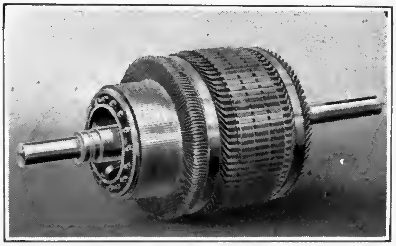


FIG. 36. Complete Armature for Crocker-Wheeler Continuous Current Dynamo.

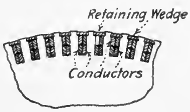


FIG. 37. Section of Slotted Armature Core.

Two or more coil sides are usually placed in one slot, one side of each coil being placed in the bottom of a slot and the other side in the top of a slot under an adjacent pole. This arrangement makes all the coils on an armature similar, and gives the finished armature a symmetrical appearance.

Continuous-current armature windings. — The principal types of armature windings used on continuous-current dynamos are: (1) lap, (2) wave.

(1) *Lap or parallel windings.* — The lap or parallel winding is used when the armature conductors are to be divided into as many groups as there are poles on the machine. Starting at any commutator segment, the armature conductor passes under one pole, across the rear end of the core, and back under an adjacent pole (which is of an opposite polarity), then to the commutator segment adjacent to the one from which the coil started. The second coil starts from the commutator segment at which the first ends, the third where the second ends, etc., until the winding closes on itself

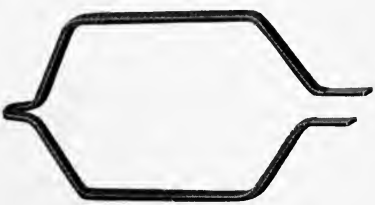


FIG. 38. Typical Armature Coil. Triumph Electric Co.

at the commutator segment from which the first coil started. Fig. 39.

Dynamos having lap-wound armatures are provided with as many brush sets as there are poles.

If, for any reason, such as the wearing of the bearings, the electromotive forces generated in the different parallel branches of a lap-wound armature are unequal, heavy currents circulate in the winding. These currents cannot be prevented but their effects are minimized by connecting, through heavy "equalizer" rings, such points of the winding as are normally of the same potential.

(2) *Wave or series windings.* — In the wave or series winding, the armature conductors are connected into two parallel groups. Starting at any commutator bar, the conductor passes under one pole, across the rear end of the core, and back under an adjacent pole

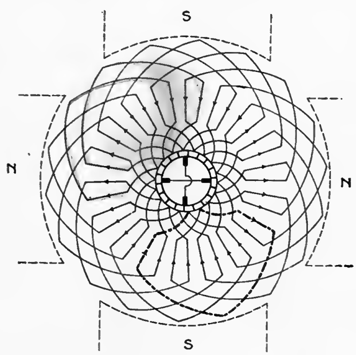


FIG. 39. Elementary (Lap) Armature Winding.

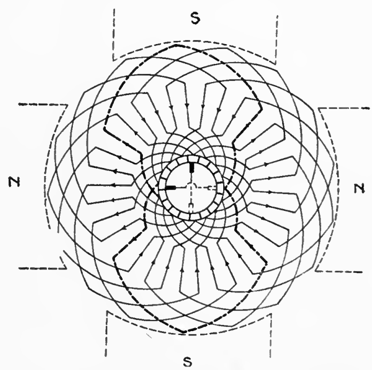


FIG. 40. Elementary (Wave) Armature Winding.

(of opposite polarity) to a commutator bar which is separated from the one from which the coil started by a distance slightly greater or slightly less than $\frac{\text{total number of commutator bars}}{\text{number of pairs of poles}}$. If a dynamo has four poles, the total number of commutator bars required is

$$C = 2\,Y \pm 1, \tag{1}$$

when Y is the number of commutator bars between the terminal connections of any given armature coil. If $Y = 9$ (terminals of first coil connected to commutator bars 1 and 10), the total number of commutator bars must be either 17 or 19. By reference to Fig. 40, it will be seen that the winding closes on itself, as in the lap winding, by making a final connection to the commutator bar from which it started.

Since the wave winding offers only two paths for the current flow, only two brush sets are required and their proper position is shown in Fig. 40. Additional brushes may be used, the total brush equipment acting like two brushes having a total contact area equal to the sum of the areas of the individual brushes. The additional brushes do not affect the voltage to any appreciable extent.

The relative positions of the brushes on a multi-polar wave-wound armature make it especially adapted for street car and other enclosed motors which are accessible from one side only.

Alternating-current armature windings. — The armature winding of an alternator does not differ, essentially, from that used in continuous-current machines, but the winding does not usually form a closed circuit. An elementary winding for alternators is shown in Fig. 41. If the armature rotates, the terminals are connected to insulated copper rings mounted on the shaft; if the armature is stationary, the terminals are connected to insulated blocks from which connection is made to the switchboard.

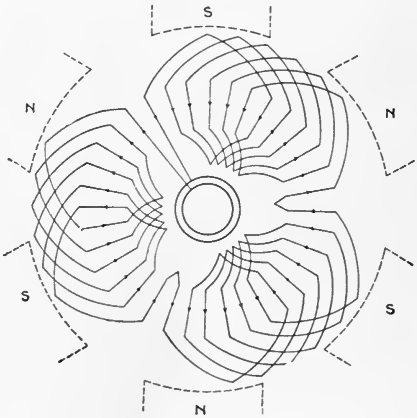


FIG. 41. Elementary Armature Winding for Alternators.

Alternator armature windings in common use are: (1′) chain, (2′) basket.

(1′) *Chain windings.* — In the chain winding one coil side only is placed in a slot. This winding is, therefore, especially adapted to high-voltage machines, the coils being easily insulated. The objection to the chain winding (but this objection is not serious) is that the coils are of different shapes and the individual conductors of different lengths. Fig. 42 shows a simple chain winding.

(2′) *Basket windings.* — In the basket winding two coil sides per slot are used. The coils are, therefore, similar, their form and arrangement being shown in Fig. 43. Basket windings are largely used for low- and medium-voltage alternators, and for induction motors.

In both chain and basket windings all the conductors are connected in series (single-phase windings) so that the current flow is

confined to a single path, *i.e.*, the terminals of one coil are connected to the terminals of adjacent coils.

(*f*) *Yoke.* — The yoke is that part of the frame connecting the

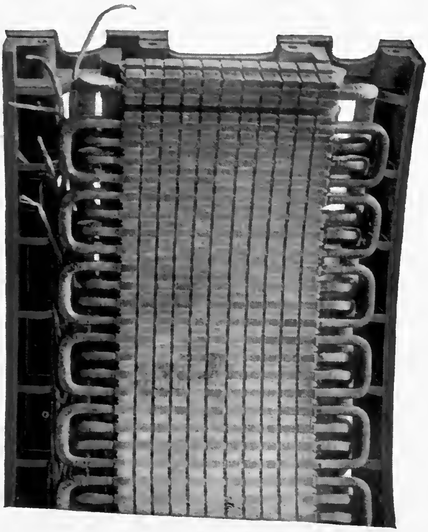


FIG. 42. Chain Armature Winding. General Electric Co.

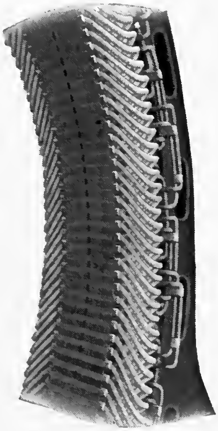


FIG. 43. Basket Armature Winding. Triumph Electric Co.

pole pieces, and serves the double purpose of a mechanical support for the field poles and their windings, and of a path for the flux to pass from the south to the north pole. Fig. 44 shows cross sections of several common forms. The ridges on *c* are added to give additional mechanical strength.

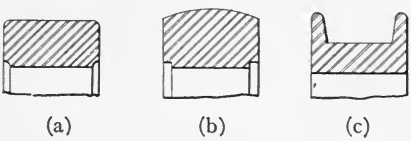


FIG. 44. Cross Sections of Typical Yokes.

(*g*) *Commutator.* — The commutator is a cylindrical structure made up of copper bars insulated from each other and from the supporting structure. It is fastened to the armature shaft with which it rotates. Its purpose is to rectify (make unidirectional) the alternating current which flows in the armature winding of a continuous-current generator, or to periodically reverse the direction of the current flowing in the armature coils of a continuous-current motor. The best insulation obtainable is used to insulate the bars from each other and from the supporting frame. Mica is generally used for this purpose. A cross section and a sectional elevation

of a commutator are shown in Fig. 45, and a typical commutator in Fig. 46.

(*h*) *Collector rings.* — In an alternator with rotating armature,

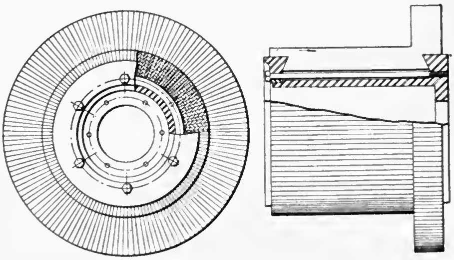


FIG. 45. Commutator Structure.

FIG. 46. Commutator. Triumph Electric Co.

the terminals of the armature winding are connected to insulated copper rings mounted on the shaft. In an alternator with rotating field, similar rings (but not always of copper) are provided for the terminal connections of the field windings. Through these rings continuous current is supplied to the field. Fig. 47.

FIG. 47. Collector Rings. General Electric Co.

(*i*) *Brushes.* — The brushes are those parts of a dynamo which make sliding contact with the commutator or the rings, and through which current is taken from or supplied to the rotating armature, or supplied to the rotating field of an alternator with stationary armature. Brushes are made of: (1) carbon, (2) copper.

(1) *Carbon brushes.* — Carbon brushes are in very extensive use on commutating machines at the present time as they offer a material help in the prevention or the reduction of sparking. Also, being set radially, the armature may be rotated in either direction without danger of injury to the brushes. Carbon brushes are sometimes coated with a deposit of metallic copper to give a reduced contact resistance between the brush and the line connection.

(2) *Copper brushes.* — Various forms of copper brushes are still used on alternating-current apparatus where no commutation takes

place, and where it is undesirable to have the large contact resistance characteristic of carbon brushes. A common form of copper brush consists of a number of thin leaves of spring copper soldered together at one end.

(*j*) *Brush holders.* — A brush holder is the arrangement by means of which the brush is supported and held in contact with the commutator or the ring. It is usually connected to a rocker by means of which the angular position of the brushes may be changed. A typical brush holder for carbon brushes is shown in Fig. 48.

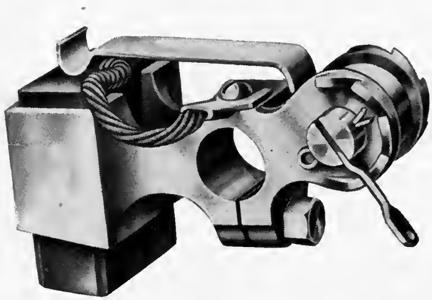


FIG. 48. Carbon Brush and Brush Holder. Triumph Electric Co.

2. Classes of dynamos. — Dynamos are divided into two general classes according to the nature of the current they produce or use: (*a*) continuous current, (*b*) alternating current.

(*a*) *Continuous-current dynamos.* — Continuous-current dynamos are sub-divided into four divisions according to the character of the field excitation: (1) shunt, (2) series, (3) compound, (4) separately excited.

(1) *Shunt dynamos.* — In the shunt dynamo, the field is excited by means of a winding composed of a large number of turns of insulated copper wire connected to the terminals of the armature circuit so that the field winding and the load circuit are in parallel when the dynamo is operated as a generator, and the field winding and the armature circuit are in parallel when the dynamo is operated as a motor.

The field current is only a small percentage of the rated current capacity of the dynamo, and is controlled by means of an adjustable resistance or field rheostat connected in series with the field winding. Since, for a given magnetization, it is required that the product of the current (amperes) flowing in the winding and the number of turns in the winding be constant, the larger the number of turns the smaller is the required current.

Fig. 49 shows the schematic and the conventional wiring diagrams of a shunt dynamo.

(2) *Series dynamo.* — The field of a series dynamo is excited by a few turns of heavy wire through which flows the entire armature current, or a constant part of this current. The field excitation, instead of being approximately constant, as in the shunt dynamo, is proportional to the current flowing in the armature, *i.e.*, to the load on the dynamo. The field flux, however, may not increase or decrease in proportion to the change in the excitation because of properties of the iron parts of the magnetic circuit.

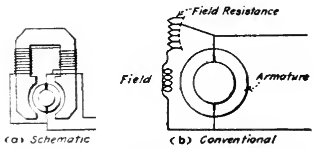


FIG. 49. Wiring Diagrams for Shunt Dynamo.

The schematic and conventional wiring diagrams of a series dynamo are shown in Fig. 50. The resistance, shown in the conventional diagram as shunting the field winding, is for the purpose of varying the field excitation for a given armature current. This resistance is made of German silver or other high-resistance material, and determines the characteristic of the dynamo. It may not be changed while the dynamo is in operation, as may the resistance in the field circuit of a shunt dynamo, but is permanently connected to the field terminals and is changed only when the characteristic of the dynamo is to be altered.

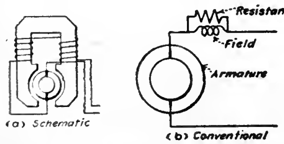


FIG. 50. Wiring Diagrams for Series Dynamo.

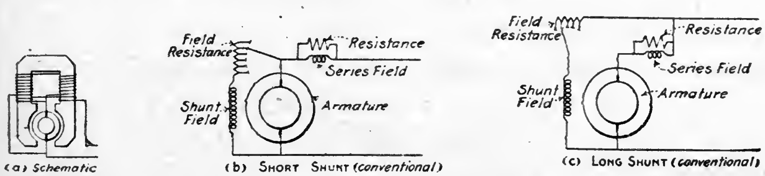


FIG. 51. Wiring Diagrams for Compound Dynamo.

(3) *Compound dynamo.* — The compound dynamo is, as the name implies, a dynamo having both a shunt and a series field winding. When the shunt field winding is connected to the terminals of the armature circuit, it is termed a "short shunt" compound dynamo; when the shunt field winding is connected so that the voltage between its terminals is the line voltage, it is termed a "long shunt" compound dynamo. Conventional and schematic diagrams are shown in Fig. 51.

(4) *Separately excited dynamo.* — If the field windings of a generator are supplied from some other source than its own armature, or the field and armature of a motor from different sources, it is said to be "separately" excited. Continuous-current dynamos are seldom separately excited while alternators are always excited in this way. The field winding of a separately excited dynamo does not differ from that of a shunt dynamo, and any dynamo may be separately excited by connecting its field winding to a continuous-current circuit of the proper voltage. Fig. 52.

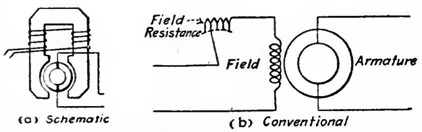


FIG. 52. Wiring Diagrams for Separately Excited Dynamo.

(*b*) *The alternator.* — The only essential difference between an alternator and a continuous-current dynamo is the substitution of continuous copper rings for the commutator, since alternating electromotive forces are induced in a continuous-current armature. These rings serve as a means of connecting the armature winding and the outside or load circuit. The alternator is not self-exciting.

However, in most alternators of the present day, the field instead of the armature is the rotating part, the armature winding being placed in slots on the inside of a cylindrical iron core, as shown in Fig. 43. In this type of alternator, rings are provided for connecting the rotating field winding to its continuous-current supply.

The field excitation of an alternator is often supplied by a small continuous-current generator, the armature of which is mounted on or belted to the alternator shaft. In large installations, the fields of all the alternators are supplied from one or more (usually not less than two) continuous-current units, each of which is driven by its own prime mover.

Attempts have been made to provide alternators with series (compound) windings by rectifying a portion of the armature current. The operation of such alternators (composite alternators) is not entirely satisfactory * and their manufacture has been discontinued.

3. Speed and frequency of an alternator. — The speed, the frequency and the number of poles of an alternator have a fixed rela-

* The degree of compounding changes with the power factor, and excessive sparking takes place unless the brushes are adjusted so as to pass from one segment to another at the instant the current is passing through its zero value.

tion to each other. An alternating-current cycle means the passage of the current, or the electromotive force, through all its values, both positive and negative, *i.e.*, starting at zero the value rises to maximum, decreases to zero, rises to maximum in the opposite direction, and again decreases to zero. To complete one cycle, a conductor must pass across a north and a south pole, or through 360 electrical degrees.* Then, for any given frequency, the speed is *inversely* proportional to the number of field poles,

$$n = \frac{2f}{p} \tag{2}$$

and for any given speed, the frequency is *directly* proportional to the number of field poles.

$$f = \frac{np}{2}, \tag{3}$$

when f = the frequency of the alternating current or electromotive force,
n = the speed (revolutions per second),
p = the number of field poles.

In America two frequencies have become standard — twenty-five cycles for exclusive power service, sixty cycles for exclusive lighting service or for service which supplies both lamps and motors. In Europe a frequency of fifteen is largely used. The lower frequencies, while preferable in many respects for motor operation, are entirely unsatisfactory for lamps. If the frequency of an alternating current supplied to incandescent lamps is reduced to about forty, a distinct variation in the light intensity (which tires the eye) becomes noticeable. The same trouble is experienced with arc lamps.

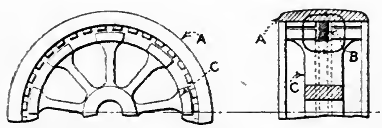


FIG. 53. Schematic Diagram of Inductor Alternator.

4. **The inductor alternator.** — An alternating electromotive force may be induced in a conductor without moving either the conductor or the exciting coil of the field magnet. In Fig. 53 let A be a cylindrical body of laminated iron similar to the armature structure of any rotating field alternator, except the central part is cut

* An electrical degree is the 360th part of the angle subtended, at the axis of the machine, by radial lines through the centers of alternate field poles.

away to receive a coil B of insulated wire, and C is a body of iron having radial projections as shown.

When coil B is supplied with continuous-current, a magnetic flux is set up as indicated by the heavy lines in the cross-sectional view. This magnetic flux is distributed in tufts over approximately one-half the inner surface of part A and moves around the cylinder as part C rotates. The conductors on the surface of A are cut by the flux in the same manner as if C were a permanent magnet. The frequency of an inductor alternator is twice that of a rotating field alternator having the same number of poles and operating at the same speed.

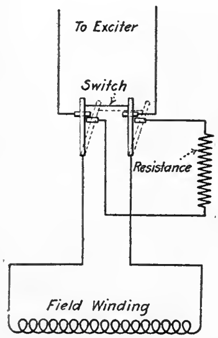


FIG. 54. Field Discharge Resistance.

This construction, while very simple, fails to show either good regulation or high efficiency.

5. Field discharge resistance. — When the field circuit of a dynamo is opened there is induced in the field windings an electromotive force, due to the rapid decrease in the flux threading the windings. This induced electromotive force may be so large as to rupture the insulation unless an auxiliary resistance, in which the energy stored * in the magnetic field may be dissipated, is provided. Such a resistance is known as a "field discharge" resistance, and is commonly used with machines having a capacity greater than 100 kw. Fig. 54 shows the essential arrangement and operation of a discharge resistance.

CHAPTER III — PROBLEMS

1. Find the resistance of a 6-pole lap-wound armature, the winding of which consists of 3000 feet of No. 10 copper wire (= 10,400 circular mils).

Note. — The resistance of any armature winding is equal to the resistance of the total length of conductor on the armature, divided by the *square* of the number of parallel paths into which the conductors are connected.

2. The armature in Problem 1 is to be wave wound, the same total weight of copper to be used and the armature to have the same number of series conductors between positive and negative brush contacts. Find: (*a*) the size and the length of the wire required, (*b*) the resistance of the armature winding.

3. Plot a development † of a 4-pole, wave-wound armature having 35 slots, 105 commutator bars and 630 conductors.

* See Appendix B, Section 6.

† Armature windings should be studied by means of wooden models on which the different windings may be placed.

4. Plot a development of a 4-pole, lap-wound armature having 44 slots, 88 commutator bars and 352 conductors.

5. The frequency of an alternator is 60. Find the speed when the number of poles equals: (*a*) 2, (*b*) 4, (*c*) 6, (*d*) 8, (*e*) 10, (*f*) 25, (*g*) 40.

6. The frequency of an alternator is 25. Find the speed when the number of poles equals: (*a*) 2, (*b*) 4, (*c*) 6, (*d*) 8, (*e*) 10, (*f*) 25, (*g*) 40.

CHAPTER IV

THE CONTINUOUS-CURRENT GENERATOR

1. **The fundamental equation.** — The voltage induced in the armature windings of a continuous-current generator is directly proportional to: (*a*) the flux ϕ entering or leaving the armature at *each* pole, (*b*) the number of poles p in the field structure, (*c*) the total number of conductors N on the armature, (*d*) the speed n (revolutions per second) at which the armature rotates. It is inversely proportional to the number of parallel circuits p' into which the armature conductors are connected. The equation representing the above statement is known as the fundamental equation of the continuous-current generator.

$$E = \frac{\phi N n p}{p' \, 10^8} \text{ volts.} \qquad (1)$$

The constant (10^8) represents the ratio between the volt and the C.G.S. unit (the abvolt) of electromotive force.

2. **Voltage characteristic.** — The voltage characteristic of a generator is a curve showing the relation between the voltage of the generator and the load or the armature current, and is: (*a*) external, (*b*) internal.

(*a*) *External characteristic.* — The external characteristic shows the relation between the *terminal* voltage of a generator, and the current flowing in the *load* circuit. This is the experimental curve, and is the one usually referred to when the term "voltage characteristic" is used.

(*b*) *Internal characteristic.* — The internal or total characteristic shows the relation between the *total* voltage induced in the armature winding, and the *armature* current. When current flows in the armature circuit, the resistance of the circuit makes the terminal voltage of a generator less than that induced in the winding. The total voltage is, then, the terminal voltage plus the voltage drop in the armature circuit and that in the series-field windings, if the dynamo is either series or compound.

$$E_a = E_t + R_a I_a + R_s I_s + E_b, \qquad (2)$$

when E_a = the total electromotive force induced in the armature winding,

E_t = the terminal electromotive force,

I_a = the armature current,

I_s = the current in the series-field circuit,

R_a = the resistance of the armature circuit,

R_s = the resistance of the series-field circuit,

E_b = brush contact drop.*

The total current flowing in the armature circuit is, evidently, the sum of that in the load circuit and that in the shunt field circuit. It may be measured directly or calculated by adding these two quantities. The current in the series field circuit of a compound generator is equal to that in the armature circuit or to that in the load circuit, as the generator is long or short shunt.

3. **Voltage regulation.** — When the speed of the armature, the resistance of the armature winding,† and the resistance of the field circuit are constant, the voltage regulation of a generator is the ratio of the maximum deviation of the actual characteristic, between full load and no load, from the ideal characteristic, which is always a straight line, and the full-load (rated) voltage.

4. **Building up.** — " Building up " is that process by which the field flux of a self-excited generator is increased to its normal value. When the iron in the magnetic circuit is once magnetized it retains some of its magnetic property (residual magnetism), and when the armature is rotated in this weak magnetic field a small electromotive force is induced in the armature windings. This electromotive force causes a current to flow in the field windings and, if the windings are properly connected, the flux is increased. The

* It has been experimentally determined that the voltage drop between the brushes and the commutator is a function of the current density in the area of contact. The drop per pair of ordinary carbon brushes is represented, with sufficient accuracy for general calculations, by the formula:

$$E_b = 0.8 + 0.2\,D, \tag{3}$$

when D = the current density (amperes per square centimeter) in the contact area between the brush and the commutator. Present practice allows five to six amperes per square centimeter of brush contact area at rated load. For densities less than one ampere per square centimeter, the values obtained by equation (3) are too large. A close approximation is obtained by assuming the brush-contact drop equal to the current density.

† The brush-contact resistance is not constant.

increased flux causes a larger current to flow in the field windings, and increases the flux still further.

A self-excited generator can not build up if the flux set up by the field windings opposes the residual magnetism of the iron in the magnetic circuit. If the magnetism due to the field windings opposes the residual magnetism, the proper relations are obtained by: (*a*) reversing the field connections, (*b*) reversing the direction of armature rotation.

(*a*) *Reversing the field connections.* — If the terminal connections of the field circuit are reversed, the direction of the current flow in the field coils is reversed, and the two fluxes are no longer opposed.

(*b*) *Reversing the direction of armature rotation.* — If the direction of armature rotation is reversed, the polarity of the armature terminals is changed, causing the direction of the current in the field coils to reverse, and the two fluxes are no longer opposed.

5. **Commutation.** — Commutation is the process of rectifying the alternating currents which flow in the armature conductors. Unlike the elementary (single loop) generator described in Chapter 2, Section 23, the conductors of commercial armatures may carry maximum current even though no electromotive force is being induced in the conductors themselves. Also, since the current in a given coil periodically reverses in direction, the current in the conductor must be reduced to zero and a current of like value established in the opposite direction during the very short time that the brush is in contact with the two commutator segments to which the terminals of the armature coil are connected. The inductance of the circuit tends to maintain any current that may be flowing in the coil at the time the brush short-circuits the coil, and to prevent the establishment of a current in the opposite direction.

The current flowing in the coil may be reduced to zero and a current in the opposite direction established during the time the brush is in contact with two commutator segments by: (*a*) changing the relative resistances of the paths through which the current may flow, (*b*) causing the conductors to generate an electromotive force opposite to that which established the original current.

(*a*) *Resistance commutation.* — Since the resistance of an armature coil, even when composed of several turns, is very low, a comparatively small additional resistance reduces the current very materi-

ally. Inherent properties of the circuit itself are used for this purpose.

The contact resistance between a carbon brush and the commutator is many times the resistance of the armature coil and increases as the area of contact decreases. Let Fig. 55 represent the relations of the coils, the commutator segments and the brush of a two-pole generator just before the brush makes contact with segment 4. One-half of the line current flows through coils *a*, *b* and *c*, unites with the other half flowing in coils *d*, *e* and *f*, and passes to the brush through the radial connection and segment 3. An instant later the brush makes contact with segment 4, the current in coils *e* and *f* naturally seeks the shorter path through segment 4, and current is diverted from coil *d*. As stated above, the inductance of coil *d*, as well as the relative resistances of the two paths now provided for the passage of the current to the brush, prevents an instantaneous diversion of all the current from the coil. As the armature continues to rotate, the area of the brush in contact with segment 4 increases, and the area in contact with segment 3 decreases, changing the relative resistances of the two paths, so that current in coil *d* has decreased to approximately zero when the area of the brush in contact with segment 4 is equal to the area in contact with segment 3.

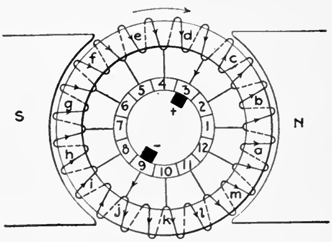


FIG. 55.

As the area of contact between the brush and segment 3 decreases still further, the resistance of this path increases, that through segment 4 decreases, and a reversed current is gradually established in coil *d*. When contact between the brush and segment 3 is broken, the entire current should be flowing through coil *d*, and no sparking occur.

(*b*) *Voltage commutation.* — Since the electromotive force induced in an armature conductor is zero only at one point (the neutral), if the brushes are advanced in the direction of armature rotation, commutation is delayed until an opposing electromotive force is induced in coil *d*. This opposing electromotive force tends to reduce the current flowing in the coil, and to establish a current in the opposite direction. The magnitude of the opposing electro-

motive force depends on the angular advance of the brushes. Therefore, to produce sparkless commutation the position of the brushes must be changed as the load on the generator increases or decreases.

To make the commutating flux proportional to the current in the armature coil, the construction shown in Fig. 56 is used. The small poles (commutating or interpoles) are excited by means of a winding connected in series with the load. The flux is, therefore, always of the proper value to produce sparkless commutation, and no shifting of the brushes is necessary for changing load.

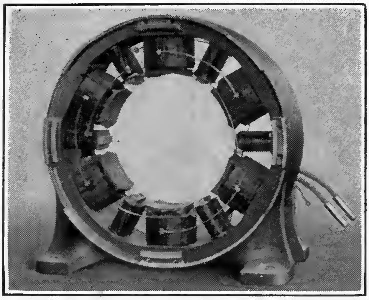


Fig. 56. Commutating Pole Field Structure. Crocker-Wheeler Co.

In nearly all dynamos of recent design, a combination of (*a*) and (*b*) is used to produce good commutation, high-resistance carbon brushes being used in connection with an angular advance of the brushes or with interpoles. Interpoles are extensively used in present day dynamos, but satisfactory commutation, from no load to 25 per cent overload, is accomplished without their use and without changing the position of the brushes.

6. Armature reaction. — When current flows in an armature winding, a magnetic flux is set up, as indicated in Fig. 57. The magnetomotive force of the armature is proportional to the current in the winding, and the direction of the flux is at right angles to the polar axis. At one tip of the pole, the magnetism thus set up decreases the flux due to the field winding, and increases it at the other tip, its main effect being to change the symmetrical distribution of the flux in the air gap* shown in Fig. 58, to the unsymmetrical distribution shown in Fig. 59, and to shift the neutral (the

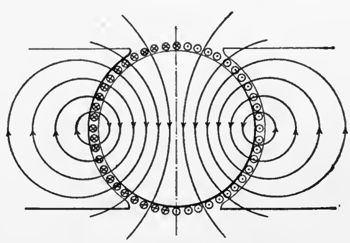


Fig. 57. Magnetic Field Due to Armature Current.

* The effect of armature teeth on the distribution of the flux is here neglected.

position in which zero electromotive force is induced in a conductor) in the direction of armature rotation.

To produce good commutation the brushes must be moved foreward,* as indicated in Fig. 60. This movement of the brushes pro-

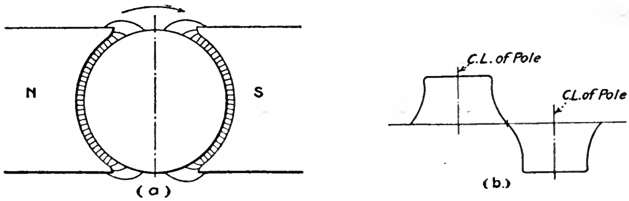


FIG. 58. Symmetrical Distribution of Flux in Air Gap.

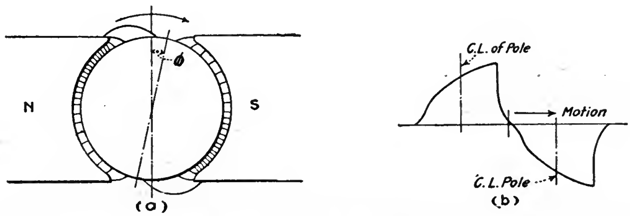


FIG. 59. Unsymmetrical Distribution of Flux in Air Gap Due to Armature Currents.

duces a corresponding change in the direction of the flux set up by the armature winding, and the armature magnetomotive force may be resolved into two components at right angles to each other. One of these components is proportional to the sine of the angle of brush advance and tends to set up a flux opposite in direction to that produced by the field windings, hence the term "demagnetizing action." The other component of armature magnetomotive force is proportional to the cosine of the angle of brush advance, and has the distorting effect described above.

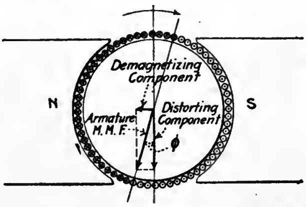


FIG. 60. Components of Armature Reaction.

The effect of armature reaction is, then, twofold: (*a*) Cross-magnetization or distortion which is proportional to the current flowing in the armature conductors and to the cosine of the angle of brush advance. Because the angle of brush advance never exceeds a few degrees, for which the cosine is

* The brushes of dynamos having commutating poles are set on the geometrical neutral (midway between the pole tips), and the effects of armature reaction are reduced to a minimum.

approximately equal to unity, and the reluctance of the air gap between the pole tips is high, the distortion in any given generator may be assumed to be proportional to the armature current. (*b*) Demagnetization which is proportional to the armature current and to the sine of the angle of brush advance.

Let N = the number of armature conductors on any given armature,
p = the number of poles on the dynamo,
I = the current flowing in each armature conductor.

Then the armature ampere-turns per pole

$$= \frac{NI}{2p}. \tag{4}$$

But the conductors are uniformly disturbed over the surface of the armature, and the magnetomotive force set up by the winding is, therefore, proportional to the average cosine (over 180 degrees).

$$\mathfrak{F} = \frac{1.257\,NI}{2p} \text{ av. cos} \Big]_{-90^\circ}^{+90^\circ} \tag{5}$$

$$= \frac{1.257 \times 0.636\,NI^*}{2p} \tag{6}$$

$$= \frac{0.4\,NI}{p} \text{ gilberts.} \tag{7}$$

7. The shunt generator. — The terminal voltage of a shunt generator decreases as the load (armature current) increases, the speed of the armature and the resistance of the field circuit remaining constant. This decrease in voltage is due to: (*a*) armature and brush-contact resistance, (*b*) armature reaction, (*c*) decreased field current.

(*a*) *Armature and brush-contact resistance.* — According to Ohm's Law, the resistance drop in any current-carrying conductor is equal to the product of the current and the resistance of the conductor. Assuming a constant temperature, the resistance of the armature winding is constant, and the resistance drop is proportional to the armature current. The drop due to brush-contact resistance is calculated by means of equation (3). At no load, when only the field current flows in the armature circuit, the armature current is negligibly small, and the terminal voltage is equal to the electromo-

* See Appendix A, Section 8.

tive force induced in the armature. As the current in the armature increases, the resistance drop in the armature circuit becomes appreciable, and the terminal voltage decreases.

(*b*) *Armature reaction.* — As explained in Section 6, armature reaction neutralizes part of the flux produced by the field winding at no load, and changes the distribution of the flux in the air gap. The total electromotive force induced in the winding, and, therefore, the terminal voltage, decreases as the armature current increases.

(*c*) *Reduction of field current.* — When the resistance of the field circuit is constant, the current flowing in the circuit is proportional to the electromotive force at the terminals of the armature, and the decreased voltage due to resistance in the armature circuit and to

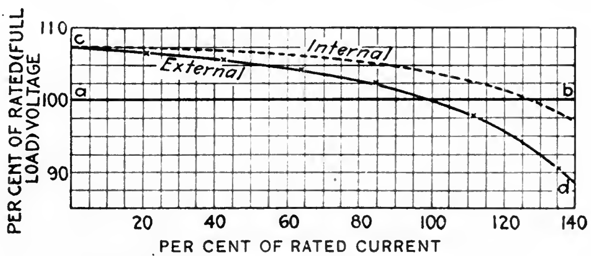


FIG. 61. Shunt Generator Characteristics.

armature reaction, causes a decrease in the field current, and a further decrease in the terminal voltage of the generator.

The ideal characteristic of a shunt generator is a horizontal straight line (*ab* in Fig. 61), while the actual characteristic rises as the load decreases (*cd* in Fig. 61). The percentage regulation of a shunt generator is, therefore,

$$\text{per cent regulation} = \frac{(E_0 - E)\,100}{E}, \tag{8}$$

when E = the full-load (rated) voltage,
E_0 = the no-load voltage.

The regulation of the average shunt generator is too large for the satisfactory operation of lamps when the load varies greatly, although it may be entirely satisfactory for motors. To give satisfaction, incandescent lamps must have approximately constant voltage applied between their terminals. Since the heating of a conductor is proportional to the square of the current flowing in it,

the light given by an incandescent lamp is approximately proportional to the square of the applied voltage. The inherent regulation of a shunt generator, therefore, cannot be depended on to maintain the proper voltage as the load fluctuates, and manipulation of the field rheostat must be resorted to when incandescent lamps form all or part of the load.

Automatic adjustment of the field resistance is made by the Tirrill and other regulators, which are largely used for the maintenance of constant voltage either at the generator terminals or at some center of distribution.

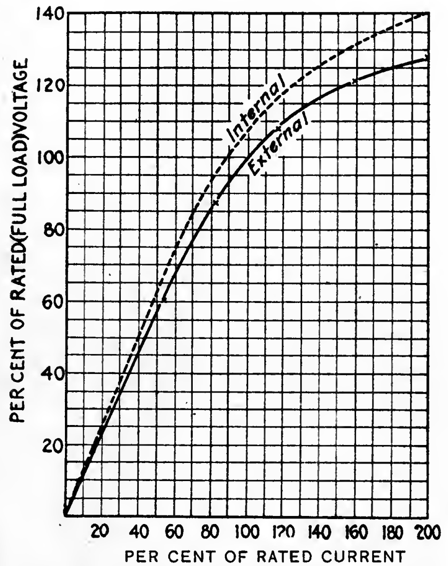


Fig. 62. Series Generator Characteristics.

8. The series generator. — The voltage characteristic of a series generator is, practically, the magnetization curve for a combined iron and air circuit. Since the load current, or a constant part of it, flows through the series-field windings, the field excitation increases as the load increases. For low excitations, the flux is very nearly proportional to the field current, but as the excitation increases, the flux increases at a constantly decreasing rate until the iron becomes "saturated." Beyond the point of saturation, the permeability of iron is little greater than that of air, and a large increase in field current produces only a small increase in flux. The voltage of a series generator is, therefore, approximately proportional to the armature current over a considerable range, the characteristic bending to the right, as indicated in Fig. 62.

Armature, brush-contact and field resistances make the terminal voltage less than the induced voltage, while armature reaction reduces the total flux set up by a given field excitation and causes an unsymmetrical distribution of the flux in the air gap as in other types of generators.

Since the series generator is used almost exclusively as a constant-current generator, its voltage regulation is of little consequence. The term "regulation," when applied to a series generator, means the ratio of the maximum deviation of the current, between rated load and short circuit, from the rated current at full load, and the rated current.

When used as a constant-current generator, the series dynamo is provided with an automatic regulator which causes the terminal voltage to increase or decrease in proportion to the increase or decrease in the resistance of the load circuit, so that the current is maintained at an approximately constant value. One of the simplest of these automatic regulators is that used on the Brush arc-light generator. The field winding is shunted by a carbon pile, the resistance of which varies inversely as the pressure between the discs. The pressure between the discs is varied by means of an electromagnet, the winding of which is connected in series with the load. When normal current flows in the circuit, the current divides, part flowing in the field windings and part through the carbon pile. If the current falls below normal, by reason of an increase in the resistance of the load circuit (an increase in the number of lamps in the circuit), the pressure on the carbon pile is decreased. The decreased pull of the magnet increases the resistance of the carbon shunt, and causes a larger current to flow in the field windings. This increased field excitation causes a larger voltage to be induced in the armature windings, and the load current rises to its normal value.

Since no current flows in the field coils of a series generator on open circuit, it can "build up" only when the load circuit is closed.

9. The compound generator. — The inherent tendency for the terminal voltage of a shunt generator to decrease as the load increases is counteracted by the addition of a series-field winding so proportioned that the total flux increases as the current in the armature winding increases. If the effect of the series-field winding is just sufficient, at full load, to compensate for the decrease in voltage due to armature resistance and armature reaction (*i.e.*, if the no-load and the full-load voltages are equal), the generator is *flat* compounded; if the effect of the series-field winding is such that the full-load voltage is greater than the no-load voltage, the generator is *over* compounded. The characteristic curves of a flat-

compounded generator and of an over-compounded generator are shown in Fig. 63.

When armature speed, field resistance and armature resistance are constant, the compounding of a generator is the ratio of the increase in voltage, between no load and full load, to the no-load voltage.

$$\text{Per cent compounding} = \frac{(E - E_0)\,100}{E_0}, \tag{9}$$

when E_0 = the no-load voltage,
E = the full-load voltage.

It is impracticable to build a compound generator, the voltage characteristic of which is a straight line. From the definition given in Section 3, the regulation of a compound generator is the maximum deviation e (Fig. 63), between no load and full load, of the characteristic from the straight line connecting the no-load and the full-load voltage points, divided by the full-load voltage. The maximum deviation is measured perpendicularly to the axis of abscissa, and only for the flat-compound generator is it perpendicular to the ideal curve.

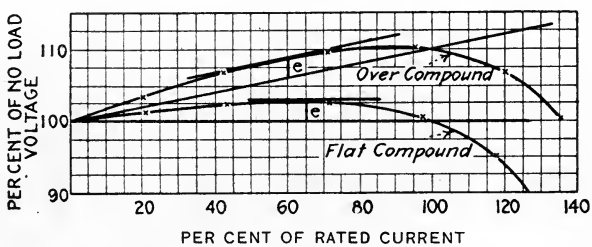


FIG. 63. Compound Generator Characteristics.

$$\text{Per cent regulation} = \frac{100\,e}{E}, \tag{10}$$

when E = the full-load voltage.

A compound generator builds up in the same manner as a shunt dynamo since the excitation due to the series-field winding is negligible at no load. If the series winding is improperly connected, the two windings oppose each other magnetically, and the terminal voltage drops very fast as the resistance of the load circuit is decreased.

The degree of compounding of a given generator may be changed, without altering its structure, by: (*a*) changing the resistance of the shunt around the series-field winding, (*b*) changing the speed.

(*a*) *Changing the resistance of the series-field shunt.* — The effect of changing the resistance shunting the series-field winding is to cause a greater or a less percentage of the total load current to flow in the series-field coils, thus increasing or decreasing the field excitation for a given armature current. This change in field excitation causes the full-load voltage to be increased or decreased.

(*b*) *Changing the speed.* — If the no-load voltage of the generator remains constant, the effect of an increase in the speed of its armature is to increase the full-load voltage. The effect of a decrease in the speed of a given generator, the no-load voltage of which is constant, is to decrease the full-load voltage.

The truth of these statements is evident from a study of the operating principles of the electric generator. The rated full-load voltage of a flat compounded generator is 100. At no load, 1,000,000 magnetic lines pass into the armature from each north pole, and this flux is increased, at full load, to 1,100,000 lines by the magnetic action of the series-field winding. The electromotive force induced in the armature at full load is, therefore, 110, and 10 volts are required to compensate for armature resistance and armature reaction.

If the speed of the armature is increased 20 per cent above its rated value, the flux required to produce the no-load voltage is reduced to 800,000 lines, while the effect of the series winding does not decrease. Assuming the magnetic effect of the series winding to be the same at all speeds, the induced electromotive force, at the higher speed, is 112.5 and the terminal voltage, at full load, is 102.5, an over compounding of 2.5 per cent.

But the compounding is greater than the above because of the fact that as the shunt-field excitation is reduced, the flux produced by a given series-field excitation increases. The last statement will be made clear by an examination of the magnetization curve of a generator. Referring to Fig. 64, it is found that 500 ampere turns are required to produce a flux of 1,000,000 lines, and that 850 ampere-turns are required to produce 1,100,000 lines. Consequently, the shunt-field winding must consist of 500 ampere-turns and the series-field winding of 350 ampere-turns. To produce

800,000 lines requires only 260 ampere-turns in the shunt-field winding, but the series-field winding still produces, at full load, 350

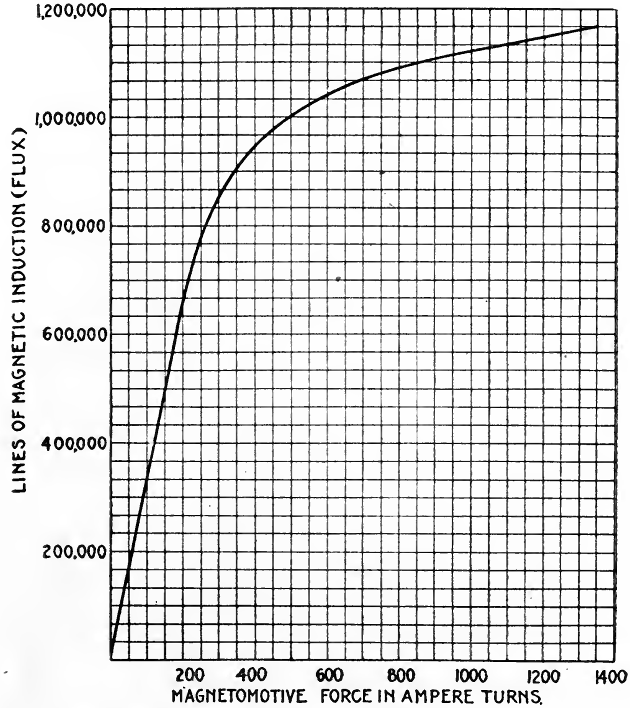


FIG. 64. Characteristic magnetization curve showing the large increase in magnetizing force for a small increase in magnetic induction after passing the "knee" of the curve.

ampere-turns or a total of 610 ampere-turns at full load. This excitation sets up a flux of 1,030,000 lines, the electromotive force induced in the armature at full load is 128.5, and the terminal voltage is 118.5, an over compounding of 18.5 per cent.

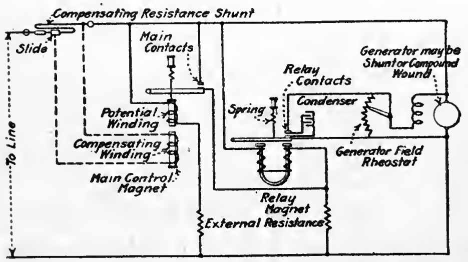


FIG. 65. Diagram of Connections for Tirrill Regulator.

10. The Tirrill regulator. — An elementary diagram of the Tirrill regulator, as applied to a shunt or a compound generator, is shown in Fig. 65. By means of electromagnetically operated contacts, which open and close as the electromotive force

rises above or falls below a certain value at the generator terminals, or at some center of distribution, the field rheostat is periodically short-circuited.

The winding of the main contact magnet is connected directly to the bus bars, and is so proportioned that it opens the main contacts, against the action of a spring, when the voltage rises above the desired value. The relay is differentially wound, one coil being connected directly to the bus bars and the other in series with the main contacts. The relay contacts are connected to the terminals of the field rheostat. The function of the condenser is to prevent excessive sparking when the relay contacts open.

The regulator operates as follows: If the main contacts are open, the left-hand coil of the differentially-wound relay is energized and the relay contacts opened. This action increases the resistance of the field circuit by opening the short circuit over the field rheostat, the voltage drops below the required value, and the spring closes the main contacts. Closing the main contacts energizes the right-hand coil of the relay and closes the relay contacts, thus raising the voltage above the value at which the main contacts open. In practice, the two sets of contacts are continually opening and closing, and the voltage varies from the desired value by only a very small amount.

When it is desired to maintain constant voltage at some distant center of distribution, a differential series winding is added to the main control magnet, thus increasing the bus-bar voltage at which the main contacts are opened. The drop in the transmission system may thus be compensated for.

11. Parallel operation of generators. — It is often desirable to supply an electric system from two or more generators instead of from one because: (*a*) of the greater efficiency of a machine when operated at or near its rated load, (*b*) continuity of service can be more easily maintained.

(*a*) *Efficiency.* — Because certain losses of a generator are practically constant and these losses are roughly proportional to the size of the generator, the ratio of the total output to the total input (all-day efficiency) is greater for a generator running at full load than for one operating at half its rated capacity, the total output being constant.

(*b*) *Continuity of service.* — It is practically impossible to maintain continuous service from a single generator because adjustments

and repairs, which require the machine to be shut down, must be made from time to time. With two or more generators, these matters may be attended to during periods of light load when all the machines are not required. It is thus possible to give continuous service and, at the same time, keep the machinery in repair.

Parallel operation of generators, then, means dividing the total load between two or more generators, and the operation of such a number of generators at a time as will cause each generator to operate at its highest efficiency. This necessitates connecting and disconnecting generators and the load circuit as the load varies.

Before two continuous-current generators are connected to the same load circuit, their terminal voltages should be approximately equal and similar terminals, either positive or negative, *must* be connected together. If the voltages are not approximately equal, an undesirable surge of current takes place in the system when the switch is closed. When dissimilar terminals of the generators are connected together, conditions identical with a short-circuit exist and an excessive current, which either operates the protective devices or overheats the armatures, flows around the circuit formed by the two armature windings.

Shunt generators. — The connections for the operation of two shunt generators in parallel are indicated in Fig. 66. If generator

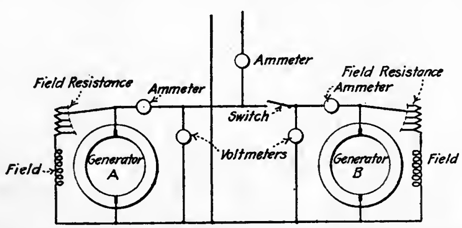


Fig. 66. Parallel Operation of Shunt Generators.

A is carrying the load and it is desired to divide the load between A and B, drive generator B at its rated speed and regulate its field to give a voltage equal to or slightly greater than that of A. When the voltage across the open switch is zero, the switch may be closed, after which the field rheostats should be manipulated until the voltage of the system is the rated or required value, and the generators divide the total load in proportion to their ratings.* After being properly regulated, two or more generators having similar characteristics will automatically divide the load in proportion to their ratings as the total load on the system varies.

* The load on any one of two or more continuous-current generators operating in parallel is reduced by reducing *either* its field excitation or its speed.

In connecting two generators as described above, a voltage across the switch indicates that the terminal connections of generator B should be reversed. When any voltage is indicated across this switch, it is equal to approximately twice the voltage of each generator, so that in testing, care should be taken that the voltmeter is not injured.

A shunt generator, operating in parallel with other generators, should be disconnected from the load circuit by opening the line switch or circuit-breakers after reducing *either* its speed or its field excitation until the current flowing in its armature is a minimum.

Compound generators. — The connections for the parallel operation of compound generators are shown in Fig. 67, the only change from the shunt diagram being a third connection between the two generators. This connection is called the equalizer, and is made between the brush and the series-field winding of each generator.

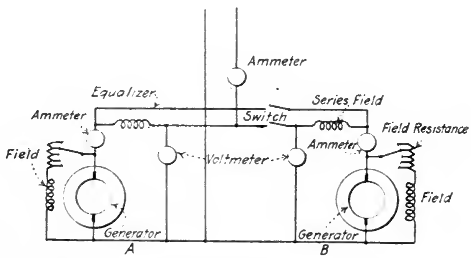


FIG. 67. Parallel Operation of Compound Generators.

The effect of the equalizer is to divide the load properly between the two generators. Consider the action of two compound generators without an equalizer connection. If, for any reason, the speed of one generator increases slightly, its voltage increases and it takes a greater part of the total load. If the total load on the system remains constant, the increased load on one generator causes a larger current to flow in its series-field winding and reduces the current flowing in the series-field winding of the other generator. This change in field excitation disturbs the equilibrium of the generators and will, by its cumulative effect, cause one generator to take the entire load and to operate the other as a motor. Compound generators connected to the same circuit and operating without equalizer connections are, therefore, in unstable equilibrium.

With an equalizer connection, the same increase in the speed of generator A causes its voltage and its current output to increase, but instead of all the increased current flowing through the series-field winding of generator A, it divides at the brush, part flowing

through the series-field winding of each generator so that the voltage of each is equally increased, and their equilibrium is not disturbed.

Equalizer connections should be large as they are often required to carry heavy currents, and an appreciable resistance in this connection between generators tends to disturb their equilibrium, and causes an unequal division of the load. Fuses or other circuit interrupting devices should *not* be placed in an equalizer circuit.

The process of disconnecting a compound generator, operating in parallel with other generators, from its load circuit, is similar to that for a shunt generator.

12. Connection to load circuits. — The load units supplied by shunt and compound generators are connected in parallel, *i.e.*, the terminals of each unit, as a lamp or a motor, are connected directly to the mains leading from the terminals of the generator or to branches of these mains.

The shunt generator is in common use under the following conditions:

(*a*) Where the load is practically constant.

(*b*) Where the load changes slowly and infrequently.

(*c*) Where the load consists exclusively of motors which do not require close voltage regulation.

(*d*) Where an attendant is employed to manipulate the field rheostat and maintain constant voltage.

(*e*) Where the voltage is controlled by a Tirrill regulator or other automatic device.

Compound generators are in common use under these conditions:

(*a′*) Where an approximate compensation is to be automatically made for the internal drop of the generator.

(*b′*) Where the drop in a long transmission line or in a long feeder is to be automatically compensated for, *i.e.*, where the voltage at the terminals of a load some distance from the generator is to be automatically maintained at an approximately constant value as the load varies.

(*c′*) Where the terminal voltage is to increase with the load.

The series generator is used almost exclusively for supplying current to series arc-lamp systems, the generator being provided with a regulator which automatically maintains the current in the system at an approximately constant value as the load changes.

The terminals of the load units in a series circuit, instead of being

connected to the mains running from the generator, are connected to each other, one terminal of the first and one terminal of the last unit in the series being connected to the mains. Consequently, the same current flows in each part of the circuit and the voltage changes as the load changes.

The series generator for the operation of arc-lamp circuits, has been largely replaced by alternating-current apparatus.

CHAPTER IV — PROBLEMS

1. The electromotive force of a shunt generator is 118 volts when operated without load, and the rated output is 11 kw. at 110 volts. Calculate: (*a*) the voltage regulation, (*b*) the resistance of the armature circuit, if one-half the drop in terminal voltage between no load and full load is due to armature resistance.

2. The terminal voltage of a series generator is 450 when the armature current is 10 amperes. Calculate the electromotive force induced in the armature winding if the resistance of the armature circuit is 1 ohm and that of the field winding is 1.5 ohms.

3. A shunt generator requires, at zero load, a field current of 2 amperes to induce the voltage at which it is rated; at full load (= 100 amperes) the field current required to give rated terminal voltage is 2.33 amperes. The shunt winding on each pole consists of 4000 turns. Find the number of turns required in a series-field winding to make the generator flat compounded, 57 per cent of the load current to flow in the series-field windings.

4. A 4-pole, 125-volt, lap-wound (drum) armature has 400 conductors. The flux per pole is 2,500,000. Find the speed at which the armature rotates to induce rated voltage in the armature conductors.

5. A 16-pole, lap-wound generator operates at 80 r.p.m., has a flux per pole of 7,500,000, and 2304 conductors. Find the voltage induced in the armature.

6. Find the number of conductors required on a wave-wound armature to be used in the generator of Problem 4, the rated voltage to remain the same.

7. Compare the resistance of a 6-pole, lap-wound armature with that of a 6-pole wave-wound armature, the rated voltages, the flux per pole, the kw. outputs, and the current densities in the armature conductors being the same.

8. Allowing 500 circular mils per ampere, find the area of the armature conductors in: (*a*) a 6-pole lap-wound dynamo, the armature current of which is 100 amperes, (*b*) a 6-pole wave-wound dynamo, the armature current of which is 100 amperes.

9. Find the voltage induced in an 8-pole lap-wound armature when:

$$\text{r.p.m.} = 300,\ \phi = 2{,}000{,}000,\ N = 784.$$

10. A 550-kw. generator has a terminal voltage of 550 at no-load and is 5 per cent overcompounded. Find: (*a*) the size of wire required to transmit 100 amperes a distance of 600 feet, the voltage between the terminals of the load

apparatus to be 550, (*b*) the watts lost in the line, (*c*) the total output of the generator.

11. The rated output (line) of a compound generator = 150 amperes, and its no-load voltage is 220. The current is transmitted over a line, the resistance of which is 0.166 ohm, and the voltage at the load terminals is the same at full load as at no-load. Find: (*a*) the terminal voltage of the generator at full load, (*b*) percentage overcompounding, (*c*) watts lost in line, (*d*) watts output of generator.

12. A 4-pole, wave-wound armature has 105 commutator bars. Each armature coil consists of two turns of No. 10 double cotton-covered wire. Find the flux (per pole) required when the induced voltage is 125, and the speed = 1000 r.p.m.

13. A 4-pole, lap-wound armature has 88 commutator bars. Each armature coil consists of four turns of No. 11 double cotton-covered wire. Find the voltage when the flux per pole = 1,000,000 lines and the speed of the armature = 1000 r.p.m.

CHAPTER V

THE CONTINUOUS-CURRENT MOTOR

1. **The fundamental equation of the motor.** — Rotation of the armature of a motor induces in the armature winding a counter-electromotive force which is dependent on the same quantities as is the induced electromotive force of a generator.

$$\text{Counter e.m.f.} = \frac{\phi N n p}{p' \, 10^8} \text{ volts,} \tag{1}$$

when ϕ = the total flux entering or leaving the armature at *each* pole,

N = the total number of conductors on the surface of the armature,

n = the speed of the armature (revolutions per second),

p = the number of poles,

p' = the number of parallel paths into which the armature conductors are connected.

The counter-electromotive force is the difference between the applied electromotive force and the resistance drop in the armature circuit. Therefore,

$$\frac{\phi N n p}{p' \, 10^8} = E - R_a I_a - E_b, \tag{2}$$

when E = the applied electromotive force,

R_a = the resistance of the armature winding,

I_a = the current flowing in the armature circuit,

E_b = the drop due to the so-called contact resistance between the brushes and the commutator.*

* As noted in Chapter 4, Section 2, the voltage drop per pair of carbon brushes may be calculated by the formula:

$$E_b = 0.8 + 0.2\,D, \tag{3}$$

when D = the current density (amperes per square centimeter) in the contact area between the brush and the commutator.

Transposing the quantities in equation (2),

$$n = \frac{k\,(E - R_a I_a - E_b)}{\phi} \tag{4}$$

$$= \frac{kE_c}{\phi} \tag{4a}$$

since p, p' and N are constant for any given motor. Equation (4) is the usual form of the fundamental equation for the continuous-current motor.

2. Torque. — The force tending to rotate the armature of a motor is termed its torque and is usually expressed in pounds at one foot radius, *i.e.*, in foot-pounds. It is proportional to the product of the flux in the air gap and the current flowing in the armature circuit. Let

T = the torque in foot-pounds developed in the armature of any motor,
n = the speed (revolutions per second) at which the armature rotates,
E_c = the counter-electromotive force (volts) induced in the armature conductors,
I_a = the current (amperes) flowing in the armature circuit.

Then $$\frac{746 \times 2\,\pi n T}{550} = E_c I_a \tag{5}$$

$$= \frac{\phi N n p I_a}{p' 10^8} \tag{6}$$

and $$T = \frac{550\,Np}{746 \times 2\,\pi p'\,10^8} \times \phi I_a. \tag{7}$$

The quantity

$$\frac{550\,Np}{746 \times 2\,\pi p'\,10^8} \left(= 11.7 \times 10^{-10} \times \frac{Np}{p'}\right)$$

is constant for any given motor.

The torque delivered at the pulley of the motor is less than that indicated by equation (7), because part of the torque developed in the armature is required to overcome the windage, friction and iron losses (stray power) of the motor itself.

3. Speed-torque characteristic. — The operation of a motor is represented graphically by a speed-torque curve, which shows the

relation between the speed of the armature and the torque developed in the armature, or that delivered at the pulley.

The relations between the speed, the torque, and the armature current of a continuous-current motor are such that a change in one of these quantities produces a change in one or both of the others. In any given motor, the speed is automatically adjusted to the value which allows the required current to flow in the armature conductors. If the armature current is larger than that necessary to produce the required torque, the speed of the armature automatically increases until equilibrium is established; if the current is too small, the speed of the armature decreases, allowing a larger current to flow and a larger torque to be exerted.

4. **Commutation.** — The phenomena of commutation occur in a motor just as in a generator, since the direction of the current in each armature coil is periodically reversed.

5. **Armature reaction.** — The direction of current in the armature conductors of a motor is opposite, for a given field polarity and direction of armature rotation, to that in the conductors of a generator. The field is, therefore, distorted as indicated in Fig. 68, and the brushes must be shifted backward, or opposite the direction of armature rotation, to obtain good commutation.

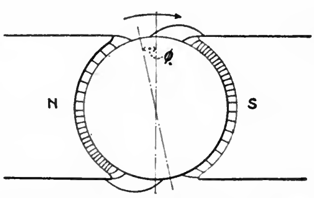


FIG. 68. Flux Distribution in the Air Gap of Motor.

6. **The shunt motor.** — With constant applied voltage, the field current and the armature flux of a shunt motor are constant, neglecting armature reaction which is small in a well-designed motor, and the torque developed in the motor is proportional to the current flowing in the armature circuit.

$$T \propto I_a. \tag{8}$$

Characteristic. — As the load on a shunt motor increases, the counter-electromotive force must decrease so that the required current may flow in the armature circuit. Armature reaction reduces the flux of a shunt motor slightly, but if the magnetic circuit is properly designed this reduction is small, and may be neglected.*

* Any decrease in the flux of a shunt motor by reason of armature reaction, tends to increase its speed and improve the regulation of the motor.

The speed of a shunt motor must, therefore, decrease as the load increases, and the speed-torque characteristic is a slightly drooping curve as indicated in Fig. 69.

Well-designed shunt motors operate with only a small change in speed between no-load and full (rated) load, and are termed con-

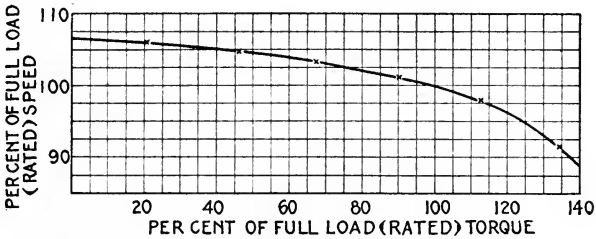


FIG. 69. Speed-torque Characteristic of Shunt Motor.

stant-speed motors. They are largely used where a small variation of speed is of little consequence.

Regulation. — The regulation of a shunt motor is the ratio of the difference between the no-load and the full-load (rated) speeds, and the full-load speed.

$$\text{Per cent regulation} = \frac{(n_0 - n)\ 100}{n} \tag{9}$$

when n_0 = the speed at no load,
n = the speed at full (rated) load.

The resistance of the armature circuit is the controlling factor in the regulation of a shunt motor. If the armature resistance is small, the variation of the product, $R_a I_a$, between no load and full load is small, and the required variation of the counter-electromotive force is correspondingly small. But the counter-electromotive force is proportional to the speed of the armature. Therefore, a small armature resistance produces, or tends to produce, good regulation.

*Speed control.** — Speed regulation is an inherent property of a motor; speed control is the variation in speed obtained by external means.

The speed of a shunt motor is varied, within certain limits, by: (*a*) changing the resistance of the armature circuit, (*b*) changing the

* For an extended discussion of motor speed control, see "Electric Motors" by Crocker and Arendt.

resistance of the field circuit, (*c*) changing the reluctance of the magnetic circuit, (*d*) changing the electromotive force between the armature terminals.

(*a*) *Changing the resistance of the armature circuit.* — The speed of a motor is changed by varying a resistance connected in series with the armature as indicated in Fig. 70. A resistance, the current-carrying capacity of which is equal to or greater than the maximum current that will flow in the circuit, must be used. This method is effective, cheap in first cost, and easily applied, but the power wasted in heating the rheostat makes it very inefficient. As pointed out above, a small change in load causes a large change in speed, and the motor acts as if the resistance of the armature winding were greatly increased.

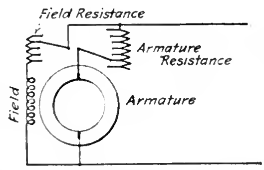


FIG. 70. Speed Control by Armature Resistance.

(*b*) *Changing the resistance of the field circuit.* — The resistance of the field circuit is changed by manipulating the field rheostat. Increasing the resistance of the field circuit decreases the field current and increases the speed; decreasing the resistance of the field circuit increases the field current and decreases the speed.*

This method is cheap, both as to first cost and as to operation, but its range of operation is limited. Very low speeds are unattainable because of the fact that above a certain point a large increase in field excitation produces only a small increase in flux. High speeds are also unattainable because, with a weak field, armature reaction becomes so pronounced as to cause excessive sparking which burns, and would ultimately destroy, the commutator.

Armature reaction and commutation difficulties are materially reduced by means of interpoles which provide a local commutating flux, and make possible the operation of shunt motors having very weak fields. Interpole motors having a maximum speed six times the minimum speed, and controlled by manipulation of the field rheostat, are in successful operation.

(*c*) *Changing the reluctance of the magnetic circuit.* — With constant field excitation, the flux in the magnetic circuit of a dynamo is inversely proportional to the reluctance of the circuit.† A number

* This statement is evident from equation (4).

† See Chapter 2, Section 10.

of variable-speed motors have been designed in which the reluctance of the magnetic circuit is varied by changing the length of the air gap between the pole and the armature. The operation of such motors is satisfactory, but they are expensive in construction and more or less complicated in operation.

(*d*) *Changing the electromotive force applied to the armature terminals.* — This method of speed control may be sub-divided into two parts: (1) when the change in voltage is made in steps, (2) when the range of voltage is continuous.

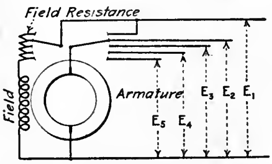


FIG. 71. Multi-voltage Speed Control.

(1) *Voltage changed in steps.* — This method of speed control necessitates the use of several supply mains having different voltages, and a generating system capable of supplying current at these different voltages. The field winding is permanently connected across one pair of the supply mains and the flux is, therefore, approximately constant. Fig. 71.

The operating characteristics of this system are good and the efficiency high, the objections being its high first cost and the fact that the speed changes by fixed steps. The latter fault is remedied by combining this method with the field rheostat method.

(2) *Voltage range continuous.* — This method, devised by Ward Leonard and generally known by his name, requires the use of three machines — a variable-speed motor, a constant-speed motor* and a generator, as shown in Fig. 72. The constant-speed motor drives the generator, and the generator, in turn, supplies current to the armature of the variable-speed motor, the field of which is connected to constant potential mains. By varying the field excitation of the generator, any desired voltage is delivered at the terminals of the motor armature, and a continuous variation of speed obtained over the widest possible range. Very

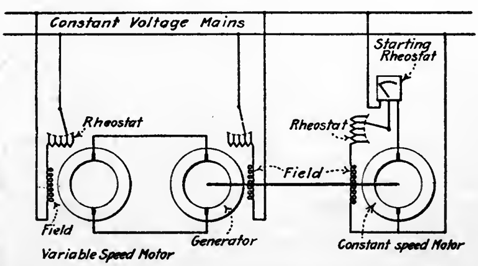


FIG. 72. Ward Leonard Speed Control System.

* It is not necessary that this be an electric motor.

simple operating mechanism is required. The efficiency of this system is the combined efficiency of three machines, the individual efficiencies of which are usually low. Its high first cost and low operating efficiency prohibit its use except where these factors are secondary considerations.

7. The series motor. — The series motor, unlike the series generator, has a very large commercial application. It is used, practically to the exclusion of all other types of motor,* for street

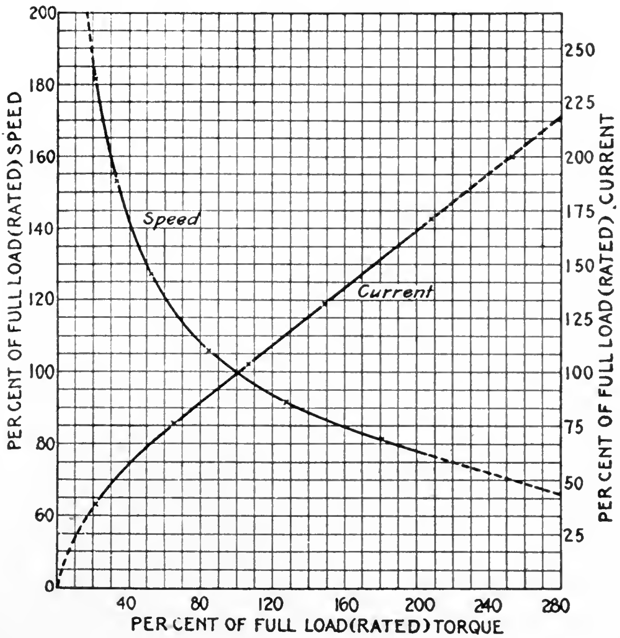


FIG. 73. Series Motor Curves.

railway work, for hoisting and for all purposes which require large starting torques, and do not require close speed regulation.

Torque. — Since the field excitation of a series motor is proportional to the armature current, the torque developed in its armature is, theoretically, proportional to the square of the armature current.

$$T \propto I_a^2. \qquad (10)$$

This equation holds good for low values of armature current, but the upper part of the current-torque curve is found to be an approximately straight line. Fig. 73. For low excitations, the flux is pro-

* Alternating-current motors are used to a limited extent for these purposes.

portional to the field current but for larger loads the iron of the magnetic circuit approaches saturation, and the flux increases more slowly than does the current in the field winding. The torque developed is, therefore, less than that indicated by equation (10).

Speed. — The speed of a series motor varies over wide limits as the load changes, the speed increasing as the load is reduced as indicated in Fig. 73. The full line shows the operating range of the motor, the dotted portion near the axis of ordinates indicates speeds above the safe operating limits, and excessive heating takes place if the motor is operated over the dotted portion at the right.

Because of the excessive speed attained by the armature of a series motor when the load is small, a series motor should never be used where the load may, accidentally or otherwise, be reduced below the safe minimum value, or the motor may be wrecked.

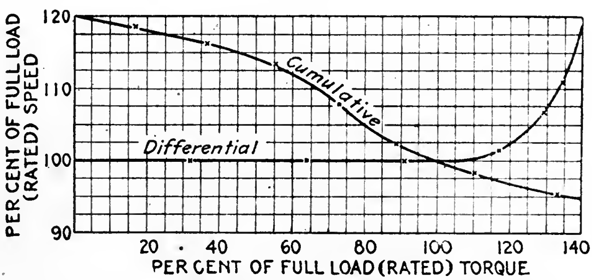


Fig. 74. Speed-torque Curve of Compound Motors.

8. Compound motors. — Compound motors are divided into two classes: (*a*) cumulative, (*b*) differential.

(*a*) *Cumulative compound motor.* — The shunt and the series-field windings of a cumulative compound motor are so connected as to produce fluxes in the same direction. Therefore, the excitation of the motor increases with the load, and the speed of the armature may be materially reduced. The shape of the speed-torque curve of a cumulative compound motor depends on the relative effects of the two field windings, but that part of the curve representing light loads is usually similar to that of a shunt motor with large armature resistance. As the load increases, the characteristic gradually reverses and finally approximates that of a series motor. Fig. 74 shows the speed-torque characteristic of a cumulative compound motor.

A cumulative compound motor having a heavy flywheel is well adapted for the operation of shears, punches and other apparatus where the load is applied suddenly. During the periods of no load, energy is stored in the flywheel and other rotating parts. When the load is applied, this stored energy is returned to the system and prevents the speed of the motor from dropping to as low a value as would otherwise be the case, and reduces the range over which the armature current varies.

(*b*) *Differential compound motor.* — The field windings of a differential compound motor oppose each other so that the total flux in the magnetic circuit decreases as the armature current increases. The speed of a differential compound motor is constant if the ratio of the counter-electromotive force and the flux is constant.

By properly designing the magnetic circuit, the speed of a differential compound motor is maintained practically constant between no load and full (rated) load, as indicated in Fig. 74. Over that portion of the curve where the speed is constant, the flux decreases slightly, and the torque is approximately proportional to the armature current. When the speed begins to rise the flux decreases rapidly and soon reaches the point where it decreases at approximately the same rate as the armature current increases. Any further increase in armature current cannot increase the torque, which is proportional to the product of flux and armature current, since the combined effect of the differential winding and armature reaction reduces the flux faster than the armature current increases. Excessive weakening of the field causes poor commutation and is indicated by sparking.

From the above, it is evident that a differential compound motor cannot be greatly overloaded. Also, because of the large current which would flow in the series winding during the starting period and largely neutralize the effect of the shunt-field winding so that the starting torque would be small, the series winding is usually short-circuited during the starting period, *i.e.*, the motor is started as a shunt motor. The commercial application of the differential compound motor is very limited.

9. Motor-starting rheostats. — Because of the very low resistance of the armature winding of a motor, an excessive current will flow if the circuit is connected directly to the supply mains to start it. It is, therefore, necessary to connect an external resistance in series

with the armature winding during the period of acceleration. As the speed of the armature increases, the counter-electromotive force opposing the flow of the current increases, and the resistance is gradually reduced until the armature terminals are connected directly to the supply mains.

Rheostats for shunt and compound motors. — The starting rheostats used on shunt and compound motors are of various designs,

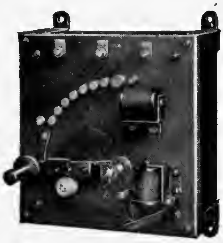


FIG. 75. Motor Starting Rheostat. General Electric Co.

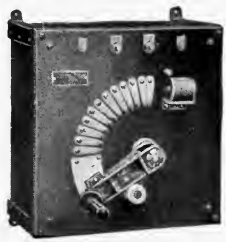


FIG. 76. Motor Starting Rheostat. General Electric Co.

typical examples being shown in Figs. 75 and 76. The internal and external electrical connections are indicated in Figs. 78 and 79.

The handle H (Fig. 78) when moved so as to make contact with stud 1, closes the armature circuit through the resistance R. This movement of the handle H also closes the field circuit. As the speed of the armature increases, the handle H is moved to studs 2, 3, etc., until the entire resistance is cut out.

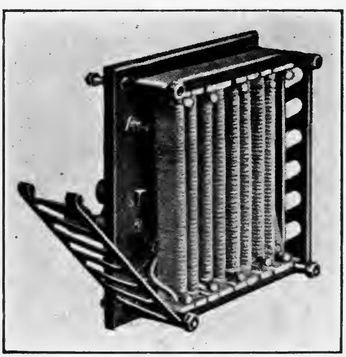


FIG. 77. Motor Starting Rheostat (Back and Side Removed). Westinghouse Elec. & Mfg. Co.

The field circuit includes the exciting coil of an electromagnet which holds the handle H, against the pull of a spring, in contact with the last stud. Should the supply circuit be opened or the field excitation, for any reason, become greatly weakened, the magnet can no longer hold the handle, the spring returns it to the position indicated in the figure and opens the armature circuit. This magnet is known as the "low-voltage" release, sometimes erroneously called the no-load release, and prevents injury to the apparatus should the supply be temporarily interrupted.

The winding of the other electromagnet shown in Fig. 78 is connected in series with the armature and so arranged that at some predetermined current value, switch S is closed, and the winding of the low-voltage release short-circuited. The low-voltage release is thus deprived of its magnetizing current and the handle returns to the open-circuit position, the supply circuit is opened, and the motor stops. This device is the "overload" release which pro-

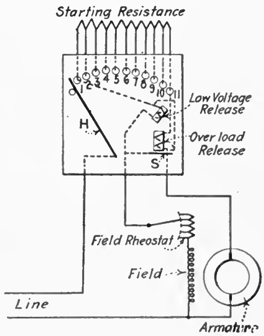


FIG. 78. Wiring Diagram for Shunt Motor and Starting Rheostat.

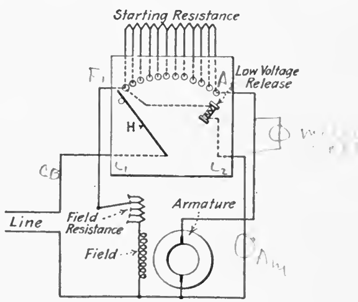


FIG. 79. Wiring Diagram for Shunt Motor and Starting Rheostat.

tects the armature from excessive currents. Circuit breakers connected in the supply lines accomplish the same purpose and are to be preferred, for when the circuit is opened by means of the over-load release, the studs of the starting rheostat are burned, becoming rough and irregular.

Fig. 79 is similar to Fig. 78 except the overload release is omitted, and the low-voltage release is connected directly across the supply circuit.

Rheostats for series motors.—The starting rheostats used with series motors are usually of the drum type. Fig. 80. Low-voltage releases are not essential on series motors as an attendant is usually near to open the circuit and return the handle to the "off" position in case the current supply fails. Circuit breakers, which should be connected in the supply leads of each motor, may be depended on to protect the armature from excessive currents.

FIG. 80. Drum Controller. Crocker-Wheeler Co.

The series-parallel system of motor control is used on street cars, which have two or more motors, start and stop frequently, and are often required to run at low speeds. Its advantage is the reduction of the losses in the starting rheostat and a correspondingly increased operating efficiency.

For starting, the two motors are connected in series with each other and with the starting resistance, which makes it possible to use a smaller resistance than would otherwise be required. Fig. 81a. As the armature speed increases, the resistance is reduced as in any other starting system. After all the starting resistance has been cut out of the circuit and the motors have attained their maximum speed with the series connection (one-half the line voltage applied to each motor), the motors are connected in parallel with each other and in series with the starting resistance. Fig. 81b. As the motors speed up further, the external resistance is reduced until each motor is connected directly across the supply lines. With this arrangement it is evident that the motors may be operated at two different speeds without any loss due to an external resistance.

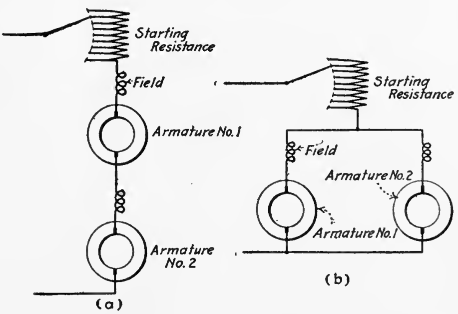


FIG. 81. Series-parallel Motor Control.

All the operations required in the series-parallel system of control are obtained by moving a handle which rotates a spindle having metal fingers or sectors. The same general type and arrangement of rheostat is used for crane and other series motors, although only one motor may be used.

Automatic motor-starting rheostats. — Motor-starting rheostats in which the resistance of the armature circuit is automatically reduced as the speed of the armature increases, have been developed. With such apparatus it is only necessary to close or open a line switch to start or stop the motor.

CHAPTER V — PROBLEMS

1. A current-carrying wire lies in a magnetic field, the intensity of which is 50,000. Find the force acting on the wire when 250 amperes flow.

2. A 4-pole, lap-wound, 220-volt shunt motor has an armature input of 200 amperes when running at 750 r.p.m.

Diameter of armature	15 inches
Length of armature	9 inches
Number of armature conductors	476
Pole enclosure	70%
Armature resistance	0.05
Flux density on pole face (average)	48,000

Find: (*a*) the torque developed in the armature, (*b*) the speed of the motor at no load (no-load armature current = 6.5 amperes), (*c*) the speed regulation, assuming 200 amperes to be rated full load.

3. Find the external resistance which must be connected in series with the armature of the motor in Problem 2 to reduce the speed to 400 r.p.m., the armature current and the field excitation being as given above.

4. Find the speed of the armature of the motor in Problem 2 when connected in series with the resistance required in Problem 3, with 100 amperes flowing in the armature circuit.

5. Calculate the per cent of total power supplied to the motor dissipated in heating the rheostats in: (*a*) Problem 3, (*b*) Problem 4.

6. Find the resistance of the starting rheostat required for the motor in Problem 2, the maximum current during the period of acceleration not to exceed 150 per cent of the rated full-load current.

7. The normal field current of a shunt motor (temperature of shunt field = 60° C.) is 10 amperes. Find the field current when first started, assuming the temperature of the air to be 20° C.

8. Two similar 15 horse-power 220-volt shunt motors are connected in series between 440-volt mains, the armatures of the motors being rigidly connected together. Resistance of each armature = 0.08 ohm. Armature current = 60 amperes. Find the speed at which the armatures rotate when: (*a*) the fields are normally (and equally) excited, (*b*) the field excitations are such that the flux density in the air gap of one motor is 25 per cent greater than that in the air gap of the other.

9. A shunt dynamo when operated as a generator at 1000 r.p.m. delivers 100 amperes (armature) current at a terminal voltage of 220. The resistance of the armature circuit is 0.05 ohm. Calculate the speed at which the armature rotates if operated as a motor, the armature current and the field excitation being the same as in the generator.

10. A 4-pole, wave-wound shunt motor is operated on a 250-volt circuit. Armature slots = 47, conductors per slot = 12, commutator bars = 141, flux = 1,660,000 maxwells per pole. Neglecting armature resistance, find the speed of the motor.

11. The armature of a 6-pole continuous-current motor is 10 inches long and 24 inches in diameter. The face of each pole shoe is 10 inches $\times$ 11 inches. The armature is lap wound and consists of 600 conductors. The average flux density on the face of the poles is 48,000 lines per square inch. Find the force acting on the armature when the total input to the armature is 300 amperes.

12. A 230-volt shunt motor has a no-load input of 30 amperes, and a full-load input of 750 amperes when operated at 400 r.p.m. Armature resistance = 0.0075; field resistance = 46. Find the speed regulation of the motor.

13. Same as Problem 12 except the armature resistance = 0.015.

14. The armature of the motor in Problem 12 is 6-pole, lap wound, and has 540 conductors. Find the flux per pole.

CHAPTER VI

LOSSES, EFFICIENCIES AND RATINGS OF CONTINUOUS-CURRENT DYNAMOS

1. **Losses.** — The losses in a continuous-current dynamo are: (*a*) resistance losses, (*b*) stray power.

(*a*) *Resistance losses.** — The resistance losses of a dynamo are those due to the resistance of: (1) the armature winding, (2) the brush contacts, (3) the field circuit.

(1) *Armature resistance losses.* — Armature resistance losses are not measured directly, but are calculated from the resistance of the armature winding and the given or required value of armature current.

$$P_a = R_a I_a^2 \text{ watts,} \tag{1}$$

when R_a = the resistance of the armature winding,
I_a = the armature current for which the loss is to be calculated.

Connections for the determination of the resistance of the armature winding are shown in Fig. 82. After opening the field circuit so that the armature will not rotate, the armature is connected to supply mains in series with a water rheostat or other suitable resistance, by means of which the current in the armature circuit may be controlled. The current in the armature circuit and the electromotive force between brush contacts are measured by means of the ammeter and the voltmeter.

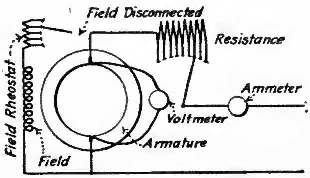


FIG. 82. Connections for Determination of Armature Resistance.

Then
$$R_a = \frac{E}{I}, \tag{2}$$

when R_a = the resistance of the armature winding,
E = the indication of the voltmeter,
I = the current flowing in the armature circuit.

* Resistance losses should be calculated for a working temperature of 75° C.

(2) *Brush-contact losses.* — As indicated in Chapter 4, Section 2, the so-called brush-contact resistance is not constant, but is a function of the current density in the contact area, and the drop per pair of carbon brushes is calculated from the formula,

$$E_b = 0.8 + 0.2\,D, \tag{3}$$

when D = the current density (amperes per square centimeter) in the contact area between the brush and the commutator.

Therefore the loss due to brush-contact resistance is

$$P_b = (0.8 + 0.2\,D)I_a \text{ watts.} \tag{4}$$

(3) *Field resistance losses.* — The resistance of a series-field winding is small and should be determined in the same way as that of an armature winding. The copper loss may then be calculated.

$$P_s = R_s I_s^2 \text{ watts,} \tag{5}$$

when R_s = the resistance of the series-field circuit,
I_s = the current flowing in the series-field circuit.

The resistance of a shunt-field circuit is comparatively large and the winding may be connected directly to the supply mains, provided the voltage of the line is not materially greater than the rated voltage of the dynamo. The losses in the shunt-field circuit, including the rheostat, may be determined by direct measurement, or by calculation.

$$P_f = EI_f \text{ watts} \tag{6}$$

$$= R_f I_f^2 \text{ watts} \tag{7}$$

$$= \frac{E^2}{R_f} \text{ watts} \tag{8}$$

when E = the voltage between the terminals of the field circuit,
I_f = the current flowing in the field circuit,
R_f = the resistance of the field circuit.

The instruments required for the determination of either the field loss or the resistance of the circuit, are an ammeter and a voltmeter.

(*b*) *Stray power.* — The stray power of a dynamo includes all the losses not specified in (*a*), and consists of: (1) iron losses, (2) frictional losses.

(1) *Iron losses.* — The iron losses of a dynamo are due to hysteresis and eddy currents, and are dependent on the speed at which the armature rotates, and on the flux density in the armature core.

Hysteresis. — Since the armature rotates in a magnetic field, the magnetism of the iron core is periodically reversed, and this rearrangement of the molecular structure of the iron requires an expenditure of energy which heats the iron. The energy required to reverse the magnetic polarity of any volume of iron has been found, experimentally, to be approximately proportional to the 1.6 power * of the maximum flux density in the iron. The power lost by hysteresis is, then, proportional to the 1.6 power of the flux density in the armature core, and to the number of magnetic reversals per unit of time, *i.e.*, to the speed of the armature.

$$P_h = k_h n B^{1.6} \text{ watts,}\dagger \tag{9}$$

when k_h = a constant dependent on the volume and the magnetic quality of the iron in the armature core, and on the number of poles on the dynamo,
n = the speed of the armature,
B = the flux density in the iron.

Eddy currents. — When the armature rotates in the magnetic field set up by the field windings, electromotive forces are induced which

* See Appendix B, Section 7.

† Professor Sheldon gives the following formulæ for the calculation of hysteresis and eddy-current losses in iron:

$$P_h = 8.3\,\eta f V B^{1.6}\,10^{-8} \text{ watts,} \tag{9a}$$

$$P_e = 4.07\,V\,(flB)^2\,10^{-17} \text{ watts,} \tag{12a}$$

when η = the magnetic (hysteretic) constant,
f = the number of magnetic reversals per second

$$\left(= \frac{\text{number of poles} \times \text{r.p.s.}}{2}\right),$$

l = thickness of laminations (in mils),
V = the volume of iron (in cubic inches),
B = the flux density in the iron (maxwells per square inch).

Because the laminations are imperfectly insulated from each other and the flux distribution in the teeth and core is not uniform, the calculation of hysteresis and eddy-current losses in armature cores is only a rough approximation. Measured values may be found to be several times the calculated values.

cause currents to circulate in the body of the core. Fig. 83. These currents cause the core to heat, and the energy lost by reason of their circulation is, by Joule's Law, proportional to the product of the square of the current and the resistance of the paths through which they flow.

Referring to Fig. 83, let

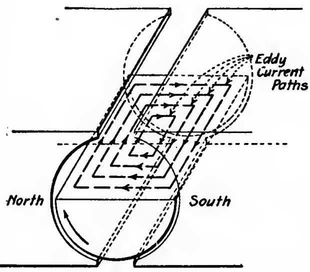


FIG. 83. Eddy Currents in Solid Armature Core.

n = the speed of the armature (revolutions per second),

B = the flux density of the magnetic field,

l = the thickness of the laminations of which the armature is built,

r = the radial distance of an element parallel to the axis of the armature core, from the axis of the armature.

k and k_e = constants proportional to the volume and the electrical quality of the iron in the armature core, and to the number of poles on the dynamo.

The electromotive force generated in an element of the lamination due to the rotation of the core in the magnetic field is

$$e = 2\,\pi r l n B\ 10^{-8} \text{ volts.} \tag{10}$$

If the resistance of the eddy current path is constant,* the losses due to the circulation of the eddy current in the iron are proportional to the square of the induced electromotive force.

$$P_e = ke^2 \tag{11}$$

$$= k_e l^2 n^2 B^2 \text{ watts,}† \tag{12}$$

* An examination of the eddy-current path shows that the length of the path does not vary appreciably for the different thicknesses of laminations used in commercial dynamos.

† Professor Sheldon gives the following formulæ for the calculation of hysteresis and eddy-current losses in iron:

$$P_h = 8.3\,\eta f V B^{1.6}\ 10^{-8} \text{ watts,} \tag{9a}$$

$$P_e = 4.07\ V\,(flB)^2\ 10^{-17} \text{ watts,} \tag{12a}$$

when η = the magnetic (hysteretic) constant,

f = the number of magnetic reversals per second

$$\left(= \frac{\text{number of poles} \times \text{r.p.s.}}{2}\right)$$

i.e., eddy-current losses are proportional to the square of the thickness of the laminations, to the square of the armature speed, and to the square of the density of the magnetic field.

In dynamos having toothed armatures, the flux density on the pole face is not uniform but is greater opposite a tooth than opposite a slot. As the armature rotates, each element of the pole face parallel to the axis of the armature shaft has an electromotive force induced in it and eddy-currents circulate in the iron. Pole-face losses are reduced by laminating the poles and by making the ratio $\frac{\text{width of slot}}{\text{length of air gap}}$ small.

(2) *Frictional losses.* — The frictional losses of a dynamo are those between the shaft and the bearings, between the brushes and the commutator, and between the air and the rotating parts.

Bearing friction and windage are seldom or never separated, but in a well-designed machine, the windage is small and may be neglected if the peripheral speed of the armature does not exceed 6000 feet per minute. This speed is seldom exceeded in continuous current machines unless they are to be direct connected to steam turbines. For bearings having ring lubrication and using light machine oil, a film of oil always separates the bushing and the shaft, and bearing friction may be assumed to be fluid friction. With this assumption, the loss due to bearing friction is calculated by means of the following empirical formula, which gives close approximations for the usual range of armature speeds.

$$P_b = 0.81\,DL\left(\frac{V}{100}\right)^{\frac{3}{2}} \text{watts}, \tag{13}$$

when D = the diameter of the bearing (in inches),
L = the length of the bearing (in inches),
V = the velocity of the rubbing surface (in feet per minute).

The above formula is independent of pressure and, therefore, of the load on the dynamo. Bearing friction, in belted machines, may

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l = thickness of laminations (in mils),
V = the volume of iron (in cubic inches),
B = the flux density in the iron (maxwells per square inch).

Because the laminations are imperfectly insulated from each other, and the flux distribution in the teeth and core is not uniform, the calculation of hysteresis and eddy-current losses in armature cores is only a rough approximation. Measured values may be found to be several times the calculated values.

vary over wide limits, depending on the tension of the belt, but the losses due to belt tension are hard to estimate and are usually neglected in making efficiency calculations.

Brush friction depends on the force with which the brushes press against the commutator, and usually amounts to only a small part of the losses in a dynamo. A pressure of one and one-half to two pounds is an average brush pressure. For constant pressure between the brushes and the commutator, the losses due to brush friction are proportional to the peripheral velocity of the commutator and may be calculated by the following empirical formula:

$$P_c = \frac{AV}{80} \text{ watts,} \tag{14}$$

when A = the area (in square inches) of the brushes in contact with the commutator,

V = the peripheral velocity (in feet per minute) of the commutator.

From the above considerations it is evident that no simple equation is available for the calculation of stray power. These losses are, therefore, determined experimentally by running the dynamo under the required conditions of speed and field excitation. While the separation of stray power into its components is not a difficult problem, it is one which is primarily of interest to the designer and the manufacturer only. The operating engineer is interested in obtaining high efficiency and good operating characteristics.

2. Experimental determination of stray power. — The stray power of a dynamo is easily determined by: (*a*) running it as a motor, (*b*) driving it as a generator by means of an auxiliary motor.

(*a*) *Running as a motor.* — When a dynamo is operated as a motor without load, the input is just sufficient to supply the losses — field copper loss, armature copper loss, brush-contact resistance loss and stray power. The resistance losses are readily determined as indicated above, and the stray power is the difference between the total input (the product of applied voltage and line current) and the resistance losses.

$$\text{Stray power} = \text{motor input} - \text{resistance losses.} \tag{15}$$

The field loss may be eliminated by placing the ammeter in the armature circuit instead of in the line. In this case the stray

power is the difference between the armature input (the product of applied voltage and armature current) and the losses due to brush contact and armature resistance. (The loss in the series-field winding of a series or compound dynamo is eliminated by separately exciting the series field.)

$$\text{Stray power} = EI_a - R_aI_a^2 - I_a(0.8 + 0.2\,D), \tag{16}$$

when E = the applied electromotive force,
I_a = the current in the armature circuit,
R_a = the resistance of the armature circuit,
D = current density in the brushes (amperes per square centimeter).

Connections for the determination of stray power in a shunt dynamo are shown in Fig. 84. A resistance of the proper size, considering the value of the current taken by the armature, is connected in series with the armature winding. The field excitation is regulated by means of the rheostat connected in series with the windings,* and the speed controlled by manipulating the resistance in the armature circuit. With this arrangement either the speed or the field excitation may be given any desired value within the operating limits of the dynamo.

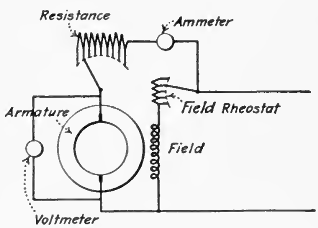


FIG. 84. Determination of Stray Power.

(*b*) *Driving by an auxiliary motor.* — It is sometimes impossible to run a dynamo as a motor under the required conditions of speed and field excitation, as when a 440-volt dynamo is to be tested and the only available source of current supply is 110 volts. Under these conditions a small 110-volt motor is connected to the dynamo to be tested, and the input to the auxiliary motor determined when driving the dynamo at the required speed and with any required excitation. Fig. 85. The losses in the auxiliary motor,

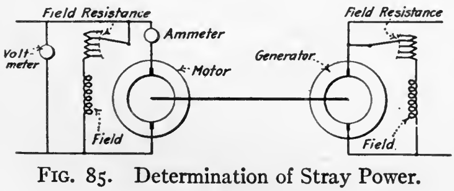


FIG. 85. Determination of Stray Power.

* In the compound dynamo each winding should be properly excited.

if not known, should be determined as in (*a*) above, and the stray power of the dynamo under test calculated.

Stray power = motor input − motor losses − resistance losses. (17)

The only current flowing in the armature of the dynamo under test is the small current required for the excitation of its shunt-field windings, and the resistance losses in the armature are usually so small that no correction need be made for them. Series-field windings may be excited from the low-voltage mains.

3. Separation of friction and iron losses. — It is evident that the friction loss of a dynamo may be determined by driving it, without field excitation, by means of an auxiliary motor. Fig. 85.

Frictional losses = motor input − motor losses. (18)

Stray power may be separated into its components in the following manner: Connect the dynamo as in Fig. 84. Keeping the speed constant, determine the stray power for different field excitations

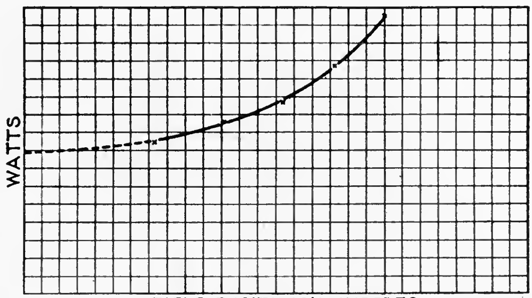


FIG. 86. Separation of Friction and Iron Losses.

and plot as in Fig. 86. Extrapolate the curve to its intersection with the axis of ordinates. The ordinate, at this intersection, represents the loss due to friction at the given speed.

4. Separation of hysteresis and eddy-current losses. — From equations (9) and (12) the iron loss for any given field excitation is

$$P_i = k_h'n + k_e'n^2. \tag{19}$$

Dividing equation (19) by n

$$\frac{P_i}{n} = k_h' + k_e'n, \tag{20}$$

which is the equation of a straight line, and may be plotted as in Fig. 87, after the iron losses at different speeds but constant field excitation have been determined. The hysteresis loss at any given or required speed n is equal to the product of the ordinate k_h' and the speed n; the eddy current loss for any given or required speed n is equal to the product of the speed and the ordinate $k_e'n$ corresponding to the given or required speed.

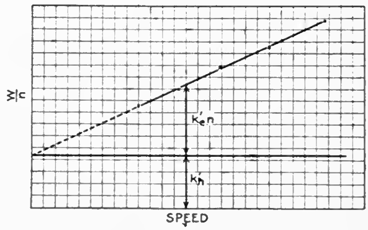


FIG. 87. Separation of Hysteresis and Eddy Current Losses.

5. **Efficiencies.** — The efficiency of an electrical machine is the ratio of the output to the input.

$$\text{Per cent efficiency} = \frac{\text{output} \times 100}{\text{input}}. \tag{21}$$

The efficiency of a dynamo is usually computed from stray power and resistance measurements rather than from an actual load test for the following reasons:

(*a*) A considerable quantity of energy must be wasted in making a load test.

(*b*) It is often inconvenient or impossible to supply the electrical energy required to run a large dynamo as a motor or to absorb its output when operated as a generator.

(*c*) Calculated efficiencies are more reliable than those determined by load test.

The equation for the efficiency of a generator, then, becomes

$$\text{per cent efficiency} = \frac{\text{output} \times 100}{\text{output} + \text{losses}}; \tag{22}$$

and that for a motor

$$\text{per cent efficiency} = \frac{(\text{input} - \text{losses})\ 100}{\text{input}}. \tag{23}$$

The efficiencies of a dynamo are represented graphically by means of an efficiency curve, per cent efficiency being used as ordinates and per cent of full (rated) load as abscissas.

The shunt dynamo. — Very simple measurements are sufficient for the calculation of the efficiency of a shunt dynamo. The copper loss in the armature is proportional to the square of the current

flowing in the armature circuit and may be calculated for any given or required current after the resistance of the armature winding has been determined as explained above.

The field loss is approximately constant, the change in value due to the decrease in field * current as the load on a generator increases is usually neglected, and is determined by measuring the current in the field circuit and the voltage across its terminals when the excitation is such as to give rated voltage when operated as a generator, or rated speed when operated as a motor.

The stray power of a shunt dynamo is approximately constant, and the no-load determination is used in the calculation of efficiencies. The dynamo is operated as a motor, without load and at rated speed, the input to the armature measured, and the calculated armature and brush contact resistance losses subtracted, as explained above. The armature and brush-contact resistance losses are small at no-load and may usually be disregarded without serious error.

The fact that the stray power of a shunt dynamo is not constant will be appreciated from the following:

In a shunt generator operating at constant speed, the field current falls off as the load increases. Therefore, the flux in the magnetic circuit is reduced, and the armature current changes the distribution of the flux in the air gap. In the shunt motor, the speed falls off and armature reaction distorts and reduces the field.

Decreased field excitation, decreased speed, and armature demagnetization cause the iron losses of a dynamo to decrease; field distortion causes the iron losses to increase. These two effects, therefore, tend to neutralize each other. No accurate and easily applied method for determining the change in the stray power of a dynamo as the load changes has been devised and, since it is usually small, no correction need be attempted.

By differentiating the equation for the efficiency of a shunt dynamo it may be shown that the efficiency is a maximum when the constant losses equal the variable losses, *i.e.*, the efficiency is a maximum when the armature resistance losses are equal to the sum of the field resistance loss and the stray power.

The series dynamo. — Since the field excitation of a series generator, and the field excitation and the speed of a series motor vary

* If the terminal voltage of a shunt generator remains constant, the field current must be increased as the load increases.

over wide limits, the stray power of a series dynamo also varies greatly. Therefore, the stray power must be determined for the particular speed and field excitation for which it is desired to calculate the efficiency. By separately exciting the field and regulating the voltage applied to the terminals of the armature, the stray power of the dynamo for any speed and field excitation may be determined.

Since the same current flows in the field as in the armature of a series dynamo, the copper losses for any load are equal to the square of the line current multiplied by the combined resistance, in series, of the two windings. The losses due to brush-contact resistance should be calculated as for a shunt dynamo.

The compound dynamo. — Since the voltage of a compound generator and the speed of a compound motor may vary widely from their no-load values, the efficiency may be accurately calculated only by determining the losses at the voltage, or the speed, at which the dynamo operates when carrying the specified load.

The approximate efficiency of a compound dynamo obtained by assuming the shunt-field loss and the stray power constant, is often sufficiently accurate, particularly if the dynamo is flat or only slightly overcompounded.

As a generator. — Given the voltage characteristic of a compound generator, very close approximations of the losses at different loads may be made from no-load measurements. Knowing the terminal voltage at any condition of loading and the resistance of the shunt-field circuit, the shunt-field loss is readily calculated. The resistance losses in the series field and in the armature are calculated in the same way as for the series dynamo.

By separately exciting the series-field winding, the stray power of a compound generator may be readily determined for any required speed and field excitation. If the stray power is determined for different excitations, but constant speed, a curve may be plotted, using stray power as ordinates and per cent of rated load as abscissa. From this curve the stray power at any load is obtained.

As a motor. — When the long shunt compound dynamo is operated as a motor from constant potential mains, the shunt-field loss is constant and the resistance loss in the armature and that in the series-field circuit may be calculated from the resistances of the windings and the armature current. The shunt-field loss in a short-

shunt compound motor, operating from constant potential mains, decreases slightly as the armature current increases because of the drop in the series-field winding which reduces the voltage between the terminals of the shunt-field circuit. Stray power should be determined for speeds corresponding to different field excitations, and a curve plotted as for a generator.

6. **Ratings.** — Aside from the greatly increased copper losses in a dynamo which materially reduce its efficiency when excessive current flows in the armature windings, the output is limited by: (*a*) heating, (*b*) sparking, (*c*) voltage considerations.

(*a*) *Heating.* — According to Joule's Law the heat liberated in a current-carrying conductor is proportional to the square of the current flowing in the circuit. The heat due to hysteresis and eddy currents also tends to raise the temperature of the armature.

Insulations used on commercial dynamos will not stand an indefinite rise in temperature without injury to the insulating qualities. The output, then, must be kept below the value which causes the insulation of the conductors to be injured. The generally recognized safe limit of temperature is 65 to 70 degrees (Centigrade).* If the atmospheric temperature is taken as 25 degrees (an average value), the allowable temperature rise is 40 to 45 degrees.

(*b*) *Sparking.* — Excessive armature currents, by reason of their reaction on the magnetic field, cause sparking at the brushes which burns and ultimately destroys the commutator, and thus limits the output of a given dynamo.

(*c*) *Voltage limitations.* — An examination of the magnetization curve of iron shows that the flux in a given magnetic circuit cannot be increased beyond a fixed value (magnetic saturation), without increasing the exciting ampere-turns beyond the economical limits. The alternative is to increase the speed, but this is limited, by mechanical considerations, to a peripheral velocity of about six thousand feet per minute.

Commutation difficulties increase as the voltage increases, and the commutation of voltages above 1200 to 1500 is practicable only on large sized machines.

* This temperature is that determined by a thermometer, and is from 5° to 15° less than the maximum internal temperature.

CHAPTER VI — PROBLEMS

1. A 500-kw., 550-volt generator has a field current of 10 amperes, an armature resistance of 0.01 ohm, and a stray power of 7500 watts. Find: (*a*) the efficiency at full (rated) load, (*b*) the maximum efficiency, (*c*) the output at maximum efficiency.

2. The dynamo in Problem 1 is run as a motor. Find: (*a*) the horse-power output when the current input to the armature is 750 amperes, (*b*) the current input to the armature when the horse-power output is 500.

Note. — Output $= EI_a - R_aI_a^2 - S - E_bI_a$.

3. Calculate the stray power of a 220-volt shunt motor when running at 800 r.p.m., the current input being 15 amperes, the resistance of the field circuit 50 ohms, the resistance of the armature circuit 0.06 ohm, the applied voltage 200 and the output zero.

4. Calculate the efficiencies of the motor in Problem 3 for the following armature currents: 25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 275 and 300. Consider the field loss and the stray-power constant.

5. Plot the efficiency curve from the values calculated in Problem 4, and from the curve determine: (*a*) the maximum efficiency, (*b*) the horse-power output at maximum efficiency.

6. The armature resistance of a 25-horse-power, 230-volt shunt motor is 0.04 ohm, and its stray power is equal to 5 per cent of its rated output. Find the current input to the armature: (*a*) at full (rated) load, (*b*) at one-half full load.

7. The rated current input to a 230-volt shunt motor is 185. Field resistance = 140. Armature resistance = 0.08. No-load speed (with rated voltage and normal field excitation) = 785. Current input at no load = 8. Find: (*a*) stray power, (*b*) field loss, (*c*) armature copper loss at rated load, (*d*) rated horse-power output, (*e*) efficiency at full load, (*f*) speed regulation.

8. A 220-volt shunt motor has a stray power of 1000 watts and an armature resistance of 0.1 ohm. At full load the armature current is 150 amperes and the speed 900 r.p.m. Find: (*a*) the no-load speed, (*b*) the full-load output in horse-power, (*c*) the armature copper loss at full load, (*d*) the armature current at no load.

9. A 230-volt shunt motor delivers 50 brake horse power when the current input is 185 amperes. The resistance of the armature = 0.06 ohm; the resistance of the field circuit = 160 ohms. Find the stray power.

10. A 220-volt, 40-horse-power shunt motor has an armature resistance of 0.11 ohm. The current input to the armature when operating at a speed of 800 r.p.m. and delivering the rated horse-power output is 156 amperes. Shunt-field current = 4 amperes. Find: (*a*) the efficiency at full load, (*b*) armature current at half load (20-h.p. output), (*c*) maximum efficiency, (*d*) output at maximum efficiency, (*e*) speed at no load.

11. A shunt dynamo, when operated as a generator at 800 r.p.m., delivers 11 kw., the terminal voltage is 110, and the field current is 2 amperes. Armature resistance = 0.05 ohm. When the same machine is operated as a motor, the

input is 102 amperes at 110 volts, and the output is 12.8 horse power. Find: (*a*) the counter-electromotive force of the motor, (*b*) the speed of the motor, (*c*) the stray power: (1) of the generator, (2) of the motor.

12. A 240-volt, 25-horse-power shunt motor is rated to run at 750 r.p.m. Armature resistance = 0.125 ohm. Field resistance = 160 ohms. Stray power (determined at no load) = 500 watts. Find: (*a*) speed at no load, (*b*) efficiency at full load, (*c*) armature current at maximum efficiency.

13. The no-load input to a 220-volt shunt motor is 1.5 kw. Armature resistance = 0.08 ohm. Field resistance = 100 ohms. Find: (*a*) the stray power, (*b*) the field loss, (*c*) horse-power output at maximum efficiency, (*d*) speed at maximum efficiency.

14. A 220-volt shunt motor is rated at 60 horse power. The maximum efficiency is 90 %, and is obtained when the output of the motor is 50 horse power. Resistance of the field circuit = 100 ohms. Find: (*a*) stray power, (*b*) resistance of the armature winding, (*c*) full load of efficiency.

CHAPTER VII

POLYPHASE ALTERNATING CURRENTS

1. **Limitations of the single-phase system.** — The single-phase system of electrical distribution has been largely displaced for the following reasons:

(*a*) Polyphase systems are more economical in the use of copper, *i.e.*, a given power may be transmitted over a smaller wire in the polyphase system.

(*b*) Single-phase induction motors are not self-starting without auxiliary apparatus which adds to their mechanical complications and increases their cost.

(*c*) The power in a single-phase system is pulsating, that in a polyphase system is constant.

Commercial polyphase systems consist of two or more single-phase circuits in which the electromotive forces or currents are out of phase. The single-phase circuits composing a polyphase system may be independent of each other, or they may be mechanically and electrically interconnected.

2. **The two-phase system.** — The two-phase system, also called four-phase and quarter-phase, consists of two single-phase circuits, the electromotive forces or currents of which are 90 degrees out of phase. The two single-phase circuits may be: (*a*) independent, (*b*) interconnected to form a three-wire system, (*c*) star connected, (*d*) mesh connected.

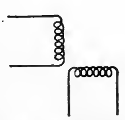


FIG. 88. Two-phase System. Phases Independent.

(*a*) *Independent phases.* — A two-phase system, the phases of which are independent, is shown in Fig. 88. Evidently, the power and the current in each phase is dependent only on the impedance of its load circuit, and each phase may be treated as an independent single-phase system.

(*b*) *Three-wire system.* — A three-wire two-phase system is shown, diagramatically, in Fig. 89, one terminal of each phase being con-

nected to a common wire. This system is little used because the voltages between wires are not equal and the current, for an equal load on each phase, is greater in the common wire than in each of the other two.

If the voltage between the terminals of each phase winding is E (= AB = BC), the values and phase relations are represented

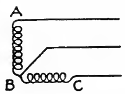


FIG. 89. Two-phase Three-wire System.

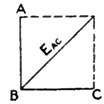


FIG. 90. Vector Diagram of Two-phase Three-wire System

in a vector diagram, and the electromotive force between A and C is the geometric sum of the phase voltages. Fig. 90.

$$E_{AC} = \sqrt{(AB)^2 + (BC)^2}. \tag{1}$$

Similarly the currents in A and C may be represented in a vector diagram, and the current in B shown to be their geometric sum.

$$I_B = \sqrt{I_A^2 + I_C^2}. \tag{2}$$

From the above it is evident that neither the mechanical nor the electrical relations are symmetrical.

(*c*) *Star connection.* — A star-connected two-phase system is shown in Fig. 91. This system is symmetrical, both mechanically

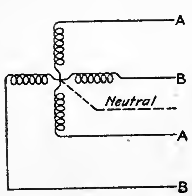


FIG. 91. Two-phase Star-connected System.

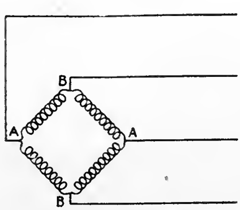


FIG. 92. Two-phase Mesh-connected System.

and electrically. The wires are all of the same size, the voltage between alternate wires is E, and that between adjacent wires is $\frac{E}{\sqrt{2}}$.

Each pair of wires, AA or BB in Fig. 91, carries the current due to the load on that phase just as if the two phases were not interconnected.

(*d*) *Mesh connection.* — The mesh-connected two-phase system is shown in Fig. 92. The mechanical and the electrical relations are obvious from the figure.

Two-phase apparatus has not attained great popularity because three-phase apparatus offers greater advantages.

3. **The balanced three-phase system.** — The three-phase system is composed of three interconnected single-phase circuits. The three component circuits may be connected: (*a*) star or wye, (*b*) delta or mesh, (*c*) open delta or V, (*d*) tee.

(*a*) *Star connection.* — If the three independent circuits shown in Fig. 93 are so related that the current in A attains its maximum positive value one-third of a cycle before the current in B, and one-

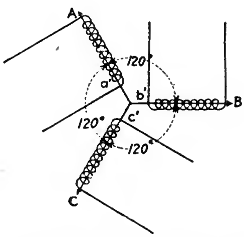


FIG. 93. Component Circuits of a Three-phase System.

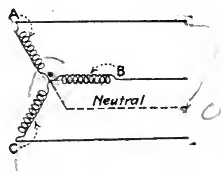


FIG. 94. Three-phase Star-connected System.

third of a cycle after the current in C, the algebraic sum of the currents in a', b' and c' is, at any given instant, zero and the terminals of coils A, B and C may be connected together as shown in Fig. 94, in which the lines a', b' and c' are omitted. The maximum values of current are assumed to be equal.

This connection is known as star or wye, and the junction of the three coils is the neutral point (or simply the neutral). A fourth wire is sometimes run between the neutral point of a generator and that of its load, but it carries little current unless unusual conditions exist in the system. A neutral wire, when run, is always smaller in cross section than are the mains. A grounding of the neutral points is usually all that is required, the earth acting as the neutral conductor.

Referring to Fig. 94, let the electrical relations of coils A, B and C be such that

$$i_A = I_m \sin \omega t, \tag{3}$$

$$i_B = I_m \sin (\omega t - 120°), \tag{4}$$

$$i_C = I_m \sin (\omega t - 240°), \tag{5}$$

$$e_A = E_m \sin \omega t, \tag{6}$$

$$e_B = E_m \sin (\omega t - 120°), \tag{7}$$

$$e_C = E_m \sin (\omega t - 240°). \tag{8}$$

Then $$i_A + i_B + i_C = 0, \tag{9}$$

i.e., the instantaneous currents in a star-connected system are positive or negative as they flow away from or toward the neutral, and the sum of the positive values is always equal to the sum of the negative values.

From equations (6), (7) and (8), the line-to-line voltage of a star-connected system is

$$e_l = E_m \sin \omega t - E_m \sin (\omega t \mp 120^\circ); \tag{10}$$

the voltage between the line terminals of coils A and B is maximum when $\omega t = 60^\circ$ or 240°; the voltage between the line terminals of coils A and C is maximum when $\omega t = 120^\circ$ or 300°; and the voltage between the line terminals of coils B and C is maximum when $\omega t = 180^\circ$ or 0°.

Therefore,

$$(E_l)_m = (E_p)_m \sin 60^\circ - (E_p)_m \sin 300^\circ. \tag{11}$$
$$= 0.866\,(E_p)_m + 0.866\,(E_p)_m \tag{12}$$
$$= 1.732\,(E_p)_m \tag{13}$$
$$= \sqrt{3}\,(E_p)_m \tag{14}$$

and $$E_l = \sqrt{3}\,E_p. \tag{15}$$

i.e., the effective line-to-line voltage of a balanced star-connected system is equal to the effective phase voltage multiplied by the square root of 3.

It is evident, from the connection, that the same current flows in the coil of a star-connected system as flows in the line to which the coil is connected.

Since the line current and the phase electromotive force of a balanced star-connected system supplying a non-reactive load cir-

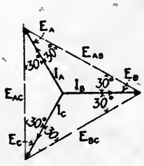


FIG. 95. Vector Diagram of Star-connected System.

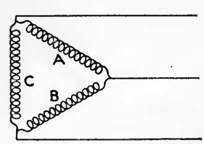


FIG. 96. Three-phase Delta-connected System.

cuit attain their maximum values at the same instant, it follows that the line current and the line-to-line voltage are out of phase by 30 degrees, as shown in Fig. 95.

(*b*) *Delta connection.* — Referring to Fig. 96, let the electrical relations of coils A, B and C be such that

$$e_A = E_m \sin \omega t, \tag{16}$$

$$e_B = E_m \sin (\omega t - 120^\circ), \tag{17}$$

$$e_C = E_m \sin (\omega t - 240^\circ), \tag{18}$$

$$i_A = I_m \sin \omega t, \tag{19}$$

$$i_B = I_m \sin (\omega t - 120^\circ), \tag{20}$$

$$i_C = I_m \sin (\omega t - 240^\circ), \tag{21}$$

Then
$$e_A + e_B + e_C = 0, \tag{22}$$

i.e., the instantaneous electromotive forces in a delta-connected system are negative or positive as they tend to cause a current to flow in a clockwise or in a counter-clockwise direction around the circuit formed by the three coils, and the sum of the positive electromotive forces is always equal to the sum of the negative electromotive forces.

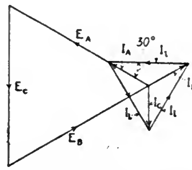


FIG. 97. Vector Diagram of Delta-connected System.

In a delta-connected system the voltage between lines is the voltage between the terminals of the coil, because the terminals of the coil are connected directly to the lines.

From equations (19), (20) and (21), the line current in a delta-connected system is

$$i_l = I_m \sin \omega t - I_m \sin (\omega t \mp 120^\circ); \tag{23}$$

the current in the line to which coils A and B are connected is maximum when $\omega t = 60^\circ$ or 240°; the current in the line to which coils A and C are connected is maximum when $\omega t = 120^\circ$ or 300°; and the current in the line to which coils B and C are connected is maximum when $\omega t = 180^\circ$ or 0°.

Therefore,

$$(I_l)_m = (I_p)_m \sin 60^\circ - (I_p)_m \sin 300^\circ \tag{24}$$

$$= 0.866\, (I_p)_m + 0.866\, (I_p)_m \tag{25}$$

$$= 1.732\, (I_p)_m \tag{26}$$

$$= \sqrt{3}\, (I_p)_m \tag{27}$$

and
$$I_l = \sqrt{3}\, I_p. \tag{28}$$

i.e., the effective line current in a balanced delta-connected system is equal to the effective current in each winding multiplied by the square root of 3.

Since the line-to-line voltage and the phase current in a balanced delta-connected system supplying a non-reactive load circuit attain their maximum values at the same instant, it follows that the line current and the line-to-line voltage are 30 degrees out of phase.

(*c*) *V-connection.* — With the open-delta or V-connection, a three-phase system is obtained by the use of only two windings. Fig. 98. The connection is essentially that of the delta with one coil omitted.

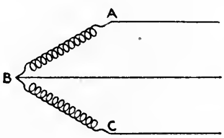


FIG. 98. Three-phase V-connection.

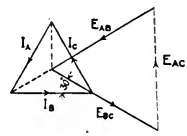


FIG. 99. Vector Diagram of V-connection.

Obviously, the line-to-line voltage is that of the winding, and the line current must flow in the winding. The vector diagram for a V-connection is shown in Fig. 99.

Let the load on a delta-connected system be 300 kw. at unity power factor. It is obvious that each winding carries one-third of the total load or 100 kw. If the same load is carried by an open-delta system, each winding must carry one-half the total or 150 kw., but the current and the electromotive force in the windings are 30 degrees out of phase and the current-carrying capacity (rating) of each open-delta winding must be 1.73 times the current-carrying capacity of each delta winding. The weight of copper required for an open-delta connection is, therefore, 15 per cent greater than that required for a delta connection.

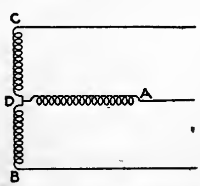


FIG. 100. Three-phase T-connection.

(*d*) *T-connection.* — A three-phase system is obtained if two coils, the electromotive forces in which are in quadrature, are connected as indicated in Fig. 100.

Referring to Fig. 100, let

$$E_{AB} = E_{BC} = E_{CA} \tag{29}$$

and

$$E_{CD} = E_{BD} = \frac{E_{BC}}{2}. \tag{30}$$

The voltages E_{AB}, E_{BD} and E_{AD} are, by construction, the hypothenuse, the base and the altitude of a right triangle. Fig. 101. Therefore,

$$E_{AD} = \sqrt{E_{AB}^2 - \frac{E_{BC}^2}{4}} \tag{31}$$

$$= 0.866\, E_{AB}, \tag{32}$$

i.e., a three-phase system having equal voltages between lines is produced by two windings connected as in Fig. 100, if the electromotive forces in the windings are in quadrature and that in AD is 0.866 times that in BC.

Let the current in winding AD be in phase with the voltage E_{AD}. Then

$$i_A = I_m \sin \omega t, \tag{33}$$

$$i_B = I_m \sin (\omega t - 120^\circ), \tag{34}$$

$$i_C = I_m \sin (\omega t - 240^\circ). \tag{35}$$

But

$$e_{BC} = (E_{BC})_m \sin (\omega t - 90^\circ). \tag{36}$$

Therefore, at unity power factor, the current in one-half of the winding BC leads the electromotive force, and that in the other half lags behind the electromotive force, as shown in Fig. 101.

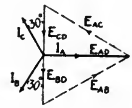


FIG. 101. Vector Diagram of T-connection.

4. Comparison of star and delta connections.—The star-connected three-phase system has the following advantages over the delta-connected system:

(*a*) The availability of a neutral point to which a ground wire, a meter connection or a load may be connected.

(*b*) Circulating currents cannot flow in the windings of a star-connected system.

(*c*) For a given line-to-line voltage, the number of turns required in a star-connected armature is only 58 per cent of the number required in a delta-connected armature.

(*d*) The electromotive-force wave of a star-connected armature winding is more nearly harmonic than that of a delta-connected winding.*

* A non-harmonic electromotive force wave may be resolved into a fundamental sine wave and the odd harmonics of this fundamental. By reason of their phase relations, some of these harmonics, particularly the third, cause currents to flow in the circuit formed by delta-connected coils, but neutralize each other in star-connected coils and do not appear in the line-to-line voltage wave.

5. Unbalanced three-phase system. — When the load on a three-phase system is unequally divided between the three phases, the system is said to be unbalanced, and the currents flowing in the different lines are no longer equal. The voltages between lines are also unbalanced to a slight degree. When the line currents are unequal, the algebraic sum of the instantaneous line currents may not be zero and an equalizing current, I_N in Fig. 102, flows in the neutral of a star-connected system, or distorts the current triangle

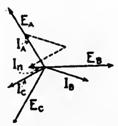


FIG. 102. Vector Diagram of Unbalanced Star-connected System.

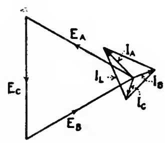


FIG. 103. Vector Diagram of Unbalanced Delta-connected System

of a delta-connected system, as indicated in Fig. 103. If the star-connected system is not provided with a neutral connection, the current relations are still further distorted.

6. Power factor of a polyphase system. — The term "power factor," when applied to a polyphase system, can have no rational meaning since each component circuit may have a different power factor.

7. Power in a polyphase system. — It has been shown* that the power in a single-phase alternating-current circuit is equal to the product of the electromotive force, the current and the cosine of the angle of phase difference.

$$P_1 = E_pI_p \cos \phi. \tag{37}$$

The power in a balanced two-phase system is, therefore, twice that in one of the component circuits,

$$P_2 = 2\, E_pI_p \cos \phi, \tag{38}$$

and that in a balanced three-phase system is three times that in one of the component circuits.

$$W_3 = 3\, E_pI_p \cos \phi, \tag{39}$$

when E_p = the phase voltage,
I_p = the phase current,
$\cos \phi$ = the power factor of each component circuit.

* See Chapter 1, Section 28.

The line quantities of a polyphase system are often more easily measured than the phase quantities, and it is convenient to have a power equation into which the line quantities may be substituted. From equation (15), the line-to-line voltage of a star-connected system is equal to $\sqrt{3}$ times the phase voltage; from equation (28), the line current in a delta-connected system is $\sqrt{3}$ times the phase current. Therefore,

$$P_3 = \sqrt{3}\, E_l I_l \cos\phi, \tag{40}$$

when

$$E_l = \text{the line-to-line voltage,}$$
$$I_l = \text{the line current,}$$
$$\cos\phi = \text{the power factor of each component circuit.}$$

8. Constancy of power in a balanced polyphase system. — One of the advantages of the polyphase system, when compared with the single-phase system, is its constancy of power, *i.e.*, the torque of a single-phase motor is pulsating; that of a polyphase motor is constant.

Let the electromotive force and current relations in one phase of a two-phase system be such that

$$e' = E_m \sin\omega t, \tag{41}$$
$$i' = I_m \sin(\omega t - \phi). \tag{42}$$

Then the relations in the second phase are

$$e'' = E_m \sin(\omega t - 90^\circ) \tag{43}$$
$$= E_m \cos\omega t, \tag{44}$$
$$i'' = I_m \sin(\omega t - 90^\circ - \phi) \tag{45}$$
$$= I_m \cos(\omega t - \phi), \tag{46}$$

and

$$p_2 = e'i' + e''i'' \tag{47}$$
$$= E_m I_m [\sin\omega t \sin(\omega t - \phi) + \cos\omega t \cos(\omega t - \phi)]. \tag{48}$$

Expanding equation (48)

$$p_2 = E_m I_m \cos\phi\,(\sin^2\omega t + \cos^2\omega t). \tag{49}$$

But

$$\sin^2\omega t + \cos^2\omega t = 1. \tag{50}$$

Therefore,

$$p_2 = E_m I_m \cos\phi, \tag{51}$$

which is a constant.

In a similar manner it may be shown that the power in a three-phase system is constant, and equal to

$$p_3 = 1.5\, E_m I_m \cos\phi. \tag{52}$$

9. **Power measurements.** — The power in a single-phase system may be measured by means of a wattmeter connected as indicated in Fig. 104. Since a polyphase system is composed of two or more single-phase circuits, the power in a polyphase system is the sum of the indications of wattmeters connected in the component single-phase circuits.

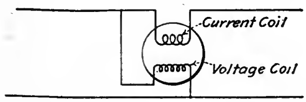


FIG. 104. Single-phase.

In the two-phase system a wattmeter connected in either phase indicates half the total power, if the system is balanced; if the system is unbalanced, a wattmeter must be connected in each phase and the sum of the readings taken as the total power of the system.

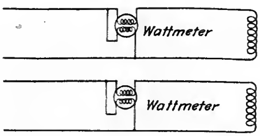


FIG. 105a. Two-phase Four-wire.

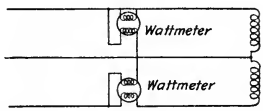


FIG. 105b. Two-phase Three-wire.

The connections for a four-wire system, either independent or interconnected, are shown in Fig. 105a, and those for a three-wire system in Fig. 105b.

The power in a balanced three-phase system may be determined from the indication of one wattmeter connected as shown in Fig. 106a. The current coil of the wattmeter is connected into any one of the three lines, and the potential coil is connected between this line and the neutral of a star-connected system. In a delta-connected system, or in a star-connected system the neutral of which is inaccessible, a so-called "artificial neutral" may be constructed by using two resistances, each of which is equivalent to the resistance of the potential circuit of the wattmeter. When connected as shown in Fig. 106b, the resistances and the potential circuit of

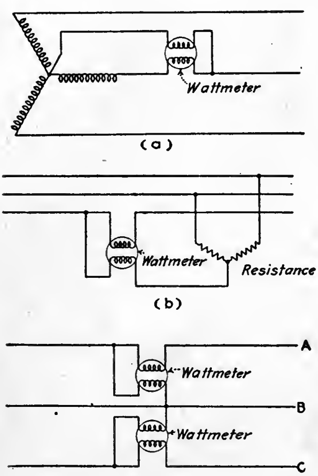


FIG. 106. Three-phase.

the meter form a star connection, and the meter indicates, for balanced load, one-third the total power of the system.

A more generally used method for determining the power in a three-phase circuit makes use of two wattmeters connected as indicated in Fig. 106c. The algebraic sum of the indications of the meters is the total power in the system, whether the system is balanced or unbalanced.* The current coils of the meters are connected into any two of the three lines, and the potential coils between these two lines and the third line. The wattmeter indications are equal only when the power factors of the load circuits are unity and the load is balanced.

In using two wattmeters for measuring the power of a three-phase system, the *sum* of their indications is to be taken if the power factors of the load circuits are greater than 50 per cent and their *difference* if the power factors are less than 50 per cent. The power factors of induction motors running at light loads are often very low. To determine whether the power factor of a circuit is greater or less than 50 per cent, disconnect the potential coil of wattmeter No. 1 from line *B* and connect it to line *A*. Fig. 106c. If the direction of torque (the direction in which the pointer deflects) is *not* reversed, the power factor is greater than 50 per cent.

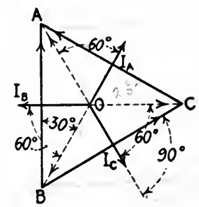


Fig. 107. Vector Diagram for Wattmeters.

Referring to Fig. 107, let *OA*, *OB* and *OC* represent the equal line to neutral voltages of a balanced three-phase system,† the power factor of which is 50 per cent (angle of lag equals 60°). The line currents, then, are represented by the vectors I_A, I_B and I_C and the line voltages by *AB*, *BC* and *AC*.

If the currents in the coils of a wattmeter remain constant, the indication of the meter is proportional to the cosine of the angle of their phase difference. Therefore, wattmeter No. 2, with a phase difference of 90 degrees, indicates zero, and wattmeter No. 1, with a phase difference of 30 degrees, indicates the total load in the three-phase system. If the power factor of the system is greater than 50 per cent, the indication of wattmeter No. 2 is positive and

* An unbalanced three-phase system with "grounded" neutral is essentially a four-wire system, and three wattmeters should be used if accurate determination of the power is required.

† It is immaterial whether the system is star- or delta-connected.

must be added to the indication of wattmeter No. 1 to obtain the total power in the system; if the power factor of the system is less than 50 per cent, the direction of current in one coil of wattmeter No. 2 must be reversed to bring the indication on the scale, and its indication regarded as negative.

From the above considerations it is evident that

$$P_1 = E_{AB} I_A \cos (30 - \phi) \tag{53}$$

and

$$P_2 = E_{BC} I_C \cos (30 + \phi), \tag{54}$$

when P_1 = the indication of wattmeter No. 1,

P_2 = the indication of wattmeter No. 2,

$E_{AB} = E_{BC}$ = the line-to-line voltage,

$I_A = I_C$ = the line currents,

ϕ = the power-factor angles.

It was shown in Section 7 that the total power in a balanced three-phase system is

$$P_3 = \sqrt{3}\, E_l I_l \cos \phi. \tag{55}$$

But

$$P_1 + P_2 = E_l I_l [\cos (30^\circ - \phi) + \cos (30^\circ + \phi)] \tag{56}$$

$$= E_l I_l (\cos 30^\circ \sin \phi + \sin 30^\circ \sin \phi + \cos 30 \sin \phi - \sin 30^\circ \sin \phi) \tag{57}$$

$$= 2\, E_l I_l \cos 30^\circ \cos \phi \tag{58}$$

$$= \sqrt{3}\, E_l I_l \cos \phi. \tag{59}$$

Therefore, the total power of a three-phase system is the algebraic sum of the indications of two wattmeters connected as shown in Fig. 106c, and the indications of the meters are equal only when the phase angle ϕ is unity, *i.e.*, when the load circuits are non-reactive.

The power factor of a balanced three-phase system may be calculated from the wattmeter readings and the following formula:

$$\cos \phi = \frac{1}{\sqrt{1 + 3\left(\frac{1 - n}{1 + n}\right)^2}}, \tag{60}$$

when $n = \frac{P_2}{P_1}$,

P_1 = the indication of that wattmeter which is always positive,

P_2 = the indication of that wattmeter which may be either positive or negative.

10. Comparison of two- and three-phase systems. — The statement was made in Section 2 that the three-phase system is superior to the two-phase system. This superiority lies chiefly in the smaller weight of copper required in the three-phase system for the transmission or the distribution of a given power, the line voltage and the power lost in the two systems being equal. Let

E = the greatest line-to-line voltage in each system,
I_2 = the current in each line of the two-phase system,
I_3 = the current in each line of the three-phase system,
R_2 = the resistance of each of the four conductors in the two-phase system,
R_3 = the resistance of each of the three conductors in the three-phase system.

Then,
$$2\,EI_2 = \sqrt{3}\,EI_3 \tag{61}$$
and
$$I_2 = \frac{\sqrt{3}\,I_3}{2}. \tag{62}$$

The copper loss in the two-phase system is
$$P_2 = 4\,R_2I_2^2 \tag{63}$$
and that in the three-phase system is
$$P_3 = 3\,R_3I_3^2. \tag{64}$$
Since the copper losses in the two systems are equal
$$4\,R_2I_2^2 = 3\,R_3I_3^2. \tag{65}$$
Substituting the value of I_2 from equation (62)
$$4\,R_2\frac{3\,I_3^2}{4} = 3\,R_3I_3^2 \tag{66}$$
and
$$R_2 = R_3, \tag{67}$$
i.e., the resistance of each conductor in the two-phase system is equal to the resistance of each conductor in the three-phase system, and the total weight of conductor in the two-phase system is one-third greater than that in the three-phase system.

When the three-wire two-phase system having a phase voltage equal to the line-to-line voltage of a three-phase system, is compared with the three-phase system, the weight of copper in the two-phase system is found to be 2.8 per cent less than that required in the

three-phase system. This small saving in copper is more than offset by the increased insulation required by reason of the 40 per cent larger voltage between two of the conductors.

11. Equivalent single-phase system. — In problems involving polyphase quantities, it is often convenient to consider the polyphase system replaced by an equivalent single-phase system, *i.e.*, by a single-phase system, the voltage of which is equal to the greatest voltage between lines in the polyphase system, and in which the total power, the losses and the power factor are equal to those in the polyphase system.

Equivalent of the two-phase system. — Since the two-phase system is composed of two single-phase circuits, the voltage of the equivalent single-phase system is the phase voltage of the two-phase system,* and the equivalent single-phase current is twice that in each of the two-phase conductors. The copper loss in the two-phase system is

$$P_2 = 2\,R_2 I_2^2, \tag{68}$$

when R_2 = the resistance of each phase circuit,
I_2 = the current in each line.

The copper loss in the equivalent single-phase circuit is

$$P_1 = R_1\,(2\,I_2)^2. \tag{69}$$

But

$$R_1\,(2\,I_2)^2 = 2\,R_2 I_2^2 \tag{70}$$

and

$$R_1 = \frac{R_2}{2}, \tag{71}$$

i.e., the resistance of the equivalent single-phase system is equal to one-half the resistance of each circuit of the two-phase system.

Equivalent of the three-phase system. — Assuming unity power factor, the power in a three-phase system is

$$P_3 = \sqrt{3}\,EI_3 \tag{72}$$

and that in the equivalent single-phase system is

$$P_1 = EI_1. \tag{73}$$

Therefore,

$$I_1 = \sqrt{3}\,I_3. \tag{74}$$

$$R_1 I_1^2 = 3\,R_3 I_3^2 \tag{75}$$

and

$$R_1 = R_3, \tag{76}$$

* Unless the phases are interconnected to form a three-wire system.

when R_1 = the resistance of the equivalent single-phase system,
R_3 = the resistance of each phase of the three-phase system,
I_1 = the current in the equivalent system,
I_3 = the current in each line of the three-phase system,

i.e., the resistance of the equivalent single-phase system is equal to the resistance of each phase of the three-phase system.

It is evident from inspection that the equivalent resistance of a star-connected system is one-half the resistance as measured by continuous-current methods between any two lines of the three-phase system. It may be shown that this is also true for the delta-connected system.

CHAPTER VII — PROBLEMS

1. A balanced 3-wire, 2-phase system delivers 50 kw. at a voltage (phase) of 220 and a power factor of 0.9. Find: (a) the current in each line wire, (*b*) the voltage between the "outside" wires.

2. The current in the common wire of Problem 1 flows in the current coil of a wattmeter, and the voltage coil of the wattmeter is connected between the common wire and one of the "outside" wires. Find the wattmeter indication.

3. The current in the common wire of Problem 1 flows in the current coil of a wattmeter and the voltage coil of the wattmeter is connected between the "outside" wires. Find the wattmeter indication.

4. A 3-phase, star-connected system has a line-to-line voltage of 6600. Find the phase voltage (line to neutral).

5. The line current in a balanced delta-connected system is 150. Find the phase current.

6. Find the power in the circuit of Problem 5 when the phase voltage is 240, and the power factor of the load circuit is 0.85.

7. The current coil of a wattmeter is connected in one line of a balanced 3-phase system, and the voltage coil of the wattmeter between the other two lines. The kw. output of the system is 100, the voltage 220 and the power factor unity. Find the indication of the wattmeter.

8. A non-inductive star-connected load and a non-inductive delta-connected load are operated from the same 3-phase system. The phase current in each load circuit is 100 amperes. Find the current in each line of the supply system.

9. Same as Problem 8 except the power factor of the star-connected loads is 0.866. Find the line currents.

10. The indications of two wattmeters connected as in Fig. 106c are 5000 and 2500. The system is balanced. Find the power factor of the load circuits.

11. The phase currents in a delta-connected 3-phase system are 50, 30 and 75 amperes, the load circuits being non-inductive. Find the current in each line. Solve graphically.

12. The phase currents of a 2-phase, 3-wire system equal 160 amperes. The power factor of the load connected to one phase is 0.85, that of the load connected to the other phase is unity. Find the current in the common wire.

13. Determine the single-phase equivalent of a balanced 3-phase system delivering 500 kw. at 2300 volts, and having a power factor of 0.92.

14. The line current in a mesh-connected 2-phase system (balanced) is 100 amperes. Find the current in each coil (Fig. 92).

15. Find the power in the 2-phase system in Problem 14 if the voltage between alternate wires (Fig. 92) is 2300.

16. Three resistances, the values of which are 10, 15 and 20 ohms, are star-connected to a 3-phase system, the line-to-line voltage of which is 220. Find: (*a*) the current in each line, (*b*) the voltage between each line and neutral, (*c*) the power input to the resistances.

17. 100 kw. are delivered to a balanced 3-phase system, the power factor of which is 0.85, and the voltage between lines is 2200. Find the current flowing in each line.

CHAPTER VIII

THE ALTERNATING-CURRENT GENERATOR

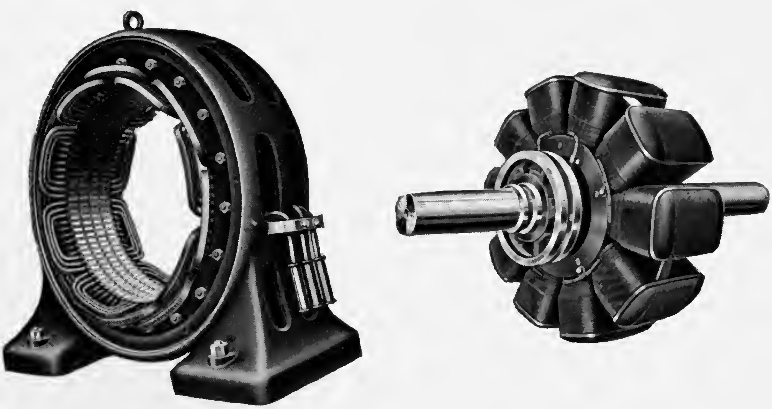
1. **Voltage.** — The voltage of an alternating-current generator is affected by the same quantities that affect the voltage of a continuous-current generator, and in addition, by the relative positions of the armature conductors. The equation for the voltage of an alternating-current generator may be written

$$E_a = k\phi npN\, 10^{-8}, \tag{1}$$

when E_a = the effective electromotive force induced in the armature winding,

k = a constant, the value of which depends on the relative positions of the armature conductors,

ϕ = the total flux passing between *each* pole and the armature,



(a) Stator (armature) (b) Field Structure

FIG. 108. Revolving Field Alternator. Triumph Electric Co.

n = the speed of the armature or of the rotating field in revolutions per second,

p = the number of poles in the field structure,

N = the number of series conductors on the surface of the armature.

Since the product of the speed and the number of field poles of an alternator is equal to twice the frequency, equation (1) may be written

$$E_a = 2\,\phi k f N\,10^{-8}, \tag{2}$$

which is often a useful form of the expression.

2. Concentrated armature windings. — Concentrated armature windings are those in which all the conductors under any given pole are placed in one slot, as indicated in Fig. 109a. It is evident that the same electromotive force is induced in each of the conductors, and that the maximum values are attained at the same instant. Therefore, the total electromotive force induced in the winding is that induced in each conductor multiplied by the number of conductors connected in series.

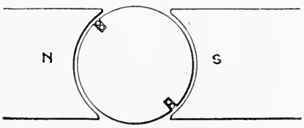


FIG. 109a. Concentrated Armature Winding.

Since the quantity $\phi n p N\,10^{-8}$ is the average electromotive force induced in the armature winding, the value of k for a concentrated winding is the ratio of the effective to the average value. For a sine wave

$$k = \frac{\pi}{2\sqrt{2}} \tag{3}$$

$$= 1.11. \tag{4}$$

3. Distributed armature windings. — Distributed armature windings are those in which the conductors under a given pole are divided into groups, and each group is placed in a different slot, as indicated in Fig. 109b. With this arrangement of the armature conductors, the electromotive force induced in each conductor is the same, but the maximum values are not attained at the same instant in the different groups, *i.e.*, the electromotive force induced in each group is out of phase with the electromotive forces induced in the other groups. Therefore, the electromotive force of an alternating-current generator, having a distributed armature winding, is the *geometric* sum of the electromotive forces induced in the several groups into which the armature winding is divided.

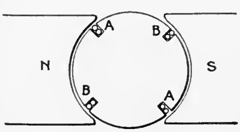


FIG. 109b. Distributed Armature Winding.

If the armature winding consists of two similar groups of conductors per pole as in Fig. 109b, the maximum in group B occurs one-quarter of a cycle (90 electrical degrees) later than the maximum in group A. The electromotive forces of the two groups are, therefore, 90 degrees out of phase, and the resultant electromotive force is $\sqrt{2}$ times that induced in one group. For any number of groups into which the armature conductors may be divided, the values and the phase relations of the electromotive forces in the different groups may be plotted, and the resultant electromotive force found graphically. Fig. 110. In calculating the effective electromotive force induced in the armature of an alternating-current generator, it should be remembered that each group is a concentrated winding, and that the effective electromotive force induced in it is the average electromotive force multiplied by 1.11, as explained for the concentrated winding.

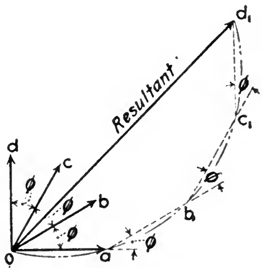


FIG. 110. Vector Diagram of Distributed Winding..

From Fig. 110 it is evident that

$$k = \frac{1.11\,Od}{Oa + Ob + Oc + Od} \tag{5}$$

$$= \frac{1.11\ (\text{chord of the group arc})}{\text{sum of chords between groups}}. \tag{6}$$

For an infinite number of groupings of the conductors, the sum of the chords becomes an arc, as indicated by the dotted arc in Fig. 110.

TABLE III

VALUES OF k FOR DISTRIBUTED ARMATURE WINDINGS

No. of slots per pole	Degrees subtended by the winding				
	180	135	90	60	45
2	0.784	0.922	1.025	1.072	1.087
3	0.739	0.893	1.012	1.065	1.084
4	0.724	0.882	1.007	1.063	1.083
Infinite	0.707	0.870	1.000	1.060	1.082

4. **Effects of distributed armature windings.** — The effects of distributing the armature winding of a alternator are: (*a*) to decrease the induced electromotive force, (*b*) to reduce the inductance* of the armature circuit and improve the regulation, (*c*) to make the induced electromotive force more nearly harmonic, (*d*) to distribute the heating over the armature surface, (*e*) to reduce the size of the armature.

The effect of a distributed winding on the shape of the electromotive-force wave is clearly shown in Fig. 111. The electromotive-

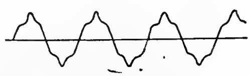


FIG. 111a. Voltage Wave with Concentrated Armature Winding.

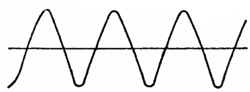


FIG. 111b. Voltage Wave with Distributed Armature Winding.

force wave of the concentrated winding (Fig. 111a) shows a very pronounced third harmonic which is entirely absent from the wave of the distributed winding (Fig. 111b). The harmonics of the different groups of a distributed winding tend to neutralize and eliminate the harmonic from the resultant electromotive-force wave.

5. **The oscillograph.** — The shape of an alternating current or electromotive-force wave is determined by means of the oscillograph. A very fine wire or strip is bent into a loop and placed between the poles of a powerful electromagnet, as indicated in Fig. 112. Attached to the loop is a small mirror *M* on which may be focused a beam of light.

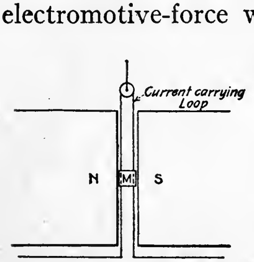


FIG. 112. Schematic Diagram of the Oscillograph.

When current flows in the looped conductor, the loop tends to move so that its plane is perpendicular to the lines of magnetic flux passing from *N* to *S*. This movement of the loop causes the mirror to be deflected, the deflection is proportional to the current flowing in the loop, and the beam of light reflected from the mirror acts as a pointer. The moving parts of an oscillograph are made very light so that the deflection of the mirror is always in time-

* The inductance of the armature is proportional to the square of the number of conductors per slot.

phase with the current flowing in the current-carrying loop, and the position of the mirror indicates the instantaneous value of the current flowing in the loop.

If the beam of light reflected from the mirror M is directed onto a photographic film having a uniform motion at right angles to the movement of the beam of light, records similar to those in Fig. 111 are obtained. When it is not desired to make photographic records a second mirror, operated by a small synchronous motor, is rotated or vibrated in such a manner that the reflection of the beam of light on a stationary screen indicates the wave form of the current or electromotive force.

6. The single-phase alternating-current generator. — In the single-phase alternating-current generator, the armature conductors are all connected in series. Fig. 113. It is evident from the considerations discussed in Section 3, that the value of k is low if the armature conductors are distributed over the entire armature surface. The terminal voltage of a single-phase alternator is only slightly greater when the conductors cover the entire armature surface than when they cover three-fourths of the surface, but the weight of copper and the armature resistance are proportional to the surface covered. It is, therefore, common practice to distribute the armature conductors over only a portion of the surface of the armature core. Because of this excess of materials, a single-phase alternator has a greater weight and is less efficient than a polyphase alternator of the same voltage and output.

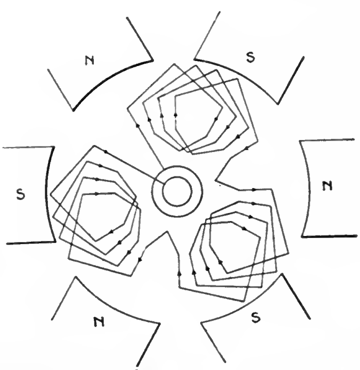


FIG. 113. Elementary Single-phase Armature Winding.

Consider an armature having four slots per pole and so wound that the average electromotive force induced in the conductors in each slot is 10 volts. The arc subtended by this winding is 180 degrees, the value of k is 0.724, and the effective electromotive force induced in this section of the armature winding is

$$E_a = 4 \times 10 \times 0.724$$
$$= 28.96 \text{ volts.}$$

If three of the four slots in the above armature core are used, the arc subtended is 135 degrees, the value of k is increased to 0.893, and the effective electromotive force induced in the coils is

$$E_a = 3 \times 10 \times 0.893$$
$$= 26.79 \text{ volts.}$$

Thus by using only three of the four slots per pole, the electromotive force induced in the armature winding is reduced only 7.5 per cent, while the weight of copper and the resistance of the armature circuit are each reduced 25 per cent. The reactance of the armature is also materially reduced.

7. The two-phase alternating-current generator. — In the two-phase alternator two armature windings are wound on the same core, the conductors being so placed that the two induced electromotive forces are in quadrature. As each armature winding covers one-half the surface of the core, the arc subtended is 90 degrees, the value of k is materially higher than for the single-phase alternator, the material is used more advantageously, and a larger output is obtained from a given weight.

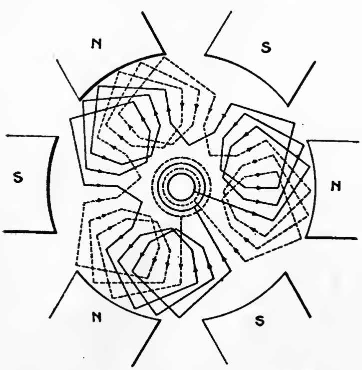


FIG. 114. Elementary Two-phase Armature Winding.

8. The three-phase alternating-current generator. — In the three-phase alternator three armature windings are wound on the same core and so placed that the induced electromotive forces are 120 degrees out of phase. Each winding covers one-third of the surface of the core, and the arc subtended by the winding is 60 degrees.

The windings of three-phase alternators are always interconnected, the star connection being more largely used than the delta.* By connecting a single-phase load between any two terminals, a star-connected three-phase alternator may be operated as a single-phase generator in which the active armature conductors cover two-thirds of the armature surface.

* In a delta-connected system harmonics cause a current to circulate around the delta and heat the conductors. Also, for a given electromotive force a delta-connected armature requires 73 per cent more conductors than does a star-connected armature.

9. Armature reaction. — As in the continuous-current dynamo, the armature current of an alternator affects both the value of the flux in the air gap and its distribution. If the current flowing in the armature windings is in phase with the induced electro-

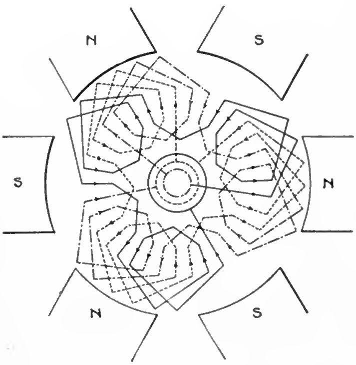


FIG. 115a. Elementary Three-phase Armature Winding (Delta Connected).

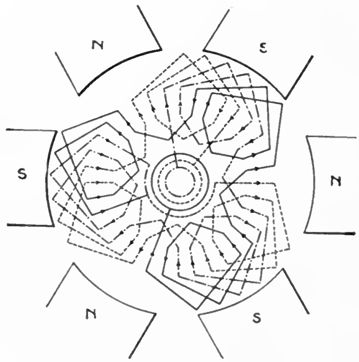


FIG. 115b. Elementary Three-phase Armature Winding (Star Connected).

motive force, the flux is distorted only; if the current and the electromotive force are not in phase, the total flux is increased or decreased as the current leads or lags behind the electromotive force.

Referring to a single-phase two-pole alternator with concentrated winding, let

N = the number of series conductors on the armature,

ϕ = the angle by which the current leads or lags behind the electromotive force.

Then

$$e = E_m \sin \omega t, \tag{7}$$

and

$$i = I_m \sin (\omega t \pm \phi). \tag{8}$$

The direction of the flux set up when current flows in the armature winding is at right angles to the plane of the coil, and its value is proportional to the product of the armature current and the number of turns in the winding. Let the instantaneous magneto-

motive force in ampere-turns due to the armature winding be represented by m. Then

$$m = \frac{Ni}{2}^* \tag{9}$$

$$= \frac{NI_m \sin(\omega t \pm \phi)}{2}. \tag{10}$$

This magnetomotive force may be resolved into components at right angles to each other, one of which (m') is parallel to the axis of the field flux, and increases or decreases the flux set up by the field windings; the other component (m'') is in quadrature with the field flux and changes the distribution of the flux in the air gap (distorts).

$$m' = \frac{NI_m \sin(\omega t \pm \phi)}{2} \cos \omega t, \tag{11}$$

$$m'' = \frac{NI_m \sin(\omega t \pm \phi)}{2} \sin \omega t. \tag{12}$$

Expanding equation (11)

$$m' = \frac{NI_m (\sin \omega t \cos \phi \pm \cos \omega t \sin \phi)}{2} \cos \omega t \tag{13}$$

$$= \frac{NI_m (\sin \omega t \cos \omega t \cos \phi \pm \cos^2 \omega t \sin \phi)}{2}. \tag{14}$$

But

$$\text{av.} \cos^2 \omega t = 0.5 \tag{15}$$

and

$$\text{av.} \sin \omega t \cos \omega t \cos \phi = 0. \tag{16}$$

Therefore,

$$\text{av. } m' = \frac{NI_m \sin \phi}{4}, \tag{17}$$

i.e., the average magnetizing magnetomotive force set up by the armature winding of a single-phase alternator is proportional to the product of the maximum value of the current flowing in the winding and the *sine* of the angle of phase difference. Similarly,

$$\text{av. } m'' = \frac{NI_m \cos \phi}{4}, \tag{18}$$

* If the armature winding is distributed, the effective ampere-turns tending to set up a flux at right angles to the plane of the coil equals $\frac{Ni}{2}$ times the average cosine of one-half the angle over which the winding is distributed.

i.e., the average distorting magnetomotive force set up by the armature winding of a single-phase alternator is proportional to the product of the maximum value of the current flowing in the winding and the *cosine* of the angle of phase difference.

The direction and relative magnitude of the armature magnetomotive force of a single-phase alternator are shown in Fig. 116 for different positions of the armature. It will be observed that the

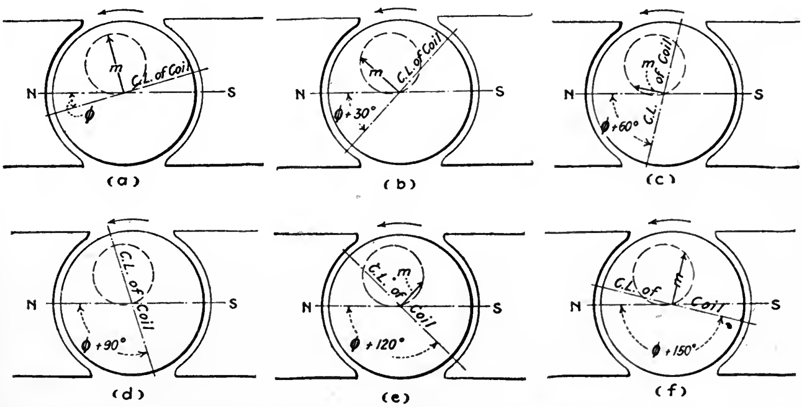


FIG. 116. Armature Magnetomotive Force. Single-phase Alternator.

direction of the magnetomotive force, as well as its magnitude, varies with the position of the armature.

In polyphase alternators with balanced load, the armature magnetomotive force is constant in value and fixed in position * for any given armature current and phase angle. For a two-phase alternator, the magnetomotive force set up by one winding is

$$m_1 = \frac{NI_m \sin(\omega t \pm \phi)}{2}, \tag{19}$$

and that set up by the other winding is

$$m_2 = \frac{NI_m \sin(\omega t \pm 90 \pm \phi)}{2} \tag{20}$$

$$= \frac{NI_m \cos(\omega t \pm \phi)}{2}. \tag{21}$$

* It is assumed that the alternator is of the rotating armature type. When the armature conductors are stationary, the flux set up by the armature winding is constant in value but rotates at a speed proportional to the frequency of the currents flowing in the armature conductors. This rotation of flux is the fundamental principle of the induction motor. See Chapter 13.

Since these two magnetomotive forces are at right angles,

$$M = \frac{N I_m \sqrt{\sin^2(\omega t \pm \phi) + \cos^2(\omega t \pm \phi)}}{2} \tag{22}$$

$$= \frac{N I_m}{2} \text{ (a constant),} \tag{23}$$

i.e., the magnetomotive force set up by the armature winding of a two-phase alternator is constant, and equal to the maximum value of the magnetomotive force set up by each phase winding.

That the direction of the magnetomotive force set up by a two-phase armature is fixed is evident from the following considerations: At the instant when the current in phase A is maximum, the armature is in the position indicated in Fig. 117a, the current

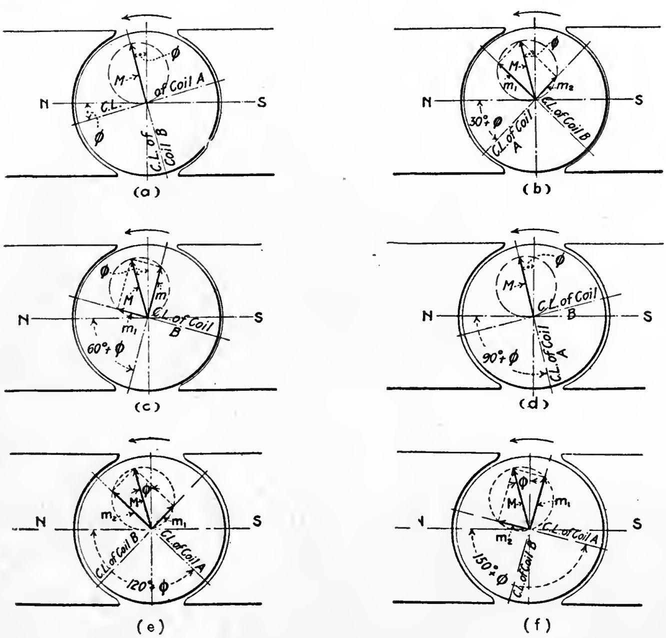


FIG. 117. Armature Magnetomotive Force. Two-phase Alternator.

in phase B is zero, and the magnetomotive force set up by the armature is at right angles to the plane of coil A. It is represented, both in magnitude and in direction, by M. When the armature has rotated through an arc of 30 degrees (one-twelfth of a revoluton), the coils are in the positions indicated in Fig. 117b, and

the magnetomotive force set up by the armature is the geometric sum of the magnetomotive forces set up by coils A and B. Similarly, the conditions existing when the armature is in other positions are shown for a half revolution of the armature which brings the coils into their first position, but with the currents flowing in the opposite direction.

That the magnetomotive force produced by the armature of a three-phase alternator is constant in value and fixed in space may be shown in a similar manner, the value of the magnetomotive force being one and one-half times the maximum value produced by each phase.

10. Voltage characteristic. — From Section 9 it is evident that the field excitation required to simultaneously produce rated terminal voltage and rated armature current varies with the character of the load, *i.e.*, armature reaction decreases or increases the flux

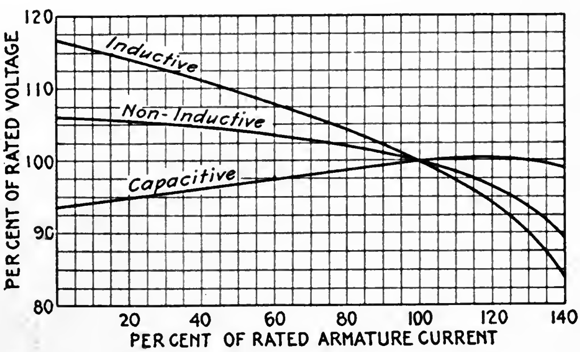


FIG. 118. Voltage Characteristics.

set up by a given field current as the armature current lags behind or leads the induced electromotive force. Therefore, the rise in voltage as the armature current decreases, is greater when the load circuit is inductive than when it is non-inductive, and the terminal voltage may decrease as the armature current decreases, if the alternator is supplying a capacitive load circuit. The voltage characteristic of an alternator is a curve showing the relations between the terminal voltage and the armature current, the speed and the field excitation remaining constant. Curves for inductive, non-inductive and capacitive load circuits are shown in Fig. 118.

11. Regulation. — The regulation of an alternating-current generator is the ratio of the increase in voltage, between rated load and no load, and the voltage at full load, the frequency and the field excitation remaining constant.

$$\text{Per cent regulation} = \frac{(\text{no-load voltage} - \text{full-load voltage})\ 100}{\text{full-load voltage}}. \quad (24)$$

It is a waste of considerable energy, and often inconvenient or impossible, to determine the regulation of a large alternator by an actual load test. The regulation of an alternator is, therefore, usually calculated from no-load tests. The data necessary for the calculation of the regulation of an alternating-current generator are: (*a*) the resistance of the armature circuit, (*b*) the open-circuit saturation (magnetization) curve, (*c*) the "short-circuit" curve, (*d*) the zero power factor saturation curve.

(*a*) *Armature resistance.* — To determine the resistance of the armature winding, connect the armature to continuous-current mains in series with a suitable resistance, so the current in the armature circuit may be controlled. The resistance of the armature winding is the ratio of the voltage between its terminals and the current flowing in the circuit.

$$R_a = \frac{E_{cc}}{I_{cc}}. \quad (25)$$

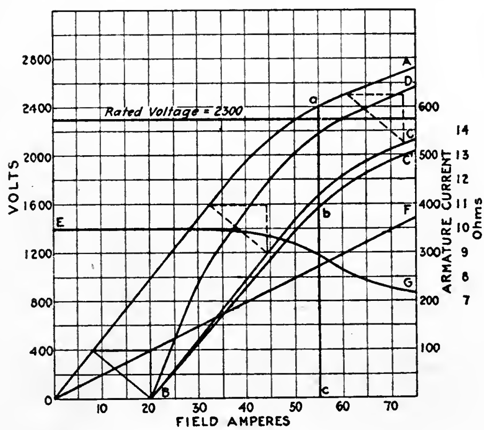


FIG. 119. Saturation, Short-circuit and Synchronous Reactance Curves for the Alternator.

OA open-circuit saturation curve.
BC approximate zero power factor curve.
BC' experimental zero power factor curve.
BD full-load saturation curve at unity power factor.
OF short-circuit curve.
EG synchronous reactance curve.

(*b*) *The open-circuit saturation curve.* — The saturation curve shows graphically the relations between the electromotive force induced in the armature winding and the field excitation. The data

from which the saturation curve is plotted are obtained by measuring the terminal voltage of the armature with the armature circuit open, for different field currents, the frequency being maintained at the rated value. Fig. 119.

(*c*) *The "short-circuit" curve.* — The short-circuit curve shows graphically the relations between the current in the short-circuited armature winding and the field excitation. Taking into consideration the rated current of the armature, short circuit the armature winding, as indicated in Fig. 120, through a suitable ammeter, and determine the field current required to cause different current

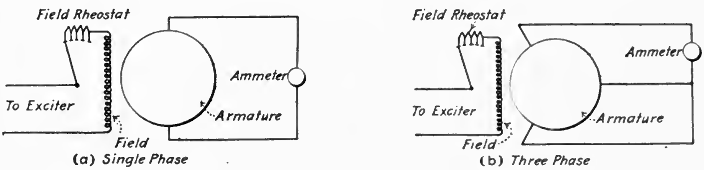


FIG. 120. Connections for Short-circuit Test.

values to flow in the armature circuit, the rotating parts of the alternator being driven at rated speed. Plot the curve as in Fig. 119. The impedance of the armature circuit is assumed to be constant and the short-circuit curve is, approximately, a straight line.

(*d*) *The zero power factor saturation curve.* — The zero power factor saturation curve of an alternator shows the relation between the terminal voltage and the field excitation when the power factor of the load circuit is zero, and rated current flows in the armature circuit. A curve approximating the zero power factor curve may be constructed from data obtained when the load on an alternator consists of idle-running under-excited synchronous motors, the power factor of which is very low.

An approximate zero power factor curve may also be constructed from the open-circuit saturation curve and the short-circuit curve. Draw the curve *BC*, in Fig. 119, parallel to the open-circuit saturation curve, beginning at the abscissa which represents the field excitation required to cause rated current to flow in the short-circuited armature. In alternators having high reactance, high saturation, and large magnetic leakage, the zero power factor curve may lie before *BC*, as indicated by the line *BC′*, and its exact location must be determined by test.

Regulation by the electromotive-force method. — In calculating the regulation of an alternator by the electromotive-force method, it is assumed that the electromotive force induced in the armature is the vector sum of two quadrature electromotive forces, one equal to the product of the armature current and the total resistance of the circuit, including that of the armature; the other equal to the product of the armature current and the total reactance of the circuit.

$$E_a = I_a\sqrt{R^2 + X^2} \tag{26}$$

$$= \sqrt{(E\cos\phi + R_aI_a)^2 + (E\sin\phi + X_aI_a)^2}, \tag{27}$$

when E_a = the electromotive force induced in the armature,
E = the terminal electromotive force of the generator,
R_a = the resistance of the armature circuit,
X_a = the reactance of the armature circuit (the synchronous reactance* of the alternator armature),
I_a = the current flowing in the armature circuit,
$\cos\phi$ = the power factor of the load circuit.

The synchronous reactance of the armature circuit, which cannot be measured directly, must be calculated. The voltage drop in the armature is the vector sum of the resistance voltage and the reactance voltage.

$$E_a' = \sqrt{(R_aI_a)^2 + (X_aI_a)^2}. \tag{28}$$

Dividing equation (28) by I_a,

$$Z_a = \sqrt{R_a^2 + X_a^2}, \tag{29}$$

and

$$X_a = \sqrt{Z_a^2 - R_a^2}. \tag{30}$$

A consideration of the open circuit and the zero power factor saturation curves, shows that the synchronous reactance of an al-

* The synchronous reactance of an alternator armature is the combined effect of: (*a*) the inductance of the armature windings, (*b*) the flux set up by the current-carrying armature conductors.

As shown in Section 9, the armature currents of a polyphase alternator set up a flux which is fixed in position, and across which the armature conductors move. The electromotive force induced in the conductors by the armature flux reduces the terminal voltage of the alternator and makes the regulation poorer in much the same manner as does the inductance of the windings.

A similar effect takes place in the single-phase alternator because of the varying value of the flux threading the armature coil.

ternator armature is not constant, but decreases as the excitation increases. It is also affected by the phase relation of the current and the electromotive force induced in the armature winding, *i.e.*, by the power factor of the load circuit.

Substituting in equation (27) the full- (rated) load armature current, the no-load voltage is calculated and

$$\text{Per cent regulation} = \frac{(E_a - E)\,100}{E}. \tag{31}$$

Example. — A single-phase alternator has the following:

$$E = 2300,\ I_a = 100,\ R_a = 1,\ X_a = 10.$$

Find the per cent regulation when the power factor of the load circuit is unity.

Solution.

$$E_a = \sqrt{(2300 + 100)^2 + (1000)^2}$$
$$= 2600 \text{ volts.}$$

$$\text{Regulation} = \frac{2600 - 2300}{2300}$$
$$= 13 \text{ per cent.}$$

Regulation by the magnetomotive-force method. — In calculating the regulation of an alternator by the magnetomotive force method it is assumed that the voltage induced in the armature windings is due to quadrature fields, one equal to that required to produce the total non-inductive drop, the other equal to that required to produce the total wattless component of electromotive force.

From the saturation curve find the field current I_f' required to induce in the armature winding an electromotive force equal to the total non-reactive drop $(E \cos \phi + R_a I_a)$ of the circuit at rated load; from the same curve find the field current I_f'' required to induce in the armature winding an electromotive force equal to the total reactive drop $(E \sin \phi + X_a I_a)$ of the circuit at rated load.

The field current I_f required to produce, simultaneously, the terminal voltage E and the current I_a in a circuit the power factor of which is $\cos \phi$, is

$$I_f = \sqrt{(I_f')^2 + (I_f'')^2}. \tag{32}$$

From the saturation curve find the electromotive force E_a in-

duced in the armature winding when current I_f flows in the field windings.

$$\text{Per cent regulation} = \frac{(E_a - E)\ 100}{E}. \tag{33}$$

Example.—Find, by the magnetomotive force method, the regulation of the 2300-volt, single-phase alternator, specified above.

Solution.—From Fig. 119,

$$I_f' = 44.5 \text{ amperes,}$$

and

$$I_f'' = 12.5 \text{ amperes.}$$

Therefore,

$$\begin{aligned} I_f &= \sqrt{(44.5)^2 + (12.5)^2} \\ &= 46.22 \text{ amperes.} \end{aligned}$$

The induced electromotive force is, from the open-circuit saturation curve, 2430 volts, and the regulation is

$$\begin{aligned} &= \frac{130}{2300} \times 100 \\ &= 5.2 \text{ per cent.} \end{aligned}$$

Regulation by the A.I.E.E. method.—The electromotive force induced in the armature winding of an alternator is the vector sum of the terminal voltage and the voltage drop in the armature circuit. Therefore, the drop in the armature, at zero power factor and rated armature current, is the difference between the ordinates of the open-circuit and the zero power factor saturation curves.

For field excitation, *Oc*, the induced electromotive force is *ca*, and the drop in the armature is *ba*. Fig. 119. For any other power factor, and the same field excitation, the terminal voltage is the vector difference of the induced electromotive force and the internal drop. Fig. 121. By calculating the terminal electromotive forces for different field excitations but constant power factor, the load saturation curve *BD*, in Fig. 119, may be plotted. From this curve and the open-circuit saturation curve, the electromotive force induced in the armature when the alternator is operating at rated voltage and armature current, and with

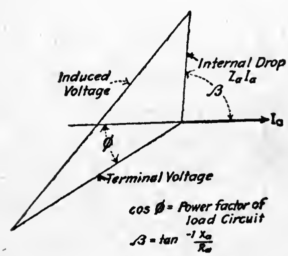


FIG. 121.

the specified power factor, is determined, and the regulation calculated.

From Fig. 119, the induced voltage is 2490 when rated current flows in the armature circuit, the terminal voltage is 2300, and the power factor of the load circuit is unity. The regulation is, therefore,

$$= \frac{190}{2300} \times 100$$
$$= 8.3 \text{ per cent.}$$

12. Limits of regulation. — From the above numerical examples it is evident that the electromotive force method and the magnetomotive-force method do not give the same result, these calculations serving only to establish the limits within which the actual regulation lies. If the saturation curve was a straight line, the actual regulation would be determined by either of these methods. Since the no-load voltage, as calculated by the electromotive-force method, is larger than that obtained by an actual load test, this method is termed "pessimistic." The no-load voltage, as obtained by the magnetomotive force method, is smaller than that obtained by an actual load test, and this method is termed "optimistic." It is worthy of note that, in a well-designed alternator, the optimistic method gives a closer approximation to the actual regulation than does the pessimistic method.

The A.I.E.E. method for the calculation of alternator regulation is essentially empirical, but gives results which approximate very closely those obtained by test, and its use is recommended in preference to either the electromotive-force or the magnetomotive-force methods.

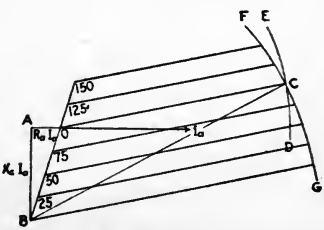


FIG. 122. Alternator Regulation.

13. Graphical determination of terminal voltage. — In Fig. 122, using O as a center and with a radius proportional to the rated electromotive force of the alternator, strike an arc ECD. Through O draw the current vector I_a. Lay off OA proportional to the full-load resistance drop R_aI_a of the armature winding, and AB proportional to the full-load reactance drop X_aI_a of the armature circuit. Draw OC to its intersection with the arc ECD, making the angle COI such that its cosine is equal to the power factor of the

load circuit. *BC* is proportional to the generated or no-load voltage.

With *B* as a center and a radius equal to *BC*, strike a second arc *FCG*. Divide *OB* into any number of equal parts and lay off equal spaces beyond *O*, the spaces representing percentages of the rated capacity (armature current) of the alternator. From these points draw lines, parallel to *OC*, to their intersection with the arc *FCG*. The lengths of these parallel lines are proportional to the terminal voltages at the given percentages of rated load, and from them the voltage characteristic may be constructed, or the regulation calculated.

14. The Tirrill Regulator. — The operation of the Tirrill regulator, when applied to an alternator, is essentially the same as for the

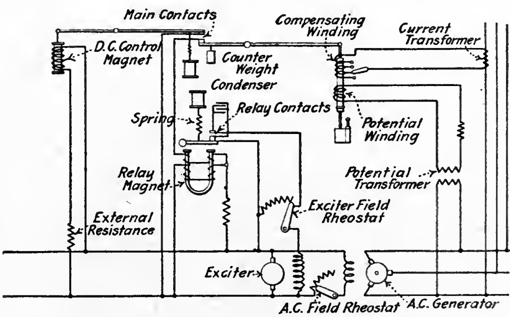


FIG. 123. Tirrill Regulator for Alternators.

continuous-current generator.* An elementary diagram showing the application of a Tirrill regulator to an alternator is shown in Fig. 123. The field rheostat of the exciter is periodically short-circuited by the closing of the main contacts, the length of time during which the short circuit exists depending on the position of the main contact. The position of this contact is controlled by the alternating-current magnet, *i.e.*, by the voltage of the alternating-current system. The function of the compensating winding is to increase the terminal voltage as the load increases, the effect of the compensating winding being proportional to the line current.

Assuming the main contacts to be open, the relay contacts are held open by the differentially wound relay magnet, and the voltage of both the exciter and the alternator falls off. The reduced voltages allow the main contacts to close. Closing the

* See Chapter 4, Section 10.

main contacts demagnetizes the relay, closes the relay contacts, and short circuits the exciter-field rheostat. The increased field excitation increases the electromotive force between the terminals of the control magnets, and causes the main contacts to open.

15. The losses in an alternating-current generator. — The losses in an alternator are: (*a*) armature copper losses, (*b*) field copper losses, (*c*) windage and friction, (*d*) iron (core) losses, (*e*) stray load losses.

(*a*) *Armature copper losses.* — The copper loss in the armature winding is that due to the resistance of the armature conductors, and increases as the square of the armature current.

(*b*) *Field copper losses.* — The field copper losses are those due to the resistance of the field winding and are proportional to the square of the field current. If the terminal voltage is to remain constant, the field excitation of an alternator must be increased as the load increases to compensate for armature reaction and resistance drop, and the field losses increase with the load.

(*c*) *Windage and friction.* — The windage and friction losses are those due to: (1) friction between the shaft and the bushings, (2) friction between the brushes and the rings, (3) the resistance offered by the air to the movement of the rotating parts. These losses are constant for any given speed.

(*d*) *Iron losses.* — The iron losses are due to: (1) hysteresis in the iron of the armature core, (2) eddy currents in the armature core. The iron losses do not vary greatly and are usually considered constant.

(*e*) *Stray load losses.* — The stray load losses, so-called for lack of a better name, include all the losses not included in (*a*), (*b*), (*c*) and (*d*) and such changes in (*c*) and (*d*) as take place when the alternator is loaded. The stray load losses are primarily additional iron losses which are due to distortion of the magnetic field as the armature current increases; to eddy-current losses in the armature conductors, *etc.;* and are directly proportional to the armature current.

*Determination of the losses in an alternating-current generator.** — The losses of an alternator are determined as follows:

Armature copper losses. — The copper loss in the armature conductors is equal to the product of the resistance of the armature circuit and the square of the armature current.

$$P_a = R_a I_a^2. \qquad (34)$$

* See the Standardization Rules of the American Institute of Electrical Engineers.

Field copper loss. —

$$P_f = R_f\,[(I_f')^2 + (I''_f)^2], \tag{35}$$

when R_f = the resistance of the field circuit,

I_f' = the field current (taken from the saturation curve) required to produce rated voltage at no load.

I_f'' = the field current (taken from the short-circuit curve) required to produce a given current in the short-circuited armature.

Windage and friction. — Drive the alternator at its rated speed but without field excitation, and determine the input to the driving motor. Determine the losses in the motor and subtract them from the total input. The difference is the loss due to windage and friction of the alternator.

$$P_{w \& f} = \text{Motor input} - \text{motor losses}. \tag{36}$$

Iron losses. — Drive the alternator at its rated speed and with such field excitation as gives rated voltage at the armature terminals. The total input to the driving motor is the sum of the losses in the motor, the windage and friction of the alternator, and the iron losses of the alternator.

$$P_i = \text{Motor input} - \text{motor losses} - W_{w \& f}. \tag{37}$$

Stray load losses. — Drive the alternator at its rated speed with the armature winding short-circuited, and with such field excitation as produces the desired current in the armature circuit. The input to the driving motor is equal to the sum of the motor losses, the windage and friction of the alternator, the copper losses in the alternator armature winding, and the load losses.

$$P_l = \text{Motor input} - \text{motor losses} - R_a I_a^2 - W_{w \& f}. \tag{38}$$

The stray load losses as measured under short circuit are greater than when the alternator is operating under load because of the low-power factor, practically zero, of the short-circuited armature. Experiment shows that when one-third the stray load loss as determined by the above method, is used in efficiency calculations, the results approximate actual load tests. This value is, therefore, recommended by the Standardization Rules of the American Institute of Electrical Engineers.

Fig. 124 shows graphically the losses in an alternator, which are so plotted that the total loss for any given armature current is obtained by reading the ordinate of the top curve corresponding to the given armature current.

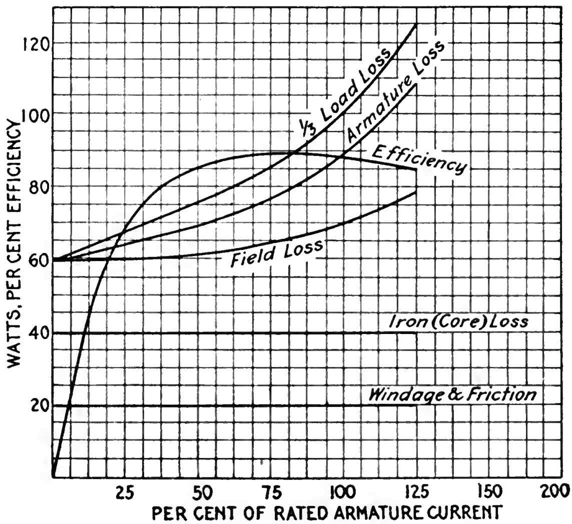


FIG. 124. Alternator Losses.

16. Efficiency. — The efficiency of an alternator is calculated from the following equation:

$$\text{Per cent efficiency} = \frac{(\text{output})\ 100}{\text{output} + \text{losses}}. \tag{39}$$

17. Parallel operation of alternators. — Parallel operation of alternators is desirable because of the greater efficiency obtainable and the assurance of continuity of service, as explained for continuous-current generators.

Alternating-current generators which are to operate in parallel should: (*a*) have equal terminal voltages, (*b*) have the same frequency, (*c*) attain their maximum positive values of electromotive force at the same instant, (*d*) have similar electromotive-force waves.

If any one of the above conditions does not exist, a current flows around the local circuit formed by the armatures of the alternators, increases their heating, reduces the capacity of the alternators, and often causes other disturbances in the system.

If the voltages are equal, the frequencies the same, the electromotive-force waves similar, and the maximum positive electromotive forces attained at the same instant, the electromotive forces are at all times equal and opposed, and current does not flow around the local circuit formed by the armatures of the generators.

If the voltages are unequal, a current, the magnitude of which is proportional to the difference of the voltages, flows between the alternators.

$$i = \frac{e_A - e_B}{Z}, \tag{40}$$

when Z = the impedance of the local circuit. This equalizing current leads the electromotive force of one alternator and lags behind that of the other. Fig. 125. Except for the resistance loss, this current is wattless and its effect is to increase the magnetization of one alternator and decrease that of the other (Section 9), so that the indicated voltage of the system is the mean of the voltages of the generators when operated separately.

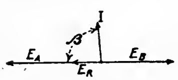


FIG. 125.

If the frequencies are not the same, the instantaneous voltages are not equal at all times, and equalizing currents flow between the alternators.

If the electromotive-force waves are not similar, or do not attain their maximum at the same instant, the instantaneous values are unequal, and equalizing currents flow between the generators as described above. If the wave forms differ greatly or the alternators are out of phase opposition by more than a few degrees, the equalizing currents are usually so large as to operate the protective devices (fuses or circuit breakers) which should be placed in every circuit.

Therefore, alternators whose operating characteristics or wave forms differ radically should not be connected in parallel, nor is it desirable to connect in parallel alternators whose prime movers have materially different speed characteristics.

Synchronizing and division of load. — The process of regulating the voltage, the frequency and the phase relation of an alternator so that it may be connected in parallel with other alternators, is termed "synchronizing."

The connections between two three-phase alternators and a

common load circuit are shown in Fig. 126. If A is carrying the load and it is desired to connect B to the system, start B and regulate its field excitation and the speed of its prime mover until:

(*a*) The indication of its voltmeter is equal to or slightly greater than the voltage across the bus bars.

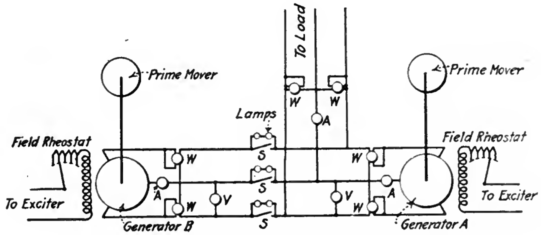


FIG. 126. Parallel Operation of Alternators.

(*b*) The lamps connected across the open switch S are dark for several seconds at a time.

Close the switch S quickly at a time when the lamps are dark. The alternators are now in parallel but A is still supplying practically all the load current. The division of the load between two alternators operating in parallel cannot be adequately explained without reference to the principles of the synchronous motor, so that only a statement of facts will be attempted here.

(*a*′) Changing the field excitation causes a wattless current to flow between the armatures without changing, to any appreciable extent, the distribution of the load.

(*b*′) The load distribution on alternators operating in parallel is changed *only* by changing the relative torques of the prime movers.

The lamps in Fig. 126 serve the double purpose of indicating when the correct phase relations are obtained, and of limiting the flow of current between the armatures during the synchronizing period. With the connections indicated in Fig. 126, all lamps are either dark or bright at the same instant. It is possible to make other lamp connections. When the lamps are arranged in a circle and two sets are cross-conneted between lines A and B, the light appears to travel around the circle, its direction of apparent rotation depending on whether the speed of the incoming machine is too fast or too slow.

Because synchronizing lamps give only an approximate indication of the time when the switch should be closed, the lamps being dark

even when a considerable difference of potential exists between their terminals, mechanical devices known as "synchroscopes" or "synchronism indicators" have been devised and are in extensive use.*

18. Ratings. — Alternators are rated in kilovolt-amperes (kv-a.) rather than in kilowatts, because the power output of a given alternator is dependent on the power factor of the load circuit; the maxi-

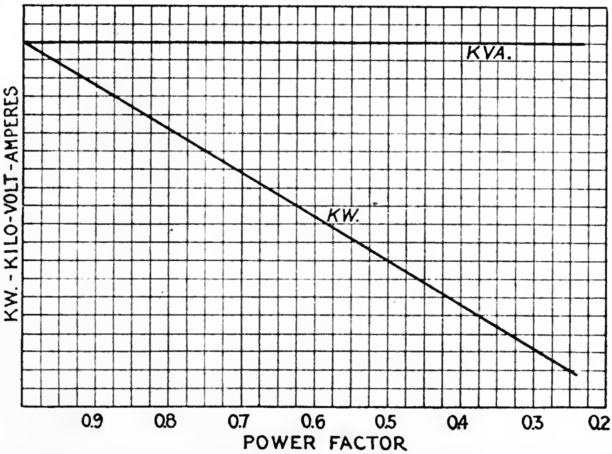


FIG. 127. Curves Showing the Relation between Kilowatts, Kilovolt-amperes and Power Factor, the Voltage and the Armature Current of the Alternator Remaining Constant.

mum load, for the allowable temperature rise, is a function of the armature current. The kilowatt output of a generator may vary over wide limits, by a variation of the power factor of the load circuit, the current output remaining constant. Fig. 127.

The output of an alternator is limited by the allowable heating of the armature, and by saturation in the iron parts of the magnetic circuit. Alternators have no commutators, and are not troubled with sparking.

CHAPTER VIII — PROBLEMS

1. The armature winding of a single-phase alternator consists of 1764 conductors (series) which are distributed uniformly over the entire armature surface, and in each of which an average electromotive force of 2 volts is induced. There are six slots per pole. Find: (*a*) the terminal voltage of the alternator at zero load, (*b*) the maximum value of armature magnetization (in effective ampere-

* See Chapter 17, Section 9.

turns) when the current output of the generator is 100 amperes, (*c*) the magnetizing and the distorting components of armature reaction when: (1) the current and the electromotive force are in phase, (2) the current lags 10 degrees behind the electromotive force, (3) the current leads the electromotive force by 15 degrees.

2. Same as Problem 1 except the armature conductors are distributed over $\frac{2}{3}$ of the armature surface.

3. Same as Problem 1 except the armature conductors are star-connected, and the current in each line is 100 amperes (3-phase).

4. Same as Problem 3 except the armature conductors are delta-connected (3-phase).

5. A test on a 2300-volt, 230-kv. a., single-phase alternator gave the following:

On Open Circuit

Field current	Terminal voltage
10	500
20	1000
30	1500
40	1950
50	2250
60	2500
70	2650

On Short Circuit

Field current	Armature current
6.25	25
12.50	50
18.75	75
25.00	100

Resistance of armature = 2 ohms.

Plot the saturation curve and the short-circuit curve, and calculate, by the electromotive force method, the percentage regulation of the alternator at: (*a*) unity power factor, (*b*) 85 per cent power factor lagging, (*c*) 85 per cent power factor leading.

6. Same as Problem 5 except regulation percentages are to be calculated by the magnetomotive force method.

7. Same as Problem 5 except regulation percentages are to be calculated by the A.I.E.E. method.

8. Using the graphical method outlined in Section 12, determine the terminal voltages of the alternator in Problem 5 when the power factor of the load circuit is unity, and the following currents flow in the armature winding: 0, 25, 50, 75, 125, 150 amperes. Plot the voltage characteristic.

9. Same as Problem 8 except the power factor of the load circuit is 85 per cent lagging.

10. Same as Problem 8 except the power factor of the load circuit is 85 per cent leading.

11. A test of a 6600-volt, 1000-kv.-a. 3-phase, star-connected alternator gave the following:

On Open Circuit

Field ampere-turns	Phase voltage
2,000	1071
4,000	2175
6,000	2990
8,000	3392
10,000	3534
12,000	3841
14,000	3979
16,000	4082
18,000	4163
20,000	4225
22,000	4280
24,000	4330

An excitation of 4310 ampere-turns is required when full-load (rated) current flows in the short-circuited armature windings.

Armature resistance (measured line to neutral) = 1 ohm.

Plot the magnetization curve.

Calculate the percentage regulation at unity power factor by: (*a*) the electromotive force method, (*b*) the magnetomotive force method, (*c*) A.I.E.E. method.

12. Same as Problem 11 except the power factor of the load circuit is 90 per cent lagging.

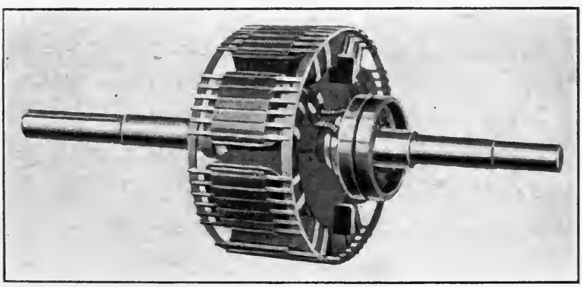
13. Same as Problem 11 except the power factor of the load circuit is 90 per cent leading.

CHAPTER IX

THE SYNCHRONOUS MOTOR

1. **Structure.** — Structurally, the synchronous motor is identical with the alternating-current generator, and the same machine may be used as a generator or as a motor.

(a) Stator (Armature).



(b) Rotor (Field).

FIG. 128. The Synchronous Motor. (Westinghouse.)

2. **Principles of operation.** — From the fundamental principles explained in Chapter 2, it is evident that a conductor carrying an alternating current may be made to move continuously in one

direction if the direction of the flux through which it moves is reversed at the same instant that the direction of the current in the conductor is reversed.

Consider the looped conductor in Fig. 129. When the current is flowing away from the observer in the part under the north pole, and towards the observer in the part under the south pole, the direction of motion is, from Fleming's left-hand rule, as indicated by the arrow. This direction of motion is made continuous by reversing the direction of the current in the loop when the plane of the loop is in the vertical position, *i.e.*, the number of revolutions per second made by the loop must be equal to the frequency of the alternating current, and the change in the direction of the current must be made when the conductors are approximately midway of the arc between the poles. Under these conditions, the relations between the field flux and the current in the moving conductor are fixed, and the torque is exerted in one direction only. Therefore, if the frequency of the supply circuit is constant, the speed of a synchronous motor is constant, but the motor is not self-starting.

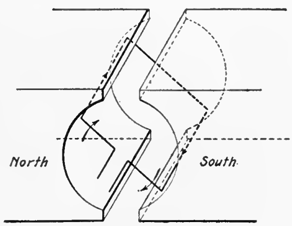


FIG. 129.

3. **Torque-load adjustment.** — It is an easily demonstrated experimental fact that when the driving torque of an alternating-current generator, operating in parallel with other generators, is reduced to zero, its moving parts continue to rotate at the same speed, and that the direction of the current in the armature windings is reversed. When the load on a continuous-current shunt motor increases, the speed of the armature decreases until the counter-electromotive force is reduced to such a value that the current necessary to produce the required torque flows in the armature circuit. Since the speed of a synchronous motor is fixed by the frequency of the system to which it is connected, the increased armature current (torque) must be produced by other means.

A synchronous motor, like a continuous-current motor, generates a counter-electromotive force which is proportional to the speed, and is dependent on the field excitation. The current in the armature of a synchronous motor may, then, be changed by changing the field excitation, but it would be practically impossible to oper-

ate, commercially, a motor the field current of which must vary with the load. Consider the conditions when the counter-electromotive force of the motor is equal to the applied electromotive force. As long as the phase difference of these equal electromotive forces is 180 degrees, no current flows in the armature of the motor, and no torque is developed in it. Consequently, the rotating parts tend to stop, and the angle of phase difference becomes greater than 180 degrees.

Let the phase relations of the two equal electromotive forces be as indicated in Fig. 130. A current proportional to the resultant electromotive force E_r flows in the armature of the motor, and the angle between the resultant electromotive force and the current is β, the tangent of which is equal to $\frac{X_a}{R_a}$.

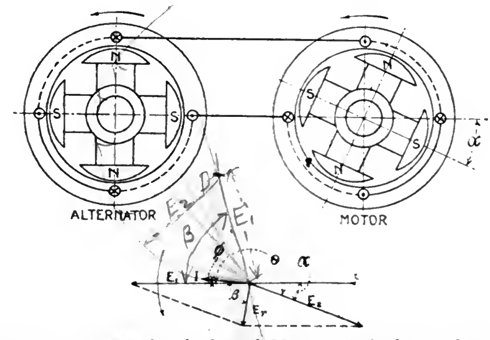


FIG. 130. Mechanical and Vector Relations of an Alternator and a Synchronous Motor.

$$\beta = \tan^{-1} \frac{X_a}{R_a}, \tag{1}$$

when X_a = the synchronous reactance of the armature circuit,
R_a = the resistance of the armature circuit.

Since R_a is always small as compared to X_a, the angle β is usually greater than 80 degrees but can never equal 90 degrees.

The input to the motor under the above conditions is

$$P = EI \cos \phi. \tag{2}$$

If the load increases, the angle between the applied and the counter-electromotive forces will, evidently, increase until equilibrium is reëstablished, *i.e.*, with constant field excitation, the torque of a synchronous motor is dependent on the phase relations of the applied and the counter-electromotive forces, and the angle between them is automatically adjusted as the load changes.

4. **Starting.**—A synchronous motor is not self-starting because the periodical reversal of the direction of the current in the station-

ary armature conductors produces a torque which periodically reverses, and tends to produce rotation of the field structure first in one direction, then in the other. The commercial methods of starting a synchronous motor are: (*a*) by means of an auxiliary motor, (*b*) as an induction motor.

(*a*) *By an auxiliary motor.* — The starting motor is a small motor, either shunt or induction, direct-connected or belted to the shaft of the synchronous machine. When a synchronous motor is started in this way, it is a generator and must be synchronized * before it is connected to the supply circuit. After the synchronous motor armature is connected to the alternating-current supply circuit, the circuit of the starting motor is opened. In case the synchronous machine is provided with a separate exciter and a source of continuous current is available, the exciter may be used as a starting motor. An induction motor is, however, more often used.

This method requires the use of synchronizing apparatus and the installation of a starting motor, but causes minimum disturbance in the alternating-current system during the starting period.

(*b*) *As an induction motor.*† — If the field circuit of a synchronous motor is opened and an alternating current supplied to the armature, the changing magnetism set up by the armature currents induces currents in the field pole shoes. These currents, reacting with the magnetism set up by the armature winding, produce a small torque, if the motor is polyphase, and the unloaded motor starts without the use of auxiliary devices. When the motor attains approximately synchronous speed, which is indicated by a violent swinging of the pointer of an ammeter connected in the alternating-current supply line, the field circuit is closed and the motor pulls into step with the supply circuit.

The starting torque of a synchronous motor, when started in this way, is increased by means of a "squirrel cage" field structure similar to that shown in Fig. 128b. This construction also tends to prevent "hunting," as explained in Section 8.

* See Chapter 8, Section 17.

† This method of starting a synchronous motor depends on the fundamental principles underlying the action of the induction motor, and will not be understood until those principles have been studied. The statements made here are to be taken simply as a mechanical process by which the synchronous motor may be started.

Because of the low power factor and the large current required during the starting period, this method of starting may cause undesirable disturbances in the system from which it is supplied. These disturbances are particularly objectionable when a single motor forms a considerable portion of the total load on the supply system and is stopped and started frequently, or when motors are operated in parallel with incandescent lamps.

5. **Stability.** — An engine or motor is in stable equilibrium as long as an increased load automatically produces a corresponding intake of power. As noted in Section 3, an increased load on a synchronous motor causes the angle between the applied and the counter-electromotive forces to increase. Assuming the applied

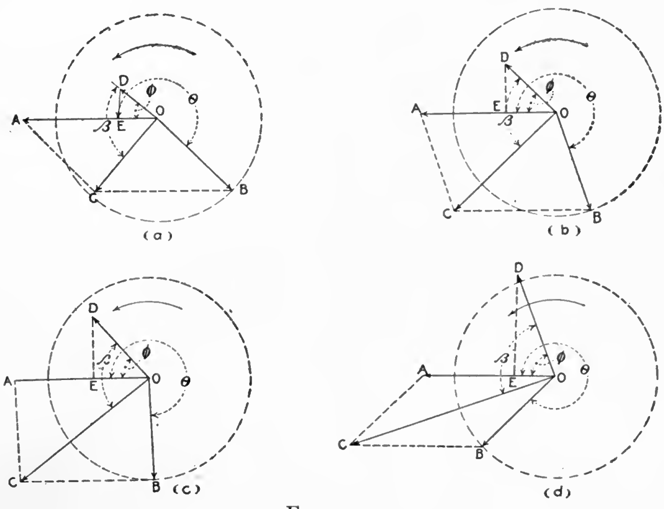


FIG. 131.

voltage to be constant, the operation of the motor is stable as long as the product of the current and the power factor increases; but it is evident that, as the angle between the two electromotive forces increases, a point is finally reached where the power factor decreases faster than the current increases. Beyond this point the motor is in unstable equilibrium, drops out of step, and stops.

The above statement is illustrated by Fig. 131, in which OA is the vector of applied electromotive force, and OB the vector of counter-electromotive force. OC is the vector of the resultant electromotive force, and OD the vector of current flowing in the armature circuit. The impedance of the circuit is approximately con-

stant, and OD is proportional to OC. The power input to the motor is proportional to the projection of the current vector on the vector of applied electromotive force, *i.e.*, to the product of the current vector and the cosine of angle AOD.

For the relation of the vectors shown in Fig. 131a, the power input to the motor is proportional to OE. As the load on the motor increases, the angle θ increases until the vector of the counter-electromotive force reaches the position indicated in Fig. 131c. If the angle θ increases still further, the projection OE decreases and the power input to the motor decreases correspondingly, as indicated in Fig. 131d.

Therefore, if a synchronous motor is loaded beyond a certain point, it cannot develop sufficient torque to carry the load, and will stop. In the commercial motor, the armature current usually becomes excessive and overheats the motor before it reaches the point of instability.

6. Maximum load. — That the maximum intake of a synchronous motor for any given counter-electromotive force (field excitation) occurs when $\theta - \beta = 180$ degrees is evident from the following considerations: Referring to Fig. 131c let

$$\theta - \beta = 180°. \tag{3}$$

If the angle θ is either increased or decreased, both the projection of the current vector on the vector of the electromotive force and the input to the motor are decreased.

7. Efficiency. — The efficiency of a synchronous motor is determined: (a) by a brake test, (b) from the losses.

(a) *Brake test.* — Measure the input and the output, and compute the efficiency from the equation

$$\text{Per cent efficiency} = \frac{\text{output} \times 100}{\text{input}}. \tag{4}$$

(b) *From the losses.* — The losses in a synchronous motor are the same as those of an alternating-current generator and are determined in the same way.*

8. Hunting. — The phenomenon known as "hunting" in a synchronous motor, or a rotary converter, consists of a periodical

* See Chapter 8, Section 15.

variation of the speed of the rotating parts, the speed being alternately too fast or too slow, and is indicated by the swinging of the ammeter pointer, and by a humming noise peculiar to this condition.

Hunting may be caused by: (*a*) a sudden change in the load, (*b*) irregularities in the speed of the prime mover driving the generator from which the motor is supplied, (*c*) faulty design of the motor. The inertia of the rotating parts of the motor tends to damp out any oscillations that may be set up, the addition of a heavy flywheel adding to this damping effect. Hunting is often guarded against, in the design of a synchronous motor, by wedging bars of copper between the tips of the pole shoes, or by a "squirrel cage" structure similar to that shown in Fig. 128.

As long as the angular velocity of the rotating parts of the motor is constant, the copper bars have no effect, but when hunting takes place the oscillatory motion causes a relative movement of the bars and the flux set up by the armature winding. This movement induces an electromotive force, and currents flow in the bars. The reaction between the currents in the bars and the flux set up by the armature winding tends to damp out the oscillations, hence the name "magnetic dampers" which is sometimes applied to this construction.*

9. Phase characteristic. — The phase characteristic, commonly called V-curve, of a synchronous motor shows the relations between the armature current and the field excitation, the load remaining constant. Referring to Fig. 132, let

OA be the vector of the applied electromotive force,
OB be the vector of the counter-electromotive force,
OC be the vector of the resultant electromotive force,
OD be the vector of the armature current,
β be the angle between the current and the resultant electromotive force,
ϕ be the power factor angle,
$I \cos \phi$ be constant, *i.e.*, the locus of the current vector is a straight line perpendicular to the vector of the applied electromotive force.

* The action of copper bars in the prevention of hunting will be more apparent after the induction motor has been studied.

When $\phi = 0$ (current and applied electromotive force in phase), the relations in the circuit are represented in Fig. 132b; when $\phi = 30$ degrees (lagging), the relations in the circuit are represented in Fig. 132a; and when $\phi = 30$ degrees (leading), the relations in the circuit are represented in Fig. 132c.

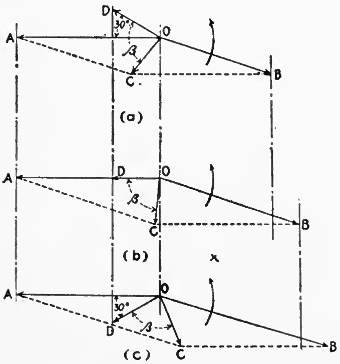


FIG. 132.

A comparison of the counter-electromotive forces in Fig. 132, shows that the power factor of a synchronous motor is dependent on its counter-electromotive force, *i.e.*, on its field excitation. The power factor of the motor is, therefore, changed by changing its field excitation.

Fig. 133 shows phase characteristics for different loads. For a constant load, the power factor of the motor is computed as the ratio of the minimum armature current to the armature current at the given or required field excitation. Because of distortion of the wave shape, unity power factor is seldom or never attained.

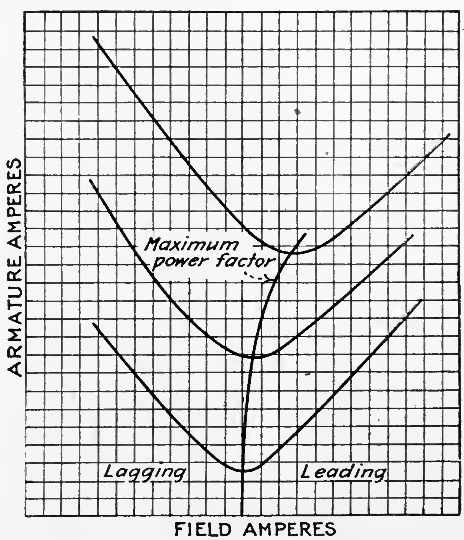


FIG. 133. Phase Characteristics.

10. The circle diagram. — In any alternating-current circuit having a constant applied voltage, a constant reactance and a variable resistance, it may be shown that the locus of the current vector is a semicircle. The phenomena of the synchronous motor may, therefore, be represented by means of a circle diagram, when it is assumed that the synchronous reactance of the motor is constant.*

* This assumption simplifies the treatment, and the error introduced is not great.

Lay off AO and OD (Fig. 134) proportional to the applied electromotive force, making the angle AOD such that

$$\beta = \tan^{-1}\frac{X_a}{R_a}, \tag{5}$$

when X_a = the synchronous reactance of the motor armature,
R_a = the resistance of the motor armature.

With D as a center and a radius proportional to the counter-electromotive force at any given or required field excitation, draw a semicircle. The semicircle is the locus of the current vector OI, and the reactance drop OC, the resistance drop BC, and the counter electromotive force AB may be drawn in their correct phase relations and magnitudes as indicated in the figure.

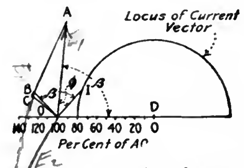


Fig. 134. Circle Diagram.

11. **The synchronous phase-modifier.** — As shown in Fig. 132c, a synchronous motor, when over excited, takes a leading current from the supply system. The wattless component of a leading current opposes and neutralizes an equal wattless component in a transmission line or a distributing system * due to a parallel inductive load such as an induction motor. When used for the purpose of improving the power factor of a circuit, a synchronous motor is termed a "synchronous phase-modifier," although it may deliver mechanical power at the same time.

Example. — Two 100-h.p., 440-volt induction motors operate at 85 per cent power factor and 87 per cent efficiency. Find the saving in transmission line copper when one of the induction motors is replaced by a synchronous motor, the efficiency of which is 85 per cent when over excited so as to compensate for the lagging current of the other induction motor. Also find the rated output of the synchronous motor, and its power factor.

Solution.

$$I = \frac{200 \times 746}{0.85 \times 0.87 \times 440}$$

$$= 465 \text{ amperes for two induction motors,}$$

* See Chapter 1, Section 38.

$$I = \frac{100 \times 746}{0.87 \times 440} + \frac{100 \times 746}{0.85 \times 440}$$
$$= 194 + 199$$
$$= 393 \text{ amperes for one induction motor and one synchronous motor,}$$

$$R' (465)^2 = R'' (393)^2$$
$$R' = 0.72 R'',$$

i.e., the weight of copper is approximately 40 per cent greater for two induction motors than for one induction motor and one synchronous motor.

$$\text{P.F. of synchronous motor} = \frac{199}{\sqrt{(199)^2 + (233 \times 0.52)^2}}$$
$$= \frac{199}{234}$$
$$= 0.85.$$

$$\text{Rating of synchronous motor} = \frac{100}{0.85}$$
$$= 119 \text{ h.p. (use 125 h.p.).}$$

12. Load division of alternators in parallel. — Let A and B be two similar alternating-current generators operating in parallel, the field excitation, terminal voltage and armature current being the same in each machine.

If the power input to the prime mover driving alternator B is reduced, the load on the alternator tends to stop its rotating parts, the angle between the electromotive forces becomes greater than 180 degrees, and a current flows between the armatures, *i.e.*, alternator B acts simultaneously as an alternating-current generator and as a synchronous motor, the current for its motor action being received from alternator A. Therefore, the load division of alternating-current generators operating in parallel is changed by changing the power input to their prime movers.

Since the input to a prime mover, *e.g.*, a steam engine, is constant at a given speed, changing the field excitation of alternator B does *not* change its output, although it causes a current to flow between the armatures. Since the resultant electromotive force is in phase with that induced in alternator A and the angle β is nearly 90 degrees, it is evident that this current is wattless except for the small

power component due to the resistance of the circuit, that it leads the electromotive force of alternator B and lags behind the electromotive force of alternator A. This quadrature current can produce no torque in either machine and the load distribution of alternators operating in parallel is, therefore, *not* disturbed by a change in field excitation.

Since the current flowing between the armatures leads the electromotive force of alternator B and lags behind the electromotive force of alternator A, its effect is to equalize the voltages induced in the armatures by reducing the flux set up by one field winding and increasing that set up by the other.

CHAPTER IX — PROBLEMS

1. Show, by means of the circle diagram, that the power factor of a synchronous motor is changed by changing its field excitation, the load remaining constant.

2. The synchronous impedance of an alternator is 1.5 ohms. When operated as a synchronous motor from 220-volt mains and without load, the armature current is 10 amperes, and the power factor is 0.866 leading. The angle $\beta = 85$ degrees. Find the counter-electromotive force of the motor.

3. Find the counter-electromotive force of the motor in Problem 2 when it power factor is unity.

4. Find the power factor of the motor in Problem 2 when the applied voltage is reduced 10 per cent.

5. Calculate the armature resistance of the motor in Problem 2.

6. Draw the vector diagram for the three-phase alternator of Problem 11, Chapter 8, when operated as a motor at unity power factor, and rated armature current.

7. From the diagram obtained in Problem 6, calculate the torque developed in the armature.

8. A 3-phase, 2300-volt, 500-kv-a. synchronous motor is overexcited so that its power factor is 0.6 leading. Calculate the capacitance to which it is equivalent when operated from 60-cycle mains, and full-load (rated) current flows in the armature circuit.

9. A 3-phase, 2200-volt induction motor having a lagging power factor of 0.85 takes 100 amperes (line), and is connected in parallel with an overexcited synchronous motor. When the power factor of the system is unity the current (line) supplied to the synchronous motor = 125 amperes. Find: (*a*) the power factor of the synchronous motor, (*b*) the line current, (*c*) the input (watts) to each motor. Draw a diagram representing the electromotive force and the currents in the system.

10. Plot the circle diagram for the alternator in Problem 11, Chapter 8, when operated as a synchronous motor, and from this diagram construct the phase characteristic (V-curve) for an input of: (*a*) 500 kw., (*b*) 100 kw.

11. Same as Problem 10 except the resistance of the armature is 2 ohms.

12. Same as Problem 10 except the resistance of the armature is 0.5 ohm.

13. The alternator in Problem 2 is used as a phase modifier. The armature current is 100 amperes and the power factor 0.1. Find: (*a*) the power input to the armature, (*b*) the wattless component of lagging current that it will neutralize.

14. The alternator in Problem 2 is driven as a synchronous motor from 220 volt mains. The counter-electromotive force is 220 volts. Find: (*a*) the current input to the motor at no load, (*b*) the power factor at which the motor operates, (*c*) the phase angle between the applied and the counter-electromotive forces.

CHAPTER X

CURRENT-RECTIFYING APPARATUS

Since electrical energy is most economically and satisfactorily transmitted by means of alternating currents, and many commercial applications require unidirectional currents, it is necessary to provide means for the rectification or conversion of alternating currents. Alternating currents may be made unidirectional by: (*a*) a mechanically-driven commutator, (*b*) a motor-generator, (*c*) the mercury arc rectifier, (*d*) the synchronous converter.

(*a*) *The Commutator*

1. A commutator driven by a small synchronous motor would appear to be a cheap and an efficient means for the rectification of alternating currents. This method is, however, of little commercial importance because of its limited capacity, excessive sparking, with a consequent destruction of the commutator, taking place whenever either the current or the electromotive force increases above very limited values.

(*b*) *The Motor-Generator*

2. The motor-generator consists of a motor, either synchronous or induction, which drives a continuous-current generator. The apparatus differs in no way from that described elsewhere in this volume. Motor-generators are in extensive use where a wide variation of the continuous voltage is required. Because it consists of two machines, the first cost of a motor-generator is high, and the operating efficiency low.

(*c*) *The Mercury Arc Rectifier*

3. **Construction.** — The mercury arc rectifier consists of a highly exhausted glass tube having two or more iron or graphite terminals to which the alternating-current supply circuit is connected, and a mercury terminal to which the load circuit is connected. Fig. 135.

It is also provided with an auxiliary mercury terminal which is used for starting purposes only. The circuits for a single-phase rectifier are shown in Fig. 136a, and those for a three-phase rectifier in Fig. 136b.

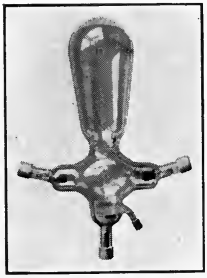


Fig. 135. Mercury Arc Rectifier Tube. Westinghouse.

The mercury arc rectifier depends, for its operation, on the fact that an electric current may flow from iron or graphite terminals to mercury, but may not flow in the reverse direction. The flow of current in the rectifier circuit is similar to that of water in the hydraulic system shown in Fig. 137, where check valves VV permit water to flow only in the direction indicated by the arrows. In the single-phase rectifier, current flows from terminal A to the mercury during one-half cycle, and from terminal B to the mercury during the other half cycle. The current in the load circuit is, therefore, a unidirectional (pulsating) current.

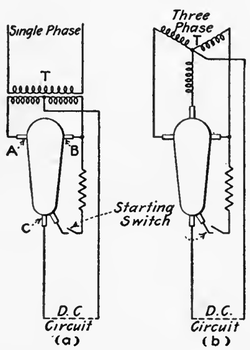


Fig. 136. Wiring Diagrams for Mercury Arc Rectifier.

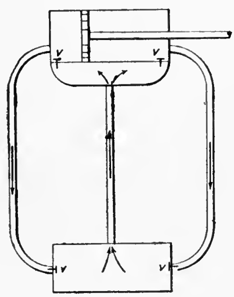


Fig. 137. Hydraulic Analogy for Mercury Arc Rectifier.

4. **Starting.**—The mercury arc rectifier cannot be started simply by connecting the iron, or graphite, terminals to an alternating-current supply circuit, because of the high initial resistance of the tube. The auxiliary mercury terminal is, therefore, necessary to break down this high resistance. When the starting switch is closed, and the bulb tilted slightly so that the mercury terminals are connected by the liquid mercury in the tube, an electric current flows through the mercury and fills the tube with vapor. The mercury vapor reduces the resistance of the main circuit so that

current may flow between the iron terminals and the mercury. The tube may now be allowed to return to its normal position and the starting switch opened, the load circuit having been closed through a suitable resistance, since the arc will "break" and the rectifier cease to operate if the current falls below a certain minimum value.

5. Operation. — Because the arc "breaks" when the load current falls below a certain minimum value, which varies from two to four amperes, an inductance must be placed in the load circuit so that the decay of the current established during one-half cycle is delayed until that from the other half cycle can be established * making the form of the wave as shown in Fig. 138. Without this inductance, which may be the secondary of the transformer supplying the rectifier, the current decreases to zero at the end of each half cycle and the operation of the arc is interrupted. Because of the overlapping of the waves in the different phases of a polyphase system, no inductance is required in the load circuit of a rectifier using polyphase currents.

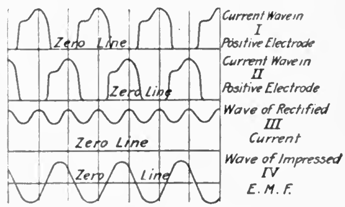


FIG. 138. Wave Forms Mercury Arc Rectifier.

The voltage E_{dc} of the load circuit bears a constant ratio to the voltage E_{ac} of the supply circuit.

$$E_{dc} = \frac{\sqrt{2}}{\pi} E_{ac} - k, \tag{1}$$

when k is the counter-electromotive force of the tube which varies, in different tubes, from thirteen to fifteen volts.

Because of the high temperatures at which they operate, the tubes of commercial rectifiers are usually immersed in oil for the better dissipation of the heat.

The mercury arc rectifier delivers constant electromotive force or constant current, as the transformer T is constant potential or constant current.

6. Efficiency. — From equation (1), it is evident that from thirteen to fifteen volts are lost in the rectifier tube, whatever the applied voltage may be. Therefore, the efficiency is greater

* This statement applies to single-phase rectifiers only.

at high voltages than at low, but is constant at all loads. At low voltages, such as are commonly used for motors, the efficiencies of mercury arc rectifiers do not compare favorably with other types of rectifying apparatus.

7. **Limitations and use.** — Because of its low efficiency at the voltages commonly used for continuous-current apparatus, and its poor power factor, the use of the mercury arc rectifier is restricted, at the present time, to special purposes. It is largely used for supplying unidirectional current to arc lamp systems where the high voltage and the small current required make it very satisfactory when operated in connection with constant-current transformers. It is also used, in small units, for charging storage batteries.

(*d*) *The Synchronous Converter*

8. **Construction.** — The synchronous, or rotary, converter does not differ essentially from a shunt or compound (continuous-current) dynamo with the addition of symmetrical connections from the armature winding to two or more continuous rings mounted on the armature shaft in the same manner as for an alternating-current generator with rotating armature. Fig. 139. Practically, the converter which has more than two rings differs from the continuous-current machine of the same rating by having a much larger commutator, and smaller yoke and field poles.

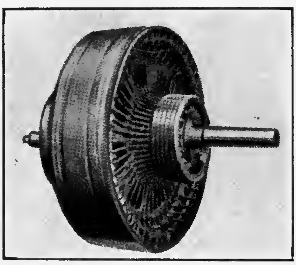


Fig. 139. Synchronous Converter Armature. Westinghouse.

9. **Operation.** — A synchronous converter may be operated as:

(*a*) A continuous-current motor.

(*b*) A continuous-current generator.

(*c*) A synchronous motor.

(*d*) An alternating-current generator.

(*e*) A double-current generator, *i.e.*, simultaneously as a continuous-current generator and an alternating-current generator.

(*f*) A converter, *i.e.*, simultaneously as a synchronous motor and a continuous-current generator. When so operated, the armature must rotate at synchronous speed, and the speed of any given

converter is determined by the frequency of the supply circuit. The converter exhibits the same phase characteristic, power factor and line-current changing with the field excitation, as the synchronous motor.

(*g*) An inverted converter, *i.e.*, simultaneously as a continuous current motor and an alternating-current generator. When so operated, the speed and, consequently, the frequency of the alternating current, varies with the field excitation, unless operated in parallel with other synchronous apparatus which controls the frequency of the circuit. Since the armature reaction of an alternating-current generator, the load circuit of which is inductive, tends to demagnetize the field, the speed of an inverted synchronous converter supplying such a circuit may become excessive, and means should be provided for preventing these high speeds which may wreck the machine. Two methods are employed to limit the speed of an inverted converter to safe values: (1) separate excitation, (2) speed-limit switches.

(1) *Separate excitation.* — If an inverted converter, instead of being excited from the supply mains, has its field windings supplied from a separate exciter direct connected, or belted, to the armature shaft, an increase or decrease in the speed of the armature causes a corresponding increase or decrease in the excitation of the converter, and tends to keep the speed constant.

(2) *Speed-limit switches.* — A speed-limit switch is attached to the shaft of the converter, and operates circuit breakers in the continuous-current supply mains when the speed reaches a predetermined value, thus stopping the machine.

10. Voltage relations. — The voltage between the brushes of a continuous-current generator is the *algebraic* sum (constant) of the instantaneous electromotive forces induced in the different coils connected in series between the brushes; the maximum alternating voltage induced in the same coils is equal to the *geometric* sum of the maximum electromotive forces induced in the separate coils. But the *geometric* sum of the maximum alternating-electromotive forces induced in the distributed windings of a synchronous converter, between adjacent continuous-current brushes, is the *algebraic* sum of the instantaneous electromotive forces. Therefore, the continuous voltage of a synchronous converter is equal to the

maximum alternating-electromotive force induced in the coils connected in series between adjacent continuous-current brushes.

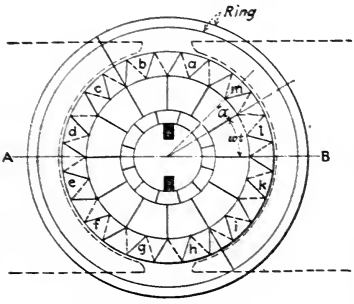


FIG. 140. Single-phase Rotary Converter.

Let an armature have twelve coils symmetrically placed, as indicated in Fig. 140. The maximum alternating electromotive forces induced in the coils, and their phase relations, are represented by a twelve-sided polygon, and the maximum alternating voltage between continuous-current brushes is equal to the geometric sum of the maximum electromotive forces induced in any six coils, *i.e.*, to the diameter of the circumscribed circle. Fig. 141. The sum of the instantaneous electromotive forces induced in the coils between continuous-current brushes is also the diameter of the circumscribed circle. If the armature winding is connected to two rings, the maximum alternating voltage between rings is equal to the voltage between the continuous-current brushes, and the effective alternating voltage is equal to the continuous voltage divided by the square root of 2. Fig. 141.

$$(E_{ac})_m = E_{cc} \qquad (2)$$

$$E_{ac} = \frac{E_{cc}}{\sqrt{2}}. \qquad (3)$$

FIG. 141.

Similarly, if the armature is provided with three rings, the maximum alternating voltage between rings is the geometric sum of the maximum voltages induced in four coils, or the side of the inscribed triangle; if four rings, the geometric sum of the maximum voltages induced in three coils, or the side of the inscribed square; if six rings, the geometric sum of the maximum voltages induced in two coils, or the side of the inscribed hexagon.

Therefore, the effective alternating voltage between adjacent rings of an n-ring synchronous converter required to give a specified continuous voltage E_{cc} is

$$E_{ac} = \frac{E_{cc} \sin \dfrac{180^\circ}{n}}{\sqrt{2}}. \qquad (4)$$

It will be remembered that the phase voltage of a four-ring (two-phase) converter is the voltage between alternate rings. Hence, the phase voltage of a four-ring converter is equal to the phase voltage of a two-ring converter, the continuous voltage remaining constant.

TABLE IV

E_{ac} for a 2-ring converter	= 0.707 times the continuous voltage
3-	= 0.612
4-	= 0.500
6-	= 0.354

From the above considerations it is evident that the ratio between the alternating and the continuous voltages of a synchronous converter is fixed, *i.e.*, the voltage at the continuous-current brushes can be changed only by changing the voltage between the alternating-current rings. The theoretical ratios developed above are obtained, within very narrow limits, in the commercial converter. The practical considerations which affect the ratios are: (1) the resistance of the armature windings, (2) the shape of the alternating electromotive-force wave.

(1) *Armature resistance.* — The theoretical ratios of voltages given above are changed by virtue of the resistance of the armature windings but this resistance is always small, and the ratios are never seriously affected.

(2) *Wave shape.* — When the electromotive force wave is not harmonic, the effective value is not equal to the maximum divided by the square root of 2. For example, for a flat-topped wave the effective value is greater than for a sine wave having the same maximum, while for a peaked wave, the effective value is less. In a well-designed converter, the wave shape is not seriously distorted.

11. Current ratios. — The ratio of the current in the alternating-current circuit to that in the continuous-current circuit of a converter is easily calculated when the losses in the converter are neglected, *i.e.*, when the output is considered equal to the input.

In an n-ring two-pole armature, the alternating voltage between the neutral and any ring is

$$E_{ac} = \frac{E_{cc}}{2\sqrt{2}}, \tag{5}$$

and
$$\frac{nI_{ac}E_{cc}}{2\sqrt{2}} = E_{cc}I_{cc}, \tag{6}$$

when the power factor of the converter is unity. Therefore, the line current

$$I_{ac} = \frac{2\sqrt{2}\,I_{cc}}{n}, \tag{7}$$

and the current in the armature conductors

$$I_{ac}' = \frac{\sqrt{2}\,I_{cc}}{n \sin \frac{180^\circ}{n}} \tag{8}$$

12. Starting. — The preferable way to start a synchronous converter is as a continuous-current motor, or by means of a small starting motor, after which the alternating-current end is synchronized* with the system from which the converter is to be supplied. When no continuous-current energy is available, the converter may be started as an induction motor, as described for the synchronous motor.† The same disturbances are caused in the alternating-current system as with the synchronous motor, and the polarity of the continuous-current brushes *must* be determined before connection is made to a storage battery, or other apparatus through which the current flows in a given direction. The polarity of the continuous-current brushes is fixed by the last pulsation of the alternating current as the armature pulls into step with the supply circuit.

13. Compounding. — Since an increased field excitation of a converter, as of a synchronous motor, is neutralized by an increased wattless component of armature current, compounding of a synchronous converter is not possible, in the sense that the term is used in connection with continuous-current dynamos. The only way to increase the continuous voltage of a converter is to increase the alternating voltage applied at the rings. A series or compound field winding may be made to increase the continuous voltage, provided the alternating-supply circuit is inductive.

If the alternating-current circuit supplying a converter is inductive, the inductive drop in the line makes the voltage at the rings less than it would be if the circuit were non-inductive.

* See Chapter 8, Section 17.

† See Chapter 9, Section 4.

When a synchronous motor is over excited, a leading current flows in the alternating-current system,* and the circuit is equivalent to a series circuit containing resistance, inductance and capacitance. The inductive line drop is thus neutralized, partially or completely, and the voltage at the rings of a converter, and consequently, the continuous voltage, is increased. Because its average power factor is inherently low, the operation of a compound-wound converter is unsatisfactory.

14. The regulating (or split) pole converter. — For automatically increasing the continuous-voltage as the load on the converter increases, the regulating pole (the so-called "split" pole) converter has been designed. In this type of converter, the field pole is divided into two parts, each part has its own winding, and may be excited to any required degree independently of the other part. If the regulating pole is unexcited, the converter operates in the usual manner and the continuous voltage of a two-ring converter is equal to the maximum induced alternating-electromotive force.

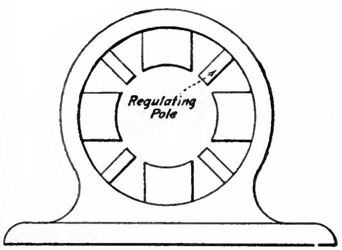


FIG. 142. The Regulating Pole Converter.

Let Fig. 142 represent a two-ring converter having two sets of field poles, the smaller (regulating) poles being placed midway between the larger (main) poles. When the converter is in operation, the maximum counter-electromotive force (OA, Fig. 143a) induced in the armature windings is equal, neglecting resistance drop, to the maximum applied electromotive force. But the counter-electromotive force is the resultant of two quadrature electromotive forces, as shown in Fig.

FIG. 143.

143a, one induced in the winding as it passes under the main poles, the other induced in the winding as it passes under the regulating poles, *i.e.*, this construction is electrically equivalent to two arma-

* See Chapter 9, Section 9.

ture windings in series, the induced electromotive forces in which are 90 degrees out of phase.

The continuous voltage of a synchronous converter is the algebraic sum of the instantaneous electromotive forces induced in the armature conductors connected in series between adjacent brushes (Section 10). The sum of the instantaneous electromotive forces induced in the armature winding in cutting the flux of the main pole is proportional to *OB* (Fig. 143a), and the sum of the instantaneous electromotive forces induced in the armature winding in cutting the flux of the regulating pole is proportional to *OC* (Fig. 143a). If the main and the regulating poles between brushes are both north or both south, the continuous voltage is proportional to the arithmetical *sum* of the lines *OB* and *OC*; if the main and the regulating poles between brushes are not of the same polarity, the continuous voltage is proportional to the arithmetical *difference* of the lines *OB* and *OC*.

In practice, the regulating poles are not placed midway between the main poles. Under this construction, the phase angle between the components of the counter-electromotive force is less than 90 degrees, as shown in Fig. 143b.

From the above considerations, it is evident that the continuous voltage of a regulating pole converter is changed by varying the field excitation. Converters of this type are in practical operation where the maximum continuous voltage is 150 per cent of the minimum value, the alternating voltage remaining constant.

During the development of the regulating pole converter, fears were expressed that its alternating electromotive-force wave would be so badly distorted as to make its use undesirable. These fears have been proven entirely groundless.

15. The Booster-Converter. — The booster-converter consists of an alternating-current generator and a synchronous converter, the armatures of which are connected in series. By means of the field rhetostat and a field reversing switch, the alternating voltage supplied to the converter armature may be either increased or decreased, and the direct voltage varied over a range proportional to twice the alternating electromotive force induced in the generator armature.

16. Hunting. — A synchronous converter, because of the light weight of its rotating parts, is more susceptible to oscillatory dis-

turbances than is a synchronous motor. The same things tend to cause hunting in a converter as in a motor, and the same methods are used to prevent oscillations, or to damp them out when once started.

17. Efficiency. — The efficiency of a synchronous converter, which is usually higher than that of either a motor or a generator having the same rating, is easily determined by loading, since both input and output are electrical quantities which are easily and accurately measured.

The approximate efficiency may be calculated from the rating, the input at no load, measured from either the continuous-current or the alternating-current end, and the resistance of the armature winding as measured between continuous-current brushes.

$$\text{Per cent efficiency} = \frac{EI \times 100}{(E + E_b + kR_a)I + W}, \tag{9}$$

when E = the rated continuous voltage,
E_b = brush-contact resistance drop.*
I = the continuous-current output for which it is desired to calculate the efficiency,
W = the no-load input,
R_a = the resistance of the armature winding as measured between continuous-current brushes,
k = the ratio of the copper loss in the armature conductors to the armature copper loss when the converter is operated as a continuous-current generator with the same output (see Section 18). The value of k depends on the number of rings with which the converter is provided and may be taken from Table V.

From the above, it is evident that the efficiency of a converter increases as the number of alternating-current rings is increased. Converters having more than six rings are seldom used, because the increased efficiency is not sufficient to justify the increased expense and complications.

The values of k given in Table V assume that the converter is operated at unity power factor. When the power factor is less than unity, the armature copper losses are increased (see Section 19), and the efficiency is decreased.

* See Chapter 4, Section 2.

TABLE V

No. rings	k
2	1.39
3	0.56
4	0.37
6	0.26
8	0.21

18. Armature reaction. — It was shown in Chapter 4, Section 6, that the magnetomotive force set up by the current in the armature of a two-pole continuous-current generator is

$$M = \frac{NI}{4} \text{ av. } \cos \Big]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \tag{10}$$

$$= \frac{NI}{2\,\pi} \text{ ampere-turns.} \tag{11}$$

The alternating current flowing in the coils of an n-ring two-pole converter, the continuous-current output of which is I, is, from equation (8),

$$I_{ac}' = \frac{\sqrt{2}\,I}{n \sin \frac{\pi}{n}}, \tag{12}$$

and the ampere-turns per ring are

$$\frac{NI_{ac}'}{2\,n} = \frac{\sqrt{2}\,NI}{2\,n^2 \sin \frac{\pi}{n}}. \tag{13}$$

Therefore, the maximum magnetomotive force per ring is

$$M_1 = \frac{NI}{n^2 \sin \frac{\pi}{n}} \text{ av. } \cos \Big]_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} \tag{14}$$

$$= \frac{NI}{\pi n} \text{ ampere-turns.} \tag{15}$$

When the current and the electromotive force of an n-ring armature are in phase, the armature magnetomotive force is in quadrature with the field flux,* and is equal to

* See Chapter 8, Section 9.

$$M_n = \frac{NI}{2\pi} \text{ ampere-turns,*} \qquad (16)$$

i.e., when the continuous-current brushes of a polyphase converter are set midway between the pole tips, the armature reaction due to the continuous current is equal and opposite to that due to the power component of the alternating current. The armature reaction in a polyphase converter is, therefore, that due to the current required to supply the losses in the converter, and to the wattless component of current when the power factor is less than unity.

From equation (15) it is evident that the maximum armature magnetomotive force of a two-ring† converter is

$$M_2 = \frac{NI}{\pi} \text{ ampere-turns,} \qquad (17)$$

i.e., the maximum value of the armature magnetomotive force of a two-ring converter, due to the power component of the alternating current, is twice that due to the continuous current. The armature magnetomotive force of a two-ring converter, therefore, varies from $+\frac{NI}{2\pi}$ to $-\frac{NI}{2\pi}$, the frequency of the variation being twice that of the alternating-current circuit.‡

The shifting of the armature flux across the pole faces causes energy losses (hysteresis and eddy currents), and tends to cause sparking at the continuous-current brushes. Therefore, commutation in a two-ring converter is inherently poor, and is *not* improved by an angular advance of the brushes.

19. Armature heating. — Since a synchronous converter acts simultaneously as a continuous-current generator and a synchronous motor, or as a continuous-current motor and an alternating-current generator, the currents actually flowing in the armature conductors must be resultants of these two actions. The instantaneous current flowing in an armature coil of a converter is, then, the algebraic sum of the constant continuous current and the instantaneous alternating current.

* Equation (16) is the resultant of n magnetomotive forces each of which has the maximum value $\frac{NI}{\pi n}$ ampere-turns, and having a displacement of $\frac{2\pi}{n}$ degrees in both space (direction) and time.

† The two-ring or single-phase converter has a very limited commercial application.

‡ See Chapter 8, Section 9.

Let the position of the armature of a two-ring two-pole converter be such that the angle ωt is zero. Fig. 140. Assuming unity power factor, the current in the alternating-current system is maximum and its direction of flow opposite to that of the current flow in the continuous-current circuit. The current flowing in the armature windings is, therefore, the *algebraic* sum of the constant continuous current and the maximum alternating current.

When the armature has advanced thirty degrees, the alternating current has decreased in value and the continuous current in coils a and g has reversed in direction, the coils having passed to the opposite sides of the brushes. The current flowing in coils a and g is, therefore, the *arithmetical* sum of the constant continuous current and the instantaneous value of the alternating current, and is greater than in the other coils. Similarly, when the armature has rotated through another thirty degrees, the direction of the continuous current in coils b and h has reversed, and the current in coils a, b, g and h is the arithmetical sum of the constant continuous current and the instantaneous alternating current. When the armature has rotated through ninety degrees, the value of the alternating current is zero and the current flowing in any coil is one-half that in the continuous-current (armature) circuit.

During the next quarter revolution the alternating current increases in the opposite direction, and coils e, f, l and m carry the larger instantaneous current values.

From the above it is evident that:

(a) The armature coils of a two-ring converter are not uniformly heated.

(b) The heating decreases as the distance from the point of connection to the ring increases.

The conditions shown to exist in a two-ring converter may be shown to exist in one having a larger number of rings, an increase in the number of rings making the heating more nearly uniform and decreasing the total quantity of heat liberated in the armature for a given value of load (continuous) current.

Taking as the axis the line AB in Fig. 140, the instantaneous current in any armature coil of an n-ring two-pole converter is

$$i = \frac{2\,I \cos(\omega t \pm \alpha)}{n \sin \frac{\pi}{n}} - \frac{I}{2}, \tag{18}$$

and

$$\text{av. } i^2 = \frac{1}{\pi}\int_0^{\pi} i^2\, d\,(\omega t) \tag{19}$$

$$= \frac{I^2}{4}\left(\frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 \cos \alpha}{n\pi \sin \frac{\pi}{n}} + 1\right), \tag{20}$$

when I = the continuous-current output,
n = the number of rings on the armature,
α = the angular displacement of the coil from a position midway between the connections to two adjacent rings.

But the heating of the coil due to the continuous current alone is proportional to $\frac{I^2}{4}$. Therefore,

$$k' = \frac{\frac{I^2}{4}\left(\frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 \cos \alpha}{n\pi \sin \frac{\pi}{n}} + 1\right)}{\frac{I^2}{4}} \tag{21}$$

$$= \frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 \cos \alpha}{n\pi \sin \frac{\pi}{n}} + 1, \tag{22}$$

when k' = the ratio of the average rate at which heat is liberated in the coil of an n-ring converter to the rate at which heat is liberated in the same coil of the armature when operated as a continuous-current generator with the same current output. Equation (22) is independent of the number of poles for which the armature is wound, and is, therefore, applicable to multipolar machines.

Integrating equation (22) with respect to α from $\alpha = 0$ to $\alpha = \frac{\pi}{n}$

$$k = \frac{n}{\pi}\int_0^{\frac{\pi}{n}} k'\, d\alpha \tag{23}$$

$$= \frac{8}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16}{\pi^2} + 1, \tag{24}$$

when k is the ratio of the average heat liberated in the armature winding of an n-ring converter to the heat liberated in the same

armature when used as a continuous-current generator, with the same current output.

20. Converter ratings. — From the above considerations it is evident that the current output (rating) of a given armature when used as a synchronous converter is to the current output (rating) when used as a continuous-current generator as 1 is to $\sqrt{k}$

$$K'W' = KW \frac{1}{\sqrt{k}} \tag{25}$$

$$= KW \frac{1}{\sqrt{\dfrac{8}{n^2 \sin^2 \dfrac{\pi}{n}} - \dfrac{16}{\pi^2} + 1}}, \tag{26}$$

when KW = the rating as a continuous-current generator,
$K'W'$ = the rating as a synchronous converter.

When the power factor of the converter is less than unity, the alternating current is increased for a given continuous-current output, the inequality of the heating of the armature is increased, the heating of the individual armature coils becomes more pronounced, and the wattless component of current tends to magnetize or to demagnetize the field as the current leads or lags behind the electromotive force.

CHAPTER X — PROBLEMS

1. The continuous voltage of a synchronous converter is 550. Find the alternating voltage (neglecting resistance drop) between adjacent rings when the converter is provided with: (*a*) 2 rings, (*b*) 3 rings, (*c*) 4 rings, (*d*) 6 rings.

2. Calculate the current in the alternating-current mains when the output of the converter in Problem 1 is 1000 amperes, the efficiency 95 per cent and the power factor 96 per cent.

3. The power factor of a 2-ring converter is unity and the continuous-current output 250 amperes. Determine the maximum (instantaneous) current flowing in a conductor: (*a*) adjacent to the connection from the armature winding to the ring, (*b*) mid-way between the connections to the rings.

4. Same as Problem 3 except the power factor of the alternating-current circuit is 0.866.

5. The continuous voltage of a 400-kw., 3-ring converter is 600, the resistance of its armature (measured between continuous-current brushes) is 0.027, and the no-load input when operated as a continuous-current motor without load (at rated speed and normal field excitation) is 12.5 kw. Calculate the efficiencies for 25, 50, 75, 100 and 125 per cent of rated output and plot the efficiency curve (using per cent of rated output as abscissas).

6. The converter in Problem 5 is provided with 6 rings. Determine its rating, calculate the efficiencies, and plot the efficiency curve as in Problem 5.

7. A compound-wound 25-cycle, 3-ring converter has an inductance connected in each alternating-current supply line. When the input to the converter is 100 kw., the power factor of the converter is unity and the voltage between continuous-current brushes is 600. When the input to the converter is 10 kw. the power factor of the converter is 0.5 and the voltage between continuous-current brushes is 550. Find: (*a*) the value of the inductance in the alternating-current lines, (*b*) the voltage of the alternating-current supply system.

8. The no-load continuous voltage of a split-pole converter is 550. The regulating pole is excited by means of a series winding, and the flux produced by the regulating pole at full load is 30 per cent of the flux produced by the main winding which is constant. The distance from the center line of the main pole to the center line of the regulating pole is 45% of the distance between the center lines of adjacent main poles. Find the full-load continuous voltage, the applied electromotive force at the rings remaining constant.

9. A 6-pole, lap-wound armature has 720 conductors. The commutator has 360 bars. Determine the commutator bars which must be connected to a given ring in: (*a*) a 2-ring converter, (*b*) a 3-ring converter, (*c*) a 4-ring converter, (*d*) a 6-ring converter.

CHAPTER XI

THE TRANSFORMER

A TRANSFORMER consists of a single magnetic circuit linking with two independent electric circuits, as shown schematically in Fig. 144. Since iron offers a path of small reluctance for the magnetic flux, the coils of commercial transformers are always wound on iron cores.

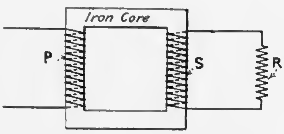


FIG. 144. Schematic Diagram of Transformer.

1. **Fundamental physical action.** — When the armature conductors of a continuous-current motor move across the magnetic field, a counter-electromotive force which opposes the flow of current is induced in the conductors, and the armature rotates at a speed which establishes equilibrium in the system. Similarly, when an alternating current flows in the windings of coil P (Fig. 144) the changing value of the flux induces* in the coil an electromotive force which opposes the flow of current, and the current is of a value which establishes equilibrium in the system. The induced electromotive force is equal to the geometrical difference of the applied electromotive force and the drop due to the resistance of the coil. The resistance drop in the primary windings of commercial transformers operating without load (secondary circuit open) is so small as to be negligible, and the applied and the counter-electromotive forces are, therefore, equal.

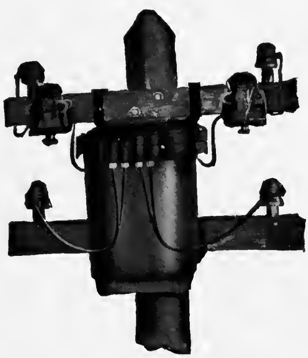


FIG. 145. Transformer in Position on Pole.

Referring to Fig. 144, let

ϕ_m = the maximum flux produced in the iron core by coil P,

f = the frequency of the alternating-current supply circuit,

N_p = the number of series turns in coil P,

N_s = the number of series turns in coil S,

* See Chapter 2, Section 13.

The flux in the iron must decrease from its maximum ϕ_m to zero in one-fourth of a cycle. The average rate of flux change is, therefore,

$$\frac{\phi_m}{\frac{1}{4f}} = 4\phi_m f, \qquad (1)$$

and the average electromotive force induced in coil P is the product of the average rate of change in the flux and the number of series turns in the coil.

$$E_{av} = 4\phi_m f N_p \, 10^{-8}. \qquad (2)$$

Since the ratio of the effective electromotive force to the average electromotive force is 1.11,

$$E_p = 4.44\phi_m f N_p \, 10^{-8}. \qquad (3)$$

Assuming that the flux set up by coil P is confined to the iron core, the electromotive force induced in coil S is

$$E_s = 4.44\phi_m f N_s \, 10^{-8}. \qquad (4)$$

If the terminals of coil S are connected through a resistance as shown in the figure, current flows in the windings of the coil. The current in coil S tends to establish in the iron core a flux opposed to that set up by coil P* and the counter-electromotive force induced in coil P is reduced. The equilibrium of the system is thus destroyed, and the current in coil P increases until it neutralizes the magnetic effect of coil S.

As the current in coil P increases, the resistance drop is no longer negligible and equilibrium is reëstablished with a counter-electromotive force less than the applied electromotive force. Equilibrium in a transformer is, then, maintained by an automatic change in the current flowing in the primary coil. The conditions are analogous to those in a shunt motor, the speed of which decreases as the load increases, so that the torque developed in the motor armature is automatically adjusted to equal the counter torque due to the load.

2. Magnetic leakage. — In Section 1 it is assumed that all the flux set up by coil P passes through coil S. Such is not the case in a transformer since the flux is not confined to the iron.† That part

* See Chapter 2, Section 14.

† See Chapter 2, Section 21.

of the flux which threads one coil but does not thread the other is termed "leakage" flux, and has the same effect as a series inductance, *i.e.*, it reduces the electromotive force induced in coil S, and causes the primary current to lag behind the applied electromotive force.

3. Electromotive force relations. — If the coils had no resistance and there were no magnetic leakage, it is evident, from equations (3) and (4), that the ratio of the voltage applied at the terminals of coil P to the voltage delivered at the terminals of coil S would be that of the number of turns in the coils.

$$\frac{E_p}{E_s} = \frac{4.44\phi_m f N_p 10^{-8}}{4.44\phi_m f N_s 10^{-8}} \tag{5}$$

$$= \frac{N_p}{N_s}. \tag{6}$$

This ratio is found to hold very closely when the secondary circuit is unloaded. Because of resistance and magnetic leakage, the secondary terminal voltage decreases as the load increases, the applied voltage remaining constant.

4. Current relations. — Neglecting the magnetizing current and the losses, which are small, the input to a transformer is equal to its output, and the product of the electromotive force and the current in the primary circuit, is equal to the product of the electromotive force and the current in the secondary circuit.

$$E_p I_p = E_s I_s. \tag{7}$$

Therefore

$$\frac{I_p}{I_s} = \frac{E_s}{E_p}, \tag{8}$$

$$= \frac{N_s}{N_p}, \tag{9}$$

i.e., the ratio of the primary and the secondary currents is equal to the *inverse* ratio of the number of series turns in the coils.

5. Vector diagram. — The electromotive forces and currents in a transformer may be represented graphically by means of a vector diagram. Fig. 146a is the diagram of an unloaded transformer. The applied electromotive force is represented by OA, the secondary electromotive force by OB and the flux, 90 degrees behind OA and 90 degrees ahead of OB, by OC. Because of hysteresis in the iron core, the no-load current is not harmonic,* and cannot,

* See Appendix B, Section 8.

therefore, be represented by a rotating vector. It is represented in transformer diagrams by what is termed the "equivalent harmonic current," *i.e.*, a harmonic current having the same frequency and effective value as the actual current. The error due to this assumption is usually negligible.

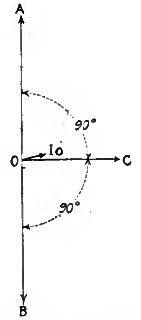


FIG. 146a. Vector Diagram of Unloaded Transformer.

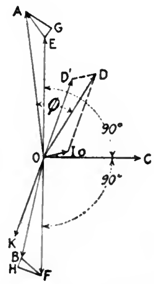


FIG. 146b. Vector Diagram of Loaded Transformer.

If the resistance of coil P were zero and there were no iron (hysteresis and eddy current) losses, the no-load current would be in phase with the flux. Because of these losses, which are practically all iron losses, there is a power component of the no-load current in phase with the applied electromotive force OA, and the vector of the no-load current is less than 90 degrees behind OA.

That the electromotive force induced in the coils of a transformer is 90 degrees out of phase with the flux set up by the primary winding is evident from the following consideration:

Let the flux in the iron core (Fig. 144) vary harmonically. The electromotive force induced in coils P and S is proportional to the rate of change in the flux threading the coils.* But the rate of flux variation is maximum when the flux is passing through its zero value.†

When the secondary circuit is loaded, the transformer quantities are represented in Fig. 146b. Let

E_p = the applied electromotive force,
E_s = the secondary (terminal) electromotive force,
I_p = the current in the primary circuit,
I_s = the current in the secondary circuit,

* See Chapter 2, Section 13.
† See Appendix A, Section 7.

R_p = the resistance of the primary circuit,
R_s = the resistance of the secondary circuit,
X_p = the reactance of the primary circuit,
X_s = the reactance of the secondary circuit,
$\cos\phi$ = the power factor of the primary circuit.

Draw

$$OA = E_p,$$
$$OD = I_p,$$
$$\phi = AOD.$$

The applied voltage E_p is equal to the geometric sum of the resistance drop R_pI_p, the reactance drop X_pI_p and the counter-electromotive force.

Draw

$AG = X_pI_p$ perpendicular to the current vector OD,
$GE = R_pI_p$ parallel to the current vector.

Then OE is the vector of the counter-electromotive force induced in the primary winding.

The load component of the primary current OD' is the geometric difference of OD and the no-load current.

The secondary current I_s is in phase opposition to the primary load current OD'.

The electromotive force induced in the secondary winding is in phase opposition to OE.

Draw

$$OK = I_s,$$
$$OF = OE\frac{N_s}{N_p},$$

$FH = X_sI_s$ perpendicular to OK,
$BH = R_sI_s$ parallel to OK.

OB is the secondary terminal voltage E_s.

6. Types and construction of transformers. — Commercial constant-potential transformers differ in the mechanical arrangement of the iron core and the windings, and are of two general types: (*a*) core, (*b*) shell.

(*a*) *Core type.* — In the core type transformer the windings are placed around the legs of the core and cover a large part of the iron. Fig. 147a.

(*b*) *Shell type.* — In the shell type transformer the windings are placed around the middle leg of the core as shown in Fig. 147b. Iron, evidently, encloses the greater part of the coils.

There is no marked superiority of one type over the other, both types being largely used. Economical considerations usually

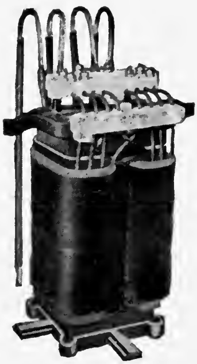


Fig. 147a. Core Type Transformer (Case Removed). Maloney Electric Co.

Fig. 147b. Shell Type Transformer (Case Removed). General Electric Co.

result in the selection of the shell type when the voltages are low and the currents relatively large, and of the core type when the voltages are high and the currents relatively small.

Transformer cores are built up of thin stampings (laminations) to reduce eddy-current losses. After the core and the windings are assembled they are placed in an iron case for mechanical protection.

7. Transformer losses.* — The losses in a transformer are: (*a*) iron losses, (*b*) copper losses.

(*a*) *Iron losses.* — The iron losses of a transformer are those due to: (1) hysteresis, (2) eddy-currents.

(1) *Hysteresis loss.* — The hysteresis loss in an iron core † is equal to

$$P_h = \eta V f B_m^{1.6}, \tag{10}$$

* See Standardization Rules of the American Institute of Electrical Engineers.

† See Appendix B, Section 7.

when η = the magnetic (hysteretic) constant of the iron used in the core,
V = the volume of the iron,
f = the frequency of the applied electromotive force,
B_m = the maximum flux density in the iron.

Since, in any given transformer, η, V and f are constant, and B_m is proportional to the applied electromotive force, equation (10) may be written

$$P_h = k_h E^{1.6}. \tag{11}$$

(2) *Eddy-current losses.* — The eddy-current loss in an iron core * is

$$P_e = bVf^2 l^2 B_m{}^2, \tag{12}$$

when b = a constant proportional to the reluctivity of the iron used in the core,
V = the volume of the iron,
f = the frequency of the applied electromotive force,
l = the thickness of the laminations of which the core is built.
B_m = the maximum flux density in the iron.

Equation (12) may be written

$$P_e = k_e E^2. \tag{13}$$

Determination of total iron loss. — The total iron loss in a transformer is determined, for any applied voltage, by connecting the transformer as in Fig. 148, and impressing the required voltage across its terminals. The wattmeter indicates the total iron loss plus the copper loss due to the no-load current. The no-load copper loss is usually so small as to be negligible, but the correction is easily made after the resistance of the primary winding has been determined.

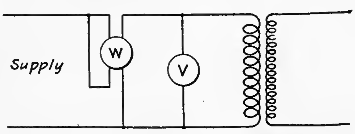


FIG. 148. Connections for Transformer Iron Losses.

Separation of hysteresis and eddy-current losses. — The iron loss of a transformer is separated into its components by the following graphical method: From equations (10) and (12)

$$P_i = k_h' f + k_e' f^2, \tag{14}$$

* See Chapter 6, Section 1.

when B is constant, *i.e.*, when the ratio $\frac{E}{f}$ is constant. Dividing equation (14) by f

$$\frac{P_i}{f} = k_h' + k_e'f, \tag{15}$$

which is the equation of a straight line, and may be plotted as *ab*, Fig. 149. The ordinate *oc* is the value of the constant k_h'.

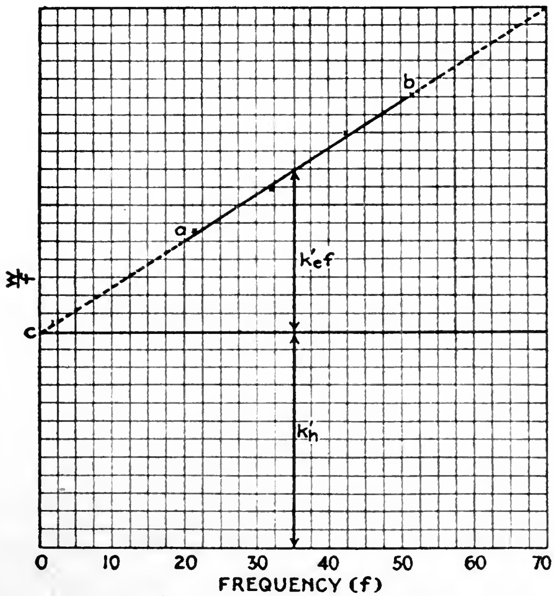


FIG. 149. Separation of Hysteresis and Eddy-current Losses.

The power lost in hysteresis at any frequency is obtained by multiplying the ordinate $oc = k_h'$ by the frequency; and the power lost in eddy currents is obtained by multiplying the ordinate $k_e'f$, corresponding to the required frequency, by the frequency.*

(*b*) *Copper losses.* — The copper losses in a transformer are those due to the resistance of the windings, and are equal to the product of the resistance and the square of the current.

$$P_c = R_p I_p^2 + R_s I_s^2, \tag{16}$$

* Compare with the separation of hysteresis and eddy-current losses in continuous-current machinery as outlined in Chapter 6, Section 4.

when P_c = the total copper loss in the transformer,
R_p = the resistance of the primary winding,
R_s = the resistance of the secondary winding,
I_p = the primary current,
I_s = the secondary current.

The resistance of a transformer winding is determined by connecting it, in series with a suitable rheostat, to a source of continuous current, and measuring the current in the circuit and the voltage between the terminals of the winding. The resistance of the winding is, from Ohm's Law,

$$R = \frac{E}{I}. \tag{17}$$

Direct measurement of the copper losses may be made by connecting a transformer as indicated in Fig. 150, where the low-voltage winding is short-circuited and the high-voltage winding connected to a source of alternating current, and the voltage between the terminals of the winding regulated to give any required current in the windings. The voltage required to produce full-load current in the windings of a short-circuited transformer is usually only a few per cent of the rated voltage of the coil. The indication of the wattmeter is the sum of the total copper loss in the transformer and the iron loss in the core. Since the iron loss in a transformer operating at rated voltage, is seldom more than 3 per cent of its rated output, the iron loss at the reduced voltage required for the short-circuit test is negligible, *i.e.*, the indication of the wattmeter is the total copper loss in the windings.

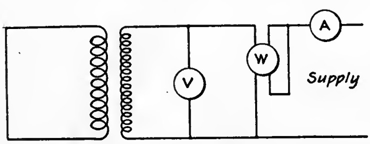


FIG. 150. Connections for Transformer Copper Losses.

8. Equivalent resistance. — The equivalent resistance of a transformer is the resistance which, when multiplied by the square of the current in the circuit, gives the total copper loss of the transformer. It will, evidently, have different values as the current used is that of the primary or of the secondary circuit. Let

R_e = the equivalent resistance of a transformer,
R_p = the resistance of the primary winding,
R_s = the resistance of the secondary winding,
I_p = the primary current,

I_s = the secondary current,
P_p = the copper losses in the primary winding,
P_s = the copper losses in the secondary winding,
P_c = the total copper losses in the transformer.

Then

$$P_c = P_p + P_s \tag{18}$$

$$= R_p I_p^2 + R_s I_s^2. \tag{19}$$

From equation (9)

$$I_p = I_s \frac{N_s}{N_p} \tag{20}$$

and

$$I_s = I_p \frac{N_p}{N_s}. \tag{21}$$

Therefore,

$$P_c = \left[R_p + R_s\left(\frac{N_p}{N_s}\right)^2\right] I_p^2 \tag{22}$$

$$= \left[R_p\left(\frac{N_s}{N_p}\right)^2 + R_s\right] I_s^2. \tag{23}$$

From equation (22)

$$R_e = R_p + R_s\left(\frac{N_p}{N_s}\right)^2, \tag{24}$$

when referred to the primary circuit. From equation (23)

$$R_e = R_p\left(\frac{N_s}{N_p}\right)^2 + R_s, \tag{25}$$

when referred to the secondary circuit.

9. **Equivalent reactance.** — The equivalent reactance of a transformer is the ratio of the voltage drop due to the inductance of the windings and to leakage flux, and the current flowing in the circuit. It is determined from the short-circuit test and the equivalent resistance of the windings.

$$X_e = \sqrt{\left(\frac{E}{I}\right)^2 - R_e^2}, \tag{26}$$

when X_e = the equivalent reactance of the transformer, referred to the primary or to the secondary circuit as the values of E, I and R are primary or secondary values,

E = the value of the primary, or the secondary, electromotive force required to cause current I to flow in the primary circuit, or in the short-circuited windings.

I = the current flowing in the primary circuit, or in the short-circuited windings.

R_e = the equivalent primary, or secondary, resistance of the windings.

10. Cooling. — The cases of small transformers are filled with oil which helps dissipate the heat liberated in the windings and in the core. In large transformers additional means of cooling are required, the heat being dissipated by means of an air blast, or by means of a system of pipes through which cold water is forced, and around which the oil circulates.

It is cheaper to cool large transformers by mechanical means than to build them of such shape and dimensions that they are self-cooling.

11. Efficiency. — The efficiency of a properly designed transformer is high, and varies from 95 per cent in small transformers to above 99 per cent in those of large size. If the applied electromotive force is constant, the iron loss is approximately constant, the copper loss is proportional to the square of the load and the efficiency may be calculated from the following equation:

$$\text{Per cent efficiency} = \frac{E_s I_s \cos\phi}{E_s I_s \cos\phi + P_o + R_e I_s^2}, \tag{27}$$

when E_s = the rated secondary voltage,

I_s = the current output at which it is desired to calculate the efficiency,

P_o = the no-load input (= iron losses),

R_e = the equivalent secondary resistance of the windings,

$\cos\phi$ = the power factor of the load circuit.

Transformer efficiencies are usually calculated for unity power factor, under which condition $\cos\phi$ in equation (27) is equal to 1.

Example. — Find the full-load efficiency of a 2200:220-volt, 15-kv-a. transformer, the no-load input to which is 200 watts, $R_p = 2$ ohms, $R_s = 0.02$ ohm.

$$E_s = 220 \text{ volts},$$

$$I_s = \frac{15{,}000}{220} = 68.1 \text{ amperes},$$

$$P_o = 200 \text{ watts},$$

$$R_e = 0.02 + 2 \times \frac{1}{100} = 0.04 \text{ ohm},$$

$$R_e I_s^2 = 0.04 \times (68.1)^2 = 185.5 \text{ watts},$$

$$\cos \phi = 1.$$

$$\text{Per cent efficiency} = \frac{15{,}000 \times 100}{15{,}000 + 200 + 185.5}$$

$$= \frac{15{,}000}{15{,}385.5}$$

$$= 97.5.$$

The "all-day efficiency" of a transformer is the ratio of the total output during twenty-four hours to the total input during the same period.

$$\text{All-day efficiency} = \frac{\text{output during 24 hours}}{\text{input during 24 hours}} \tag{28}$$

$$= \frac{\text{output during 24 hours}}{\text{output during 24 hours} + \text{losses}}. \tag{29}$$

Example. — Calculate the all-day efficiency of the above transformer, when operated at full load for 8 hours each day.

$$\text{Per cent efficiency} = \frac{15{,}000 \times 8 \times 100}{15{,}000 \times 8 + 200 \times 24 + 185.5 \times 8}$$

$$= \frac{120{,}000}{126{,}248}$$

$$= 95.$$

12. Regulation. — The regulation of a transformer is the ratio of the increase e in the secondary terminal voltage, between rated load and no load, and the secondary voltage E_s at rated load, the primary voltage remaining constant.

$$\text{Per cent regulation} = \frac{100\, e}{E_s}. \tag{30}$$

The secondary terminal voltage decreases as the load increases because of: (*a*) the resistance of the windings, (*b*) the reactance of

the windings. Since the vector sum of the secondary terminal voltage, the resistance drop and the reactance drop is constant, the regulation of a transformer becomes larger (poorer) as the power factor of an inductive load circuit decreases.

The resistance and the reactance of a transformer are determined as explained in Sections 8 and 9, and the regulation calculated by means of the following equation:

Per cent regulation

$$= \frac{\sqrt{(E_s \cos\phi + R_e I_s)^2 + (E_s \sin\phi + X_e I_s)^2} - E_s}{E_s} \times 100, \qquad (31)$$

when $E_s =$ the full-load (rated) secondary voltage,
$I_s =$ the full-load (rated) secondary current,
$R_e =$ the equivalent secondary resistance,
$X_e =$ the equivalent secondary reactance,
$\cos\phi =$ the power of the load circuit.

Example. — Determine the regulation of the transformer in Section 11, if 110 volts must be applied to the 2200-volt winding to cause rated current to flow in the short-circuited secondary winding.

$$E_s = 220 \text{ volts},$$
$$I_s = 68.1 \text{ amperes},$$
$$R_e = 0.04 \text{ ohms},$$
$$X_e = \sqrt{\left(\frac{11}{68}\right)^2 + (0.04)^2} = 0.16 \text{ ohms},$$
$$\cos\phi = 1,$$
$$R_e I_s = 0.04 \times 68.1 = 2.7 \text{ volts},$$
$$X_e I_s = 0.16 \times 68.1 = 10.9 \text{ ohms}.$$

$$\text{Per cent regulation} = \frac{\sqrt{(220 + 2.7)^2 + (10.9)^2} - 220}{220} \times 100$$
$$= \frac{300}{220}$$
$$= 1.4.$$

13. The auto-transformer. — When a single continuous winding forms both the primary and the secondary of a transformer

the arrangement is termed an auto-transformer. Fig. 151. The auto-transformer, like the independent coil transformer, is reversible and may be used to increase or to decrease the applied voltage.

The ordinary (two-winding) transformer may be used as an auto-transformer by interconnecting the windings as indicated in Fig. 152. Let

$$E_1 = 220 \text{ volts},$$
$$\frac{N_1}{N_2} = 10,$$
$$I_1 = 10 \text{ amperes}.$$

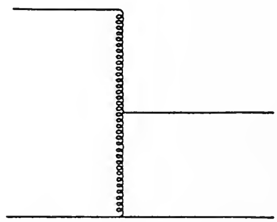


FIG. 151. Conventional Diagram of Auto-Transformer.

When the coils are interconnected as indicated in Fig. 152a, the voltage of the load circuit is the arithmetical *sum* of the applied voltage and the electromotive force induced in the secondary winding; when connected as indicated in Fig. 152b, the load voltage is the arithmetical *difference* of the applied voltage and the electromotive force induced in the secondary winding. The above statements are evident from the fact that the windings may be interconnected so that the applied and the secondary (induced) electromotive forces are either in phase or in phase opposition.

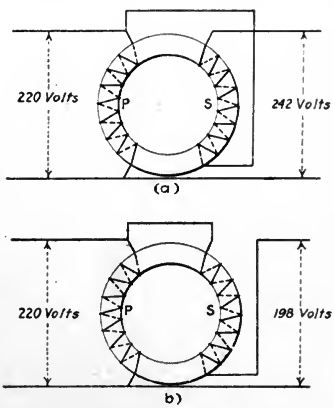


FIG. 152. The Auto-Transformer.

The ratio of the currents in the windings of an auto-transformer is the same as if the transformer were operating in the usual manner, *i.e.*, the currents in the windings are inversely proportional to the number of turns in the windings, while the current in the supply mains is the *algebraic* sum of the currents in the two windings. Assuming the load circuit to be non-inductive, the power delivered by the transformer when connected as indicated in Fig. 152a, is

$$P = 242 \times 10$$
$$= 2420 \text{ watts}.$$

The same power, neglecting the losses, is supplied to the

transformer, but the voltage of the primary circuit is only 220. Therefore,

$$I_1 = \frac{2420}{220}$$
$$= 11 \text{ amperes,}$$

and the current flowing in the primary winding is 1 ampere.

Similarly, for the connection indicated in Fig. 152b, the power delivered is

$$P = 198 \times 10$$
$$= 1980 \text{ watts}$$
$$I_1 = \frac{1980}{220}$$
$$= 9 \text{ amperes,}$$

and the current flowing in the primary winding is 1 ampere.

The copper losses in an auto-transformer are less than in a two-circuit transformer having the same output and the same primary and secondary voltages, but the interconnection of the windings makes its use inadvisable except where the difference between the supply voltage and that of the load circuit is small. One of its principal uses is to reduce the voltage applied to an alternating-current motor during the period of acceleration.

14. The constant-current transformer. — A transformer having a counterbalanced movable coil delivers approximately constant current to a load circuit of variable impedance, when the voltage between the terminals of the primary winding is constant.

When an alternating current flows in coil P the magnetic flux set up divides between the iron and the air as indicated in Fig. 153. That part of the flux which threads coil S induces in it an electromotive force which causes a current to flow in the closed secondary circuit. The flux which passes through the air forms a magnetic field, the direction of which is at right angles to the current-carrying conductors of coil S. Since the current-carrying conductors of coil S lie in a magnetic field, the coil tends to move and the direction of its motion is upward. The magnetic circuit is designed for large magnetic leakage, so that the number of magnetic lines threading coil S is materially reduced as the coil moves upward. Both the induced electromotive force and the current flowing in the coil are reduced, and the force producing, or tending to produce, motion becomes less.

By partially counterbalancing coil *S* as shown in Fig. 153, the secondary comes to rest in that position which establishes equilibrium in the system, *i.e.*, coil *S* moves upward until the reaction of the magnetic field and the current-carrying conductors is just sufficient to neutralize the effect of the unbalanced part of the coil. If the impedance of the load circuit is increased, the current in coil *S* decreases, the upward force acting on the coil decreases, and the coil moves downward until equilibrium is restored, *i.e.*, until the flux threading the secondary coil increases sufficiently to restore the original value of current flowing in the secondary circuit. If the impedance of the load circuit is reduced, the current in the secondary circuit increases, increasing the upward force on the coil, and the coil rises until equilibrium is re-established.

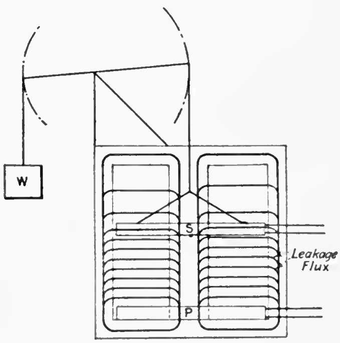


FIG. 153. The Constant-current Transformer.

As stated in Section 2, leakage flux has the same effect as a series inductance, *i.e.*, it causes the current to lag behind the applied electromotive force. The power factor of a constant-current transformer is, therefore, low when the magnetic leakage is large, *i.e.*, when the transformer is operating at partial loads. In fact, the primary current of a constant-current transformer is approximately constant, and the change in load is accomplished by a change in the power factor of the primary circuit. It is, therefore, highly undesirable that these transformers operate at loads much below their rated output.

To make the operation of constant-current transformers at partial loads commercially feasible, taps are often provided on the secondary coil, by means of which the number of current-carrying conductors in the coil may be changed. The relative position of the coil for a given load and, consequently, the leakage flux, may thus be changed, and the power factor of the system maintained at approximately full-load value.*

* The average power factor of constant-current arc lamp circuits is about 70 per cent when the transformers operate at full load.

The constant-current transformer is largely used to supply current to arc and other lamps connected in series. In large transformers the coils may be divided, and the secondary (movable) coils balanced against each other. With divided coils the size of the auxiliary weight W is reduced to a minimum, that necessary to give the required range of current.

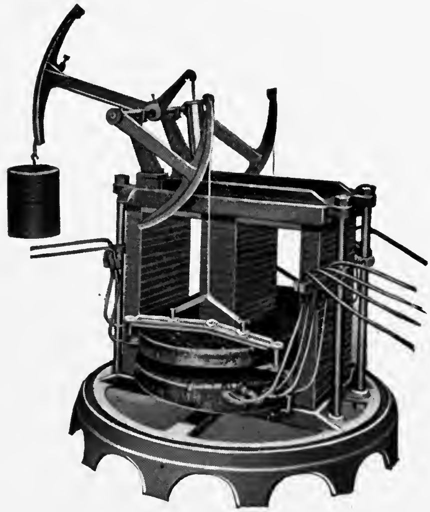


FIG. 154. Constant-current Transformer (Case Removed). General Electric Co.

15. Polyphase transformers.—Polyphase transformers are constructed by interlinking the magnetic circuits of two or more transformers which are to be connected to a polyphase system. If the flux set up by coil A is in quadrature with that set up by coil B, the weight of the iron cores is approximately 15 per cent less than the weight of the cores of two equivalent single-phase transformers, the maximum flux density being the same in each construction. Polyphase transformers are extensively used in Europe but have not attained great popularity in America.

CHAPTER XI — PROBLEMS

1. The iron losses (measured at no-load) in a 5-kv-a. 2200:220-volt transformer are 75 watts. The resistance of the primary winding is 7.5 ohms; that of the secondary winding is 0.07 ohm. Calculate the full-load efficiency: (*a*) when the power factor of the load circuit is unity, (*b*) when the power factor of the load circuit is 0.85.

2. The transformer in Problem 1 is operated during 24 hours as follows:

 4 hours at full load and unity power factor,
 6 hours at half load and 90% power factor,
 14 hours without load.

Calculate: (*a*) the all-day efficiency, (*b*) the yearly loss charge against the transformer if energy is produced for 1 cent per kw.-hour.

3. A 20:1-transformer, the regulation of which is 2 per cent, has a terminal voltage of 115 at full load. Determine the primary voltage at full load.

4. The hysteresis loss in a transformer is 125 watts when operated on 60-cycle mains. Calculate the hysteresis loss when the transformer is operated on a 25-cycle circuit, the voltage of which is equal to that of the 60-cycle circuit.

Note. — From equation (3)

$$B = k\frac{E}{f}.$$

5. The eddy-current loss in a transformer is 100 watts when operated on 60-cycle mains. Calculate the eddy-current loss when the transformer is operated on a 25-cycle circuit, the voltage of which is equal to that of the 60-cycle circuit.

6. 20 per cent of the iron loss in a 60-cycle transformer, when operated at rated frequency and voltage, is due to eddy currents. Calculate the percentage increase or decrease in the total iron loss when the transformer is operated at 25 cycles and rated voltage.

7. The hysteresis loss in a 60-cycle, 2300-volt transformer is 500 watts. Calculate the hysteresis loss when operated on a 60-cycle, 1150-volt circuit.

8. The eddy-current loss in a 60-cycle, 2300-volt transformer is 400 watts. Calculate the eddy-current loss when operated on a 60-cycle, 1150-volt circuit.

9. Two transformers have the same volume of iron of the same quality, but are so wound that the ratio of the flux densities is 2:3. Calculate the ratios of the iron losses.

10. Calculate the equivalent resistance of the transformer in Problem 1: (*a*) referred to the primary circuit, (*b*) referred to the secondary circuit.

11. Compare the iron losses in three similar transformers star-connected to a balanced 3-phase system, with the iron losses in the same transformers delta-connected to the same system.

12. Compare the copper losses in three similar transformers connected as in Problem 11, the line current in the star-connected system being the same as the line current in the delta-connected system.

13. An auto-transformer has a ratio of 2:3. Determine the relative size of the conductors to be used in the windings, the current densities to be equal.

14. The iron losses in a 15-kv-a. 2200:220-volt transformer are 135 watts (measured at no load). Resistance of primary winding = 2 ohms; resistance of secondary winding = 0.018 ohm. Find: (*a*) the copper losses at full load, (*b*) the efficiency at full (rated) load and 90 per cent power factor, (*c*) the copper losses when the efficiency is maximum.

15. The maximum efficiency of a 12.5-kv-a. 2200:220-volt transformer is 97.5 per cent, and occurs when the output is 80 per cent of the rated value. Find: (*a*) the copper losses, (*b*) the iron losses, (*c*) the equivalent resistance referred to the 2200-volt circuit, (*d*) the equivalent resistance referred to the 220-volt circuit.

16. A 2200:220-volt transformer is rated at 150 kv-a. and has a no-load input of 1.2 kw. Primary resistance = 0.16 ohm; secondary resistance = 0.0018 ohm. Find: (*a*) the total copper loss at full load, (*b*) the efficiency at full load, (*c*) the output at maximum efficiency.

CHAPTER XII

TRANSFORMER CONNECTIONS

1. **Protection.** — In connecting transformers to a supply circuit they should be protected from excessive currents by means of fuses of proper size in the primary circuits. A fuse is a short piece of soft wire which melts and opens the circuit whenever an excessive current flows in the circuit. Fuse wire is rated in amperes carrying capacity, the capacity varying, approximately, as the cross-sectional area.

2. **For single-phase circuits.** — Connections for single-phase circuits are shown in Fig. 155, where P is the primary winding and

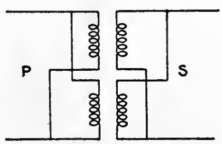


FIG. 155a. Primary and Secondary Coils in Parallel.

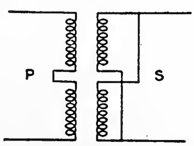


FIG. 155b. Secondary Coils in Parallel.

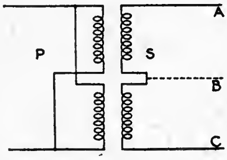


FIG. 155c. Primary Coils in Parallel.

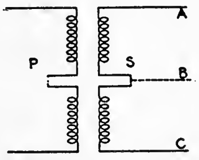


FIG. 155d. Primary and Secondary Coils in Series.

Single-phase Transformer Connections.

S is the secondary winding. In Figs. 155a and 155b, the halves of the secondary winding are connected in parallel, and the voltage of the load circuit is that of each coil. In making a parallel connection care must be taken that similar terminals of the two coils are connected together, or a short circuit may be formed. In Figs. 155c and 155d the secondary coils are connected in series and the voltage of the load circuit is twice that of Figs. 155a and 155b, the primary voltage being constant. It is impossible, in the series

connection, to form a short circuit but if the coils are improperly connected their voltages oppose and neutralize each other, and the voltage of the load circuit is zero.

Since, for a given load, the product of the current and the electromotive force is constant, it is evident that the secondary current in the parallel connections (Figs. 155a and 155b) is twice that in the series connections (Figs. 155c and 155d).

A three-wire system is formed by connecting a third wire to the junction of the series-connected coils, as indicated in Figs. 155c and 155d. The voltage between A and B and that between B and C is one-half that between A and C. This connection offers two available voltages and is much used, the load being connected between any two of the three wires. If the load connected between A and B is not equal to that connected between B and C, the halves of the transformer are unequally loaded. With a properly-distributed load, the current in line B is materially less than that in either A or C. For this reason B is usually made smaller than A and C.

Two or more transformers may be connected so that they supply the same load circuit. In making such connections the same conditions must exist as for the parallel operation of alternators. Since the primary coils are connected to the same supply circuit, the frequencies of the secondary circuits are the same, but the voltages may be in phase or in phase opposition. If the secondaries are connected so that their electromotive forces are not in phase opposition, a short circuit is formed and an excessive current flows in the windings. This large current heats the transformers to such a degree as to seriously damage or destroy the insulation, unless the protective devices work promptly.

Transformers having different characteristics should not be operated in parallel as they will not divide the load properly. For example, if two transformers of the same rating and no load voltage, but having regulations of 2 and 5 per cent, are operated in parallel, the one having the better (smaller) regulation will carry the greater part of the load. When the total load is equal to their combined ratings both transformers operate at a reduced efficiency, one being overloaded and the other underloaded. The wave shapes, or the ratios of transformation of different transformers, may be such as to make their parallel operation impossible.

3. **For two-phase circuits.** — Transformers may be operated on each phase of a two-phase system, all the connections given for single-phase circuits being available. In addition, the primaries, or the secondaries, or both, may be interconnected as explained for the polyphase system in Chapter 7.

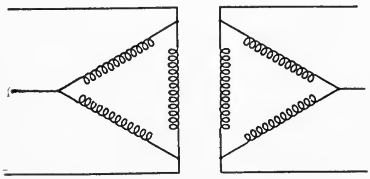


FIG. 156. Delta-delta Connection.

4. **For three-phase circuits.** — Single-phase transformers may be connected between any two lines of a three-phase system, or between any line and the neutral of a star-connected system, and operated as from a single-phase circuit. The primaries of three single-phase transformers may be connected to a three-phase supply system in either star or delta; the secondaries of the transformers may be connected to the load circuit in either star or delta. The voltage and current relations are those given for the polyphase system in Chapter 7.

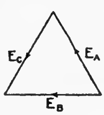


FIG. 157a. Correct Delta Connections. Instantaneous Voltages Balanced.

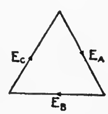


FIG. 157b. Incorrect Delta Connections. Instantaneous Voltages Unbalanced.

In making three-phase transformer connections the relations in the primary coils adjust themselves. If the secondaries are to be connected in delta, the voltage relations must be as indicated in Fig. 157a, *i.e.*, the sum of the instantaneous electromotive forces around the delta must be zero. If any one of the coils is reversed the equilibrium of voltages is destroyed, and an excessive current flows around the delta. Fig. 157b.

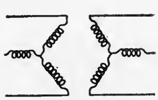


FIG. 158. Star-star Connection.

If the secondaries are to be star-connected, a short circuit is impossible because the coils do not form a closed circuit, but the voltages between lines may be unequal. The correct relations are indicated in Fig. 159a, and the voltage between any two lines is equal to that between any other two lines. If the terminals of any one of the coils is reversed, the relations shown in Fig. 159b

exist and are indicated by the fact that the voltage between two of the lines is correct, while that between either of these and the third line is equal to the voltage in the coil, *i.e.*, to the voltage between line and neutral.

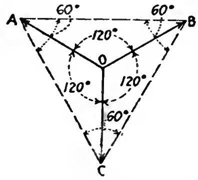


FIG. 159a. Correct Star Connections. $AB = AC = BC$.

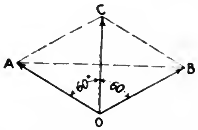


FIG. 159b. Incorrect Star Connections. $AC = CB = \frac{AB}{\sqrt{3}}$

V or open-delta connection. — If one transformer of a delta connection is omitted, a three-phase system may be operated with only two transformers. This is the V or open-delta connection. Fig. 161. While the number of transformers required for this connection is

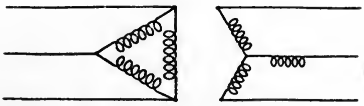


FIG. 160. Delta-star (or Star-delta) Connection.

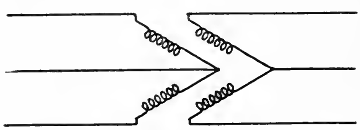


FIG. 161. Open-delta (or V) Connection.

less than in the delta connection, its use is not advised, except as a temporary expedient for the continuation of service, because of the phase relations of the current and the electromotive force in the windings of the transformers.*

5. Phase transformation. — It is sometimes desirable to change two-phase current to three-phase, or three-phase to two-phase. This may be accomplished by the "T" or Scott connection in which two transformers are connected as indicated in Fig. 162, *i.e.*, the two-phase coils are independent of each other while one terminal of one three-phase coil is connected to the middle point of the other three-phase coil. The terminals A, B and C are connected to the three-phase lines.

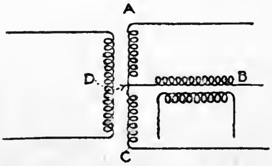


FIG. 162. Scott (or T) Connection.

* See Chapter 7, Section 3c.

The voltage of the coil AC is that between the lines to which it is connected. The voltage of the coil BD is a component of the voltage AB (or BC), the other component being $\frac{AC}{2}$ ($= AD = DC$). Fig. 163.

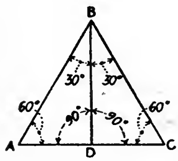


FIG. 163. Vector Diagram of T-connection.

Since ABC is, by construction, an equilateral triangle and $AD = DC$, ADB is a right triangle and

$$BD = \sqrt{(AB)^2 - (AD)^2} \tag{1}$$

$$= 0.866\ (AB). \tag{2}$$

The voltage between the terminals of BD is, therefore, 86.6 per cent of the voltage between the terminals of AC, and two similar transformers do not induce equal electromotive forces in the two-phase windings when the transformers are supplied from a balanced three-phase system. The voltages induced in the two-phase windings are equal if the ratio of the number of turns in BD to the number of turns in AC is made equal to 0.866.

$$\frac{\text{No. turns in coil } BD}{\text{No. turns in coil } AC} = 0.866. \tag{3}$$

6. For synchronous converters. — Transformer connections for two-, three- and four-ring converters differ in no way from those connections already described. Because of the increased output of a given converter armature when provided with six rings, six-ring converters are desirable. For supplying current to six-ring converters, the primary windings of transformers may be connected either star or delta to three-phase mains, but the secondary windings must be connected so as to produce six-phase currents. These connections are: (*a*) diametral, (*b*) double delta, (*c*) double star, (*d*) hexagonal.

(*a*) *Diametral.* — In this connection the terminals of transformer A are connected to rings 1 and 4 of the converter, the terminals of transformer B to rings 2 and 5, and the terminals of transformer C to rings 3 and 6. Fig. 164a.

(*b*) *Double delta.* — The windings of the transformers are connected so as to form two separate deltas, as indicated in Fig. 164b. The parallel sides of the deltas are formed by the coils of one transformer.

(*c*) *Double star.* — The six coils of a transformer may be connected into a double star, as indicated in Fig. 164c. In this connection the coils of a given transformer are diametrically opposed.

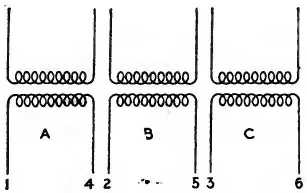


FIG. 164a. Diametral.

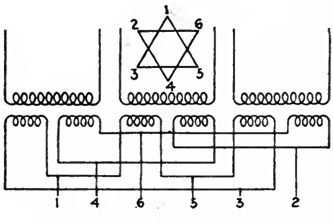


FIG. 164b. Double Delta.

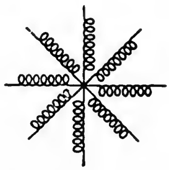


FIG. 164c. Double Star.

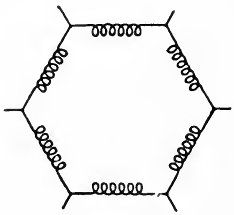


FIG. 164d. Hexagonal.

(*d*) *Hexagonal.* — The six coils may be connected as indicated in Fig. 164d, connection being made to the rings of the converter from the junction of each two coils. As in the double delta, parallel sides of the hexagon are formed by the coils of one transformer.

From the voltage relations given above it is a simple matter to make any of the above connections when three identical transformers are used. When the transformers are not similar, the relative direction of the electromotive forces in the coils must be determined.

CHAPTER XII — PROBLEMS

1. The primaries of three similar 20:1 transformers are star-connected to 2300-volt mains. Find the voltage of the secondary circuit when the secondaries are: (*a*) star-connected, (*b*) delta-connected.

2. Same as Problem 1 except the primaries are delta-connected.

3. A 5 kv-a. transformer, the regulation of which is 2 per cent is operated in parallel with another 5 kv-a. transformer, the regulation of which is 4 per cent. When the total load on the circuit is equal to 10 kv-a. it is equally divided between the transformers. Determine the load division when the total load is (*a*) 7.5 kv-a., (*b*) 5 kv-a., (*c*) 2.5 kv-a. Assume that the voltage characteristics are straight lines.

4. Three similar transformers have their primaries star-connected to balanced three-phase mains. The voltages between the terminals of the star-connected secondary windings are

$$AB = 127, \ BC = 127, \ AC = 220.$$

State what the trouble is and how it may be remedied.

5. Two 100 kv-a. transformers are V-connected. Find the load on each transformer when the balanced load on the system is equal to 150 kw., and the power factor is: (*a*) unity, (*b*) 0.866.

6. The primaries of two similar 10:1 transformers are connected to 2300-volt 2-phase mains. The secondaries are T-connected. Find the voltages between the 3-phase (secondary) terminals.

7. Draw a vector diagram showing the relations of the current and the electromotive force in the 3-phase coils of a T-connection when the power factor of the system is 0.866 and the load is balanced.

8. The primaries of three similar 10 : 1 transformers are delta-connected to 2300-volt mains. Determine the voltage of a single-phase circuit when the three secondaries are connected in series.

9. A transmission line is connected to 6600-volt generators through 1:10 step-up transformers delta-connected to the generators, and star-connected to the line. Find the line-to-line voltage.

10. A 3-phase line supplies an induction motor delivering 500 brake horse power through a bank of transformers. The primaries are star-connected; the secondaries are delta-connected. Ratio 60 : 1. Voltage between lines at transformer (primary) terminals = 45,000. Transformer efficiency = 98.7 per cent, motor efficiency = 92 per cent, power factor of motor = 87 per cent. Find: (*a*) the voltage impressed on the motor, (*b*) the current per line delivered to the motor, (*c*) the kw. supplied to the transformers.

CHAPTER XIII

THE INDUCTION MOTOR

1. Construction. — The essential parts of an induction motor are: (*a*) the stator, (*b*) the rotor.

(*a*) *The stator.* — The stator of an induction motor is the stationary part, and its structure is essentially that of the armature of a rotating field alternator. A slotted core (Fig. 166) is built up of laminations (Fig. 167), and the stator conductors, which are connected to the supply mains, are placed in the slots so as to form a distributed winding.

Fig. 165. Induction Motor with Wound Rotor. Triumph Electric Co.

(*b*) *The rotor.* — Induction motors are differentiated by the construction of the rotor, and are: (1) squirrel cage, (2) slip-ring.

(1) *Squirrel-cage rotor.* — A squirrel-cage rotor consists of copper bars or rods placed in slots on the surface of a cylindrical laminated iron core, the ends of the conductors being connected to copper rings, placed at each end of the core. Since the conductors are connected in parallel, the resistance of such a winding is small. A squirrel-cage rotor is shown in Fig. 168a.

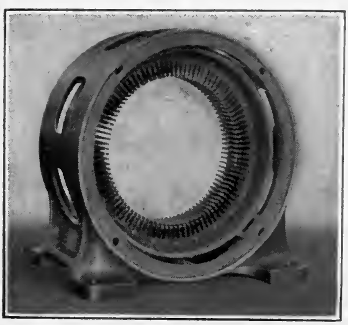


Fig. 166. Stator Core and Frame. Crocker-Wheeler Co.

(2) *Slip-ring rotor.* — A slip-ring or wound rotor is one in which distinct windings, similar to those on the stator, are interconnected, and terminals brought out to slip rings mounted on the

shaft. Through these slip rings the rotor winding is connected to an external rheostat, by means of which the resistance of the rotor circuit may be varied. Since the conductors of a wound rotor are connected in series, its resistance is much greater than that of a squirrel-cage rotor. Fig. 168b shows a wound rotor with three rings mounted on the shaft.

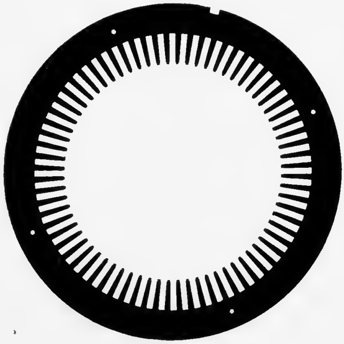


FIG. 167. Stator Lamination. Crocker-Wheeler Co.

The rheostat for regulating the resistance of the rotor circuit is sometimes placed in the rotor structure of small motors. In this case no rings are required, but a rod, by means of which the resistance of the rotor circuit is changed, projects through the hollow shaft.

2. Rotating flux and its production. — It was shown in Chapter 8, Section 9, that the magnetomotive force due to the rotating armature of a two-phase alternator is constant in value and fixed in direction. When the armature is stationary, the magnetomotive force of the armature winding is constant in value, but is momentarily changing in direction, and the flux rotates at a speed proportional to the frequency of the current flowing in the armature conductors.

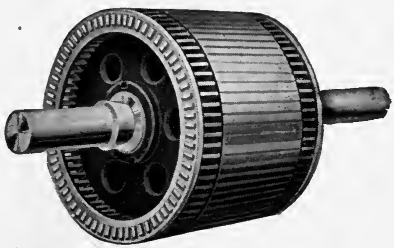


FIG. 168a. Squirrel-cage Rotor. Triumph Electric Co.

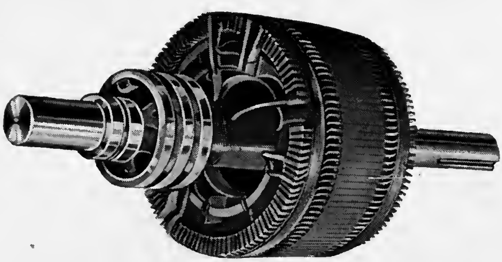


FIG. 168b. Slip-ring Rotor. Crocker-Wheeler Co.

Consider two electromagnets placed at right angles to each other as shown in Fig. 169, and excited from a two-phase alternating-current system. It is evident that the flux due to winding A is maximum when that due to winding B is zero. As flux A

decreases, flux B increases. During the quarter cycle required for flux A to decrease to zero and for flux B to increase to maximum, a resultant field is produced, the instantaneous value and direction of which is represented by the vector sum of the instanta-

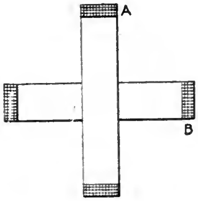


Fig. 169.

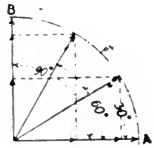


Fig. 170.

neous values of the quadrature fluxes.* The flux due to each winding of Fig. 169, and the resultant flux is represented in Fig. 170 when flux A is maximum, 30, 60 and 90 degrees later.

It is evident that the maximum value of the resultant flux is constant and equal to the maximum flux produced by each winding separately, and that its rate of rotation is proportional to the frequency of the supply circuit.

It may be shown in a similar manner that a rotating flux, the maximum value of which is 1.5 times the maximum flux set up by each phase winding, is produced by connecting properly designed coils to a three-phase supply system.

The magnetic effect when the stator windings of a polyphase induction motor are connected to a circuit supplying polyphase alternating current, is essentially that described, the difference being due to the arrangement of the windings, which is purely a mechanical detail, and to the reaction of the currents set up in the conductors of the rotor.

3. **Generator action.** — The rotating flux set up by the stator windings of an induction motor cuts across the conductors of the rotor, induces in them an electromotive force, and causes a current to flow in the rotor circuit.

4. **Motor action.** — The rotating flux set up by the stator windings reacts with the currents induced in the rotor conductors to produce a torque,† and the direction of the torque is the same as the direction of flux rotation.

* The fluxes are in quadrature as regards both time and space (direction).

† See Chapter 2, Section 14.

In a two-phase motor, the direction of flux rotation is reversed by reversing the terminal connections of *either* phase winding; in a three-phase motor, the direction of flux rotation is reversed by reversing any *two* line connections.

5. Speed of rotor.* — The torque produced by the reaction of the stator flux and the rotor currents causes the rotor to revolve. Because the rate at which the flux cuts the rotor conductors decreases as the speed of the rotor increases, both the induced electromotive force and the rotor current decrease, and the rotor runs at a speed which establishes equilibrium in the system. If the load increases, the speed decreases until the reaction between the flux and the increased current produces the required torque; if the load decreases, the speed increases until the current decreases to a value which reëstablishes equilibrium in the system.

The speed of the rotor can never attain the speed of the rotating flux because at this speed (the synchronous speed) no current flows in the rotor windings, and no torque is exerted to compensate for the frictional and other losses of the motor.

6. Slip of the rotor. — The difference between the speed of the rotating flux and that of the rotor is termed the "slip" of the rotor. The slip is approximately proportional to the load over the normal working range of the motor. At overloads the slip increases faster than the load until maximum torque is reached after which both speed and torque decrease very rapidly.

The slip of an induction motor is easily measured, unless it becomes excessive, by the stroboscopic method. On the end of the shaft or the pulley, mark as many equally-spaced radial lines as there are pairs of poles on the motor, and illuminate these lines by means of an arc lamp connected to the circuit from which the motor receives its current. When the motor is in operation, the radial lines appear to rotate in a direction opposite to that of the rotor. The speed of this apparent rotation is proportional to the slip of the rotor.

The stroboscopic method of slip measurement depends on the fact that the light from an alternating-current arc lamp is pulsating. If the rotor moved at synchronous speed, the radial lines

* The induction motor is, inherently, an approximately constant-speed machine. Large variations in speed are obtained only by a sacrifice of efficiency or of mechanical simplicity.

would advance through the angular distance of one pole pitch for each pulsation of the light, and successive pulsations would show the lines in the same relative positions. But the angular advance of the lines is less than the angular pitch of the poles, and each successive pulsation of the light shows the lines in a position slightly behind that which it occupied at the previous pulsation.

Example.—A four-pole induction motor is operated from 60-cycle supply mains. Determine the slip of its rotor when 127 radial lines pass through the field of vision in one minute,

$$\frac{127}{60 \times 60 \times 2} = 1.76 \text{ per cent.}$$

Another simple method for the determination of slip is to connect a contact maker to the shaft of the motor so that it closes, once in each revolution of the rotor, the circuit of a voltmeter connected across the mains supplying the motor. The voltmeter pointer swings back and forth, the rate of the swing being proportional to the slip of the motor. If an electrodynamometer type of voltmeter is used, the rate at which the pointer swings is twice as great as if the voltmeter is of the permanent magnet type.

7. **Torque.**—From the above it might be supposed that the maximum torque of an induction motor is exerted at zero speed, since both the induced electromotive force and the rotor currents are then maximum. But the frequency of the rotor currents is proportional to the slip of the rotor, and the rotor current, therefore, lags behind the induced electromotive force by a constantly increasing angle as the slip increases. The lagging current tends to set up a flux which is opposed to that set up by the stator windings.* When the slip becomes large, this demagnetizing action is excessive, and the flux decreases faster than the rotor current increases. Therefore the speed-torque curve of an induction motor is not a straight line, but has the general shape, for a rotor circuit of constant resistance, shown in Fig. 171.

Starting slightly below synchronous speed (the speed of the rotating flux), the torque increases as the speed decreases until the point of maximum torque is reached. At speeds less than that at which maximum torque is developed, the torque decreases very

* See Chapter 8, Section 9.

rapidly. Consequently, when an induction motor is loaded beyond the point of maximum torque, it stops.

An examination of the speed-torque curve of an induction motor shows that the characteristic, over the working range of the motor,

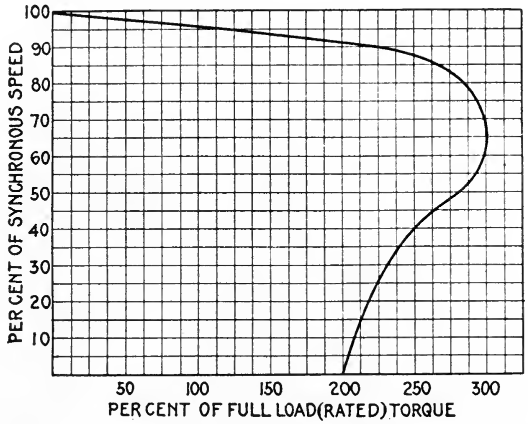


FIG. 171. Speed-torque Characteristic for Squirrel-cage Induction Motor.

is similar to that of a shunt (continuous-current) motor, *i.e.*, the speed drops slightly as the load increases. When the applied voltage is that at which the motor is rated, the maximum torque of an induction motor is usually from two to three times its rated load torque; its starting torque is from one and one-half to two times rated value, and the starting current is from five to six times that at rated load.*

The relations between the slip and the torque of an induction motor may be derived as follows: Let

E_r = the electromotive force induced in the rotor circuit at zero speed,
I_r = the rotor current,
R_r = the resistance of the rotor circuit,
X_r = the reactance of the rotor circuit at zero speed,
s = the slip of the rotor expressed as a fraction of the synchronous speed,

* The Standardization Rules of the American Institute of Electrical Engineers require that a motor intended for continuous service shall develop a maximum running torque at least 75% greater than the normal torque at rated load.

n' = the synchronous speed of the rotor in revolutions per minute,

n'' = the actual speed of the rotor in revolutions per minute,

$\cos\phi$ = the power factor of the rotor circuit.

$$\text{Rotor input} = E_r I_r \cos\phi. \tag{1}$$

But

$$I_r = \frac{sE_r}{\sqrt{R_r^2 + s^2X_r^2}}, \tag{2}$$

and

$$\cos\phi = \frac{R_r}{\sqrt{R_r^2 + s^2X_r^2}}. \tag{3}$$

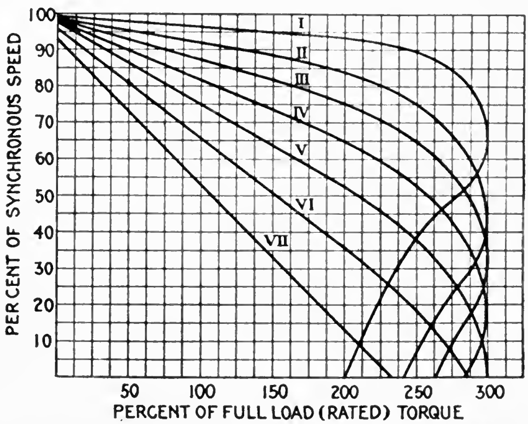


FIG. 172. Speed-torque Characteristics for Slip-ring Induction Motor. Resistance of Rotor Increased by Steps.

Substituting in equation (1),

$$\text{Rotor input} = \frac{sE_r^2R_r}{R_r^2 + s^2X_r^2}\text{ watts}, \tag{4}$$

$$\text{Rotor output} = \frac{sE_r^2R_r}{R_r^2 + s^2X_r^2} - \frac{s^2E_r^2R_r}{R_r^2 + s^2X_r^2} \tag{5}$$

$$= \frac{sE_r^2R_r\,(1 - s)}{R^2 + s^2X^2}\text{ watts}, \tag{6}$$

$$\text{Torque} = \frac{33{,}000\, sE_r^2R_r\,(1 - s)}{746 \times 2\,\pi n''\,(R_r^2 + s^2X_r^2)} \tag{7}$$

$$= \frac{33{,}000\, sE_r^2R_r}{746 \times 2\,\pi n'\,(R_r^2 + s^2X_r^2)} \tag{8}$$

$$= \frac{7.04\, sE_r^2R_r}{n'\,(R_r^2 + s^2X_r^2)}\text{ foot-pounds}. \tag{9}$$

The above formulæ disclose the following operating characteristics of the induction motor:

(*a*) Maximum torque is developed when

$$R_r = sX_r \tag{10}$$

(*b*) Maximum torque

$$= \frac{3.52\, E_r^2}{n'X_r}, \tag{11}$$

and is independent of rotor resistance.

(*c*) Starting torque

$$= \frac{7.04\, E_r^2 R_r}{n'\,(R_r^2 + X_r^2)}, \tag{12}$$

is proportional to the rotor copper losses, and is maximum when

$$R_r = X_r. \tag{13}$$

(*d*) The copper losses in the rotor circuit are equal to the product of the rotor input and the slip.

(*e*) At speeds near synchronism, the reactance of the rotor circuit is negligible, the torque is directly proportional to the slip, inversely proportional to the resistance of the rotor circuit, and

$$= \frac{3.52\, sE^2}{n'R_r}. \tag{14}$$

(*f*) At maximum torque, the power factor of the rotor circuit

$$= 0.707. \tag{15}$$

(*g*) At constant speed, the torque is directly proportional to the square of the electromotive force induced in the rotor circuit at zero speed, and, therefore, to the square of the applied voltage.

(*h*) At constant torque, the slip of the rotor is inversely proportional to the square of the electromotive force induced in the rotor circuit at zero speed, and, therefore, to the square of the applied voltage.

8. Starting. — The polyphase induction motor is self-starting when supplied with alternating current of the proper voltage, frequency, and number of phases. The squirrel-cage motor is usually started by supplying the stator with current at a voltage less than that at which the motor is rated, the line voltage being reduced by means of an auto-transformer or other step-down device. Fig. 173. After the rotor has attained a considerable speed, the line voltage is

applied and the starting device automatically disconnected from the line. Starting at a reduced voltage minimizes the line disturbances and reduces the heating of the motor windings, but decreases the starting torque.

The slip-ring induction motor is started by applying rated voltage to the stator windings, after resistance has been inserted in the rotor circuit. As the rotor speeds up, the resistance of the rotor circuit is gradually reduced until the windings are short-circuited.

Because of their large starting currents and low power factors, induction motors often cause undesirable fluctuations of the line voltage during the starting period. These fluctuations are particularly objectionable when lamps and motors are operated in parallel.

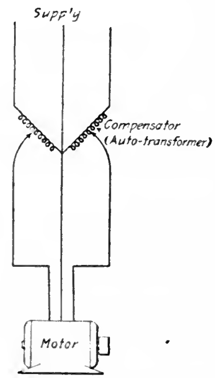


FIG. 173. Compensator Connections.

9. Power factor. — Since the air gap of an induction motor introduces considerable reluctance into the magnetic circuit, the magnetizing current, as compared to the load current, is relatively large at small percentages of the rated load. The power factor of an induction motor is, therefore, low at no-load but increases as the load increases.

The frequency of the currents induced in the rotor circuit is proportional to the slip of the rotor. The reactance of the rotor circuit is, then, proportional to the slip of the rotor.

$$X_r' = 2\pi s f L, \tag{16}$$

when X_r' = the reactance of the rotor circuit, at slip s,
s = the slip of the rotor expressed as a fraction of the synchronous speed,
f = the frequency of the supply circuit,
L = the inductance of the rotor circuit.

When the slip is small the reactance of the rotor circuit is negligible, the impedance of the circuit is practically equal to its resistance, and the rotor current is in phase with the induced electromotive force. As the slip increases, the reactance of the rotor circuit increases, the impedance becomes greater than the resistance, and the current lags behind the induced electromotive force.

The power factor of an induction motor will increase as long as

the power component of the induced voltage increases faster than the reactive component. When the slip becomes so great that the reactive component of the voltage increases faster than the power component, the power factor decreases.

10. The losses.* — The losses in an induction motor are: (*a*) copper losses, (*b*) stray power.

(*a*) *Copper losses.* — The copper losses of an induction motor are those due to the resistance of the stator windings and that of the rotor circuit. The losses in the stator windings are easily calculated for any given value of stator current when the resistance of the windings is known. The resistance is determined by continuous-current methods.

The copper losses in a slip-ring rotor are as easily determined as are those of the stator. Those in a squirrel-cage rotor cannot be determined directly, since neither the resistance of the circuit nor the value of the current flowing in it can be measured. The rotor copper loss should be calculated by means of the following formula:

$$R_r I_r^2 = \frac{\text{output} \times \text{slip}}{1 - \text{slip}}. \tag{17}$$

Let

E_r = the electromotive force induced in the rotor circuit at zero speed,
R_r = the resistance of the rotor circuit,
X_r = the reactance of the rotor circuit at zero speed,
s = the slip of the rotor expressed as a fraction of the synchronous speed,
I_r = the current in the rotor circuit at slip s.

$$I_r = \frac{\text{induced e.m.f.}}{\text{impedance}} \tag{18}$$

$$= \frac{sE_r}{\sqrt{R_r^2 + s^2X_r^2}}. \tag{19}$$

Squaring equation (19) and multiplying by R_r

$$R_r I_r^2 = \frac{s^2 E_r^2 R_r}{R_r^2 + s^2 X_r^2} = S E_2 I_2 \cos\theta_2. \tag{20}$$

$$= ks. \tag{21}$$

* See the Standardization Rules of the American Institute of Electrical Engineers.

(*b*) *Stray power.* — The stray power of an induction motor includes windage, friction, and iron losses, and is approximately constant over the working range of speeds. It is evident that windage, friction, and stator iron losses decrease as the slip (load) increases, and that the iron losses in the rotor increase. The stray power of an induction motor is usually taken as the no-load input to the

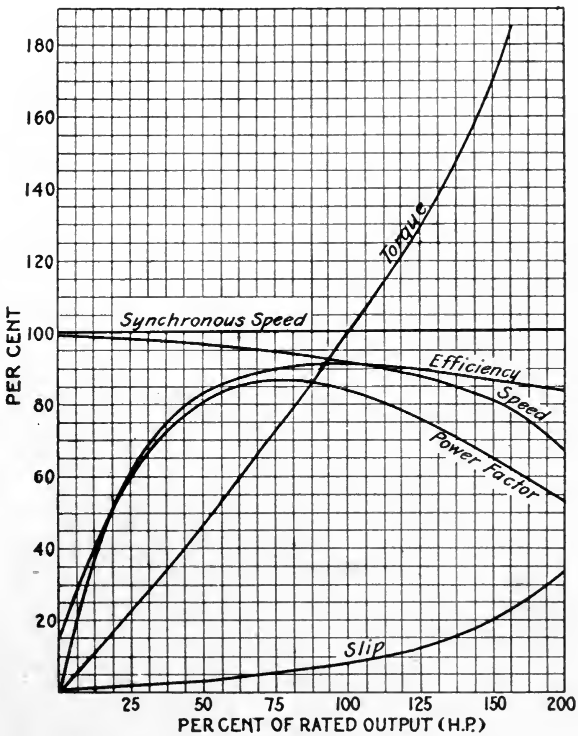


FIG. 174. Induction Motor Performance Curves.

motor minus the stator copper losses, the rotor copper losses at no-load being so small as to be negligible.

$$\text{Stray power} = \text{no-load input} - \text{stator } RI^2. \qquad (22)$$

11. Performance curves. — The performance of an induction motor is indicated by curves such as those shown in Fig. 174 data for which may be derived from: (*a*) a brake test, (*b*) the losses, (*c*) the circle diagram.

(*a*) *The brake test.* — A brake test is usually unsatisfactory because of the large number of readings required, and the difficulty

experienced in holding a brake load constant. The brake test is, therefore, little used in induction motor testing.

(*b*) *The losses.* — The set-up for the determination of the losses is the same as that used in a brake test, but the load is applied by means of an electric generator or a blower, and no measurements of output need be taken. The stray power is calculated from the no-load input and the stator copper losses as indicated above.

$$\text{Output} = \text{input} - \text{stray power} - \text{stator } RI^2 - \text{rotor } RI^2. \quad (23)$$

But from equation (21) the rotor copper losses are proportional to the slip. Hence,

$$\text{Output} = (\text{input} - \text{stray power} - \text{stator } RI^2)\,(1 - s). \quad (24)$$

(*c*) *The circle diagram.* — It may be proved both experimentally and mathematically, that the locus of the vector of the current in the rotor circuit is a semi-circle, *i.e.*, the value of the rotor current and

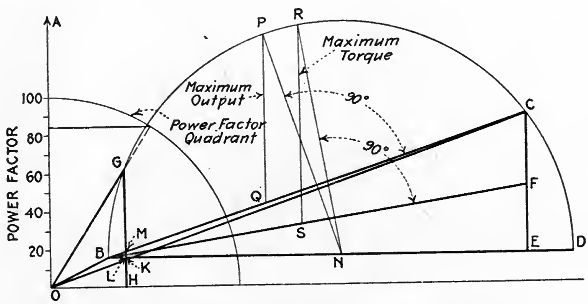


FIG. 175. Circle Diagram for Induction Motor.

the power factor of the circuit are so related that, as the current varies, the end of the vector is always on a semi-circle as indicated in Fig. 175. Therefore, the semi-circle is determined when two points are determined, the diameter being parallel to the axis of abscissa. The two points usually selected for experimental determination are: (1) when the motor is running without load (no-load test), (2) when the rotor is blocked to prevent its rotation (blocked rotor test).

(1) *No-load test.* — The motor is supplied with current at rated voltage and run without load. Measure the current and the watts input to the stator.

(2) *Blocked rotor test.* — After blocking the rotor to prevent its rotation, apply rated voltage to the stator, and determine the stator current and the watts input.

The excessive currents which flow in the windings of the motor when rated voltage is applied with the rotor blocked, may be avoided by making the blocked rotor test as follows: Wattmeter and ammeter readings are taken for several values of applied voltage less than that at which the motor is rated. (Very low values should not be used as the results are likely to be erratic. If the stator currents do not exceed twice the rated full-load value, no damage can be done to the windings or to the insulation.) A curve plotted between the power component of the stator current $\left(\frac{\text{watts}}{\text{volts}}\right)$ and volts is a straight line that may be extended to any desired point, the product of the ordinate and the abscissa of the point being equal to the watts input at this voltage. Fig. 176.

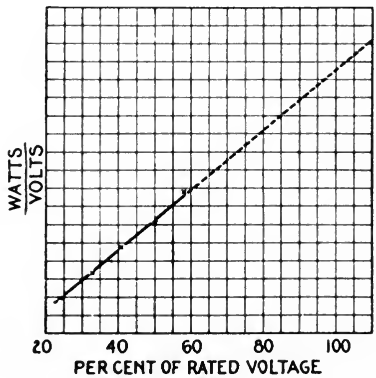


FIG. 176.

12. Construction of the circle diagram. — Draw OA (Fig. 175) proportional to the rated electromotive force of the motor, using any suitable scale. From O lay off OB proportional to the no-load current, the angle AOB being made such that its cosine is equal to the power factor of the motor at no load. Also lay off the line OC proportional to the stator current with the rotor blocked, the cosine of the angle AOC being equal to the power factor of the motor under this condition. Draw the line BD parallel to the axis of abscissa. The points B and C are two points on the semicircular locus of the current vector, the center of the circle being on the line BD.

The perpendicular from C to the axis of abscissa is proportional to the total losses in the motor. Since the rotor is blocked and can deliver no mechanical power, this loss is equal to the input. Determine the increase in the stator copper loss over that at no-load, and lay off as EF.

$$EF = R_s (I_2^2 - I_1^2), \tag{25}$$

when R_s = the resistance of the stator windings,
I_2 = the stator current with the rotor blocked,
I_1 = the stator current at no-load.

CF is proportional to the copper and iron losses in the rotor. No attempt need be made to separate these losses since the rotor iron losses, under load conditions, are negligible. (The frequency of the magnetic reversals in the rotor iron is very low.)

Draw straight lines from *C* and *F* to *B*. For any current input, as *OG*,

HK is proportional to the no-load (constant) losses,
KL is proportional to the "added" stator copper loss,
LM is proportional to the rotor copper losses,
MG is proportional to the output of the motor,
LG is proportional to the torque developed in the rotor,
$\frac{MG}{LG}$ is the ratio of the rotor speed to the synchronous speed,
$\frac{LM}{LG}$ is the ratio of the slip to the synchronous speed,
$\frac{MG}{HG}$ is the efficiency of the motor,
cos of the angle *AOG* is the power factor of the motor circuit.

The maximum power factor at which an induction motor can operate is obtained when the current vector is tangent to the circle.

The maximum output of an induction motor is obtained when the current is of such a value that a line drawn through the end of the current vector, and tangent to the circle, is parallel to *BC*.

The maximum torque possible to develop in an induction motor is exerted when the current is of such a value that a line drawn through the end of the current vector, and tangent to the circle, is parallel to *BF*.

It will be observed that the above quantities, as determined by the circle diagram, are not absolutely correct, but that the errors are small and that they tend to neutralize each other. For example, the so-called constant losses decrease slightly as the slip increases (windage and friction are dependent on the speed of the rotor and are zero when the rotor is blocked), while the rotor iron losses increase.

The circle diagram offers a simple method for determining the actions of an induction motor; the results are sufficiently accurate for commercial purposes; and the only data required are the current and watts input at no load, the current and watts input with the rotor blocked, and the resistance of the stator windings.*

13. **Proof of the circle diagram.** — Let

E_r = the electromotive force induced in the rotor circuit at zero speed,
R_r = the resistance of the rotor circuit,
X_r = the reactance of the rotor circuit at zero speed,
s = the slip of the rotor expressed as a fraction of the synchronous speed,
I_r = the current in the rotor circuit at slip s,
ϕ = the phase angle between E_r and I_r.

Then

$$I_r = \frac{sE_r}{\sqrt{R_r^2 + s^2X_r^2}}. \qquad (26)$$

But

$$\frac{sX_r}{R_r} = \tan\phi, \qquad (27)$$

and

$$R_r = \frac{sX_r}{\tan\phi} \qquad (28)$$

$$= sX_r \cot\phi. \qquad (29)$$

Substituting the value of R_r from equation (29) in equation (26),

$$I_r = \frac{sE_r}{\sqrt{s^2X_r^2 + s^2X_r^2\cot^2\phi}} \qquad (30)$$

$$= \frac{sE_r}{sX_r\sqrt{(1 + \cot^2\phi)}} \qquad (31)$$

$$= \frac{E_r \sin\phi}{X_r}. \qquad (32)$$

Equation (32) is a polar equation of the circle.†

* In plotting the circle diagram for a polyphase motor it is convenient to use the quantities for one phase, in which case the watts, torque, output, losses, etc., must be multiplied by the number of phases to obtain the quantities for the motor, or to reduce the experimental quantities to the "equivalent single-phase" system, as explained in Chapter 7, Section 11.

† The proof given above is strictly true only when the resistance of the stator windings and the magnetic leakage are negligible, but over the operating range of an induction motor very small differences are found between the theoretical and the actual locus of the current vector.

The Single-phase Induction Motor *

Structurally, the single-phase induction motor is essentially the same as the polyphase motor, but the stator windings are connected to a single-phase supply system. The chief operating difference between the single-phase and the polyphase motor is the fact that the former is not inherently self-starting.

14. Transformer action. — When the windings *AA* (Fig. 177a) are connected to a single-phase alternating-current circuit, the flux set up by the windings periodically reverses in direction, and the changing value of flux induces an electromotive force in the conductors of the squirrel-cage structure. But the directions of the currents in the conductors are such that the torque exerted on one conductor is equal and opposite to that exerted on another conductor. There is, therefore, no tendency for the conductors to rotate, the effect of the rotor currents being to largely neutralize the flux set up by the windings *AA*, the action being identical with that in a transformer, the secondary of which is short-circuited.

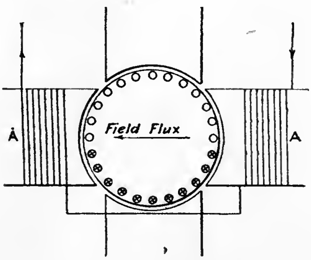


Fig. 177a.

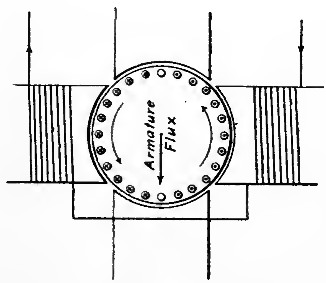


Fig. 177b.

15. Generator action. — If the structure supporting the rotor conductors is rotated by some external power, an electromotive force is generated in the conductors by reason of their movement across the magnetic field set up by windings *AA*.†

The current-carrying conductors on the rotor structure set up a quadrature flux, as indicated in Fig. 177b. During the half-cycle in which the current in *AA* flows in the direction indicated by the arrows, the flux set up by *AA* is constant in direction, though

* For a mathematical discussion of the single-phase induction motor, the student is referred to "Electric Motors" by Crocker & Arendt.

† See Chapter 2, Section 13.

varying in magnitude, and the direction of the flux set up by the rotor winding is that indicated in the figure. During the next half-cycle the flux due to the windings AA is reversed, since the current direction in the windings is reversed. The reversed direction of the field across which the rotor conductors move, reverses the direction of the rotor flux. Therefore, the flux set up by the rotor winding of a single-phase induction motor is in quadrature, both in time and in space, with the flux set up by the stator windings, and alternates in direction, the frequency of the flux reversal being the same as that of the current reversal in the windings AA.*

It is evident that the generated electromotive force and, therefore, the quadrature flux, is proportional to the speed of the rotor; and that the magnetomotive force of the rotor, when rotating at synchronous speed, is equal to the magnetomotive force of the stator windings.

16. Rotating flux. — While the simultaneous existence of quadrature magnetic fluxes in the same material is entirely imaginary, the effects are real and are due to the actual flux resulting from quadrature components. There is, then, set up in a single-phase induction motor, when once started, a rotating flux the value of which is constant only when the rotor revolves at synchronous speed. Since no electromotive force could be induced in the rotor conductors at synchronous speed (field and conductors rotating at the same speed), the operating speed of the rotor is always less than synchronous speed, and the rotating field of a single-phase induction motor is an ellipse, as indicated in Fig. 178, the short axis being proportional to the speed of the rotor.

Fig. 178. Rotating Flux of Single-phase Induction Motor.

17. Starting. — To make single-phase induction motors commercially practicable, auxiliary starting devices are required. Two methods of producing a starting torque in single-phase induction motors are in general use, and will be described: (*a*) shading coils, (*b*) split-phase windings.

* Because of the reversal of the quadrature flux, and the passage of the rotor conductors across the quadrature field, the currents flowing in the rotor conductors of a single-phase induction motor, are due to the combined action of *four* electromotive forces.

(*a*) *Shading coils.* — A shading coil is a closed winding placed around a portion of the pole, as indicated in Fig. 179. As the flux in the pole alternates, by reason of the reversal of the electromotive force of the supply circuit, an electromotive force is induced in the shading coil. The effect of the induced electromotive force is to oppose any change in the magnetic conditions existing in the space enclosed by the coil. This opposition causes the flux threading the coil to lag behind the flux in the unshaded part of the pole, and thus reach its maximum value at a later period. A flux which moves (shifts) from the unshaded to the shaded part of the pole is thus produced, and a small starting torque obtained. Shading coils are commonly used in fan and other small motors.

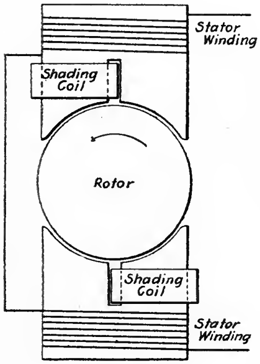


FIG. 179. Single-phase Induction Motor.

(*b*) *Split-phase windings.* — When a single-phase induction motor is to be started by the split-phase method, an auxiliary stator winding must be provided and the motor is, structurally, a polyphase motor. The connections for starting a two-phase motor from a

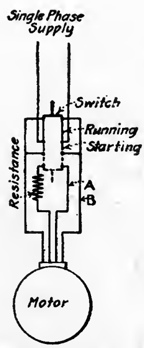


FIG. 180a. Split-phase (Two-phase Winding).

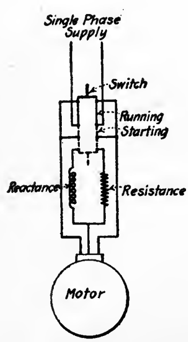


FIG. 180b. Split-phase (Three-phase Winding).

single-phase circuit are shown in Fig. 180a, and those for a three-phase motor in Fig. 180b.

Fig. 181 is a vector diagram, with stationary rotor, of the circuits represented in Fig. 180a. The impedance of circuit A is greater than that of circuit B because of the resistance connected in series

with the motor winding; the power factors of the two circuits are different, and the currents are out of phase as indicated. The fluxes set up by the windings are, therefore, out of phase, and produce a resultant rotating (or shifting) field.

It is impracticable to produce a quadrature displacement of fluxes by means of split-phase connections, but a displacement sufficient to produce a small starting torque is easily obtained. The direction of rotation is reversed by reversing the terminal connections of either winding.

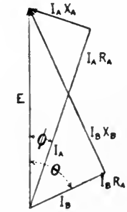


FIG. 181. Vector Diagram Split-Phase Motor.

18. Speed-torque curves. — Fig. 182 shows the speed-torque relations of a single-phase induction motor with variable rotor resistance. Both the maximum torque and the speed at which it

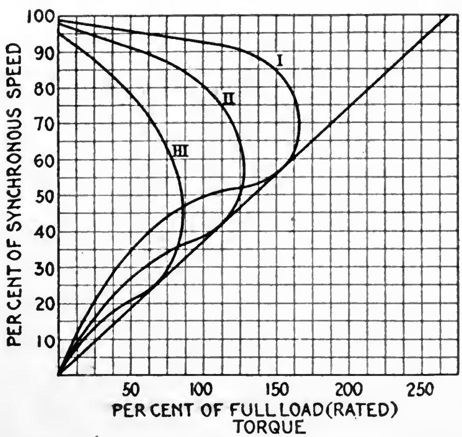


FIG. 182. Speed-torque Characteristics of Single-phase Induction Motor with Variable Rotor Resistance.

is developed, are reduced by increasing the resistance of the rotor circuit.

THE FREQUENCY CHANGER

19. The frequency of a transmission line is often less than that required for circuits supplying lamps. To operate lamps from such a system it is necessary to raise the frequency. As indicated in Section 9, the frequency of the current in the rotor circuit of an induction motor is proportional to the slip of the rotor. If, then, the

slip-ring rotor of an induction motor is driven at the proper speed a current of any desired frequency may be obtained from the rotor windings. When so used the induction motor is termed a frequency changer.*

The voltage at the terminals of the rotor circuit is proportional to the slip of the rotor. This is easily demonstrated by considering the voltage at synchronous speed and at zero speed. At synchronous speed, the voltage must be zero since there is no relative movement between the flux and the rotor conductors. At zero speed, the stator and the rotor act as the primary and the secondary of a transformer, and the voltages are to each other as the number of turns in the windings. The voltage of the rotor circuit at any given speed is, then, obtained by multiplying the voltage of the supply circuit by the ratio of the number of turns in the windings and by the slip of the rotor from synchronism.

$$E_r = ksE_s, \dagger \qquad (33)$$

when E_r = the electromotive force induced in the rotor winding,
k = the ratio of the number of turns on the rotor to the number of turns on the stator,
s = the slip of the rotor expressed as a fraction of the synchronous speed,
E_s = the voltage of the supply circuit at the stator terminals.

The ratio k of the number of turns in the windings may be determined by voltage measurements at 100 per cent slip (standstill).

The rotor of an induction motor, when used as a frequency changer, is driven by another motor (either induction or synchronous) supplied from the same system as is the stator of the frequency changer. In case the driving motor is synchronous, its fields may be over excited to compensate for the lagging current of the frequency changer, and the power factor of the supply circuit kept at a high value.

At zero speed, it is evident that no power is required from the driving motor, and that a certain electromotive force is induced in the rotor circuit by transformer action. If the rotor is driven at

* The required change in frequency may be affected by means of a motor-generator set, *e.g.*, a twenty-five cycle motor (synchronous or induction) driving a sixty-cycle alternator.

† This equation is only approximate when current flows in the windings, because of magnetic leakage and stator resistance.

synchronous speed backwards, and the current maintained at a constant value, the voltage of the circuit is doubled. But the increased voltage is due to the actual movement of the rotor (*i.e.*, to generator action), and the increased power is supplied through the motor.

Since there is no fixed relation between the number of phases supplied to the stator and the number for which the rotor may be wound, the induction machine may be used to change the number of phases as well as the frequency of a system.

The chief objections to the induction frequency or phase changer are its poor regulation and low power factor, due to the large air-gap leakage reactance.

The Induction Generator

20. When the rotor of an induction motor is driven above synchronism, the counter-electromotive force of the motor becomes greater than the applied electromotive force, and the power component of current in the supply circuit is reversed, *i.e.*, the motor acts as a generator and delivers current to the supply system. When so operated, an induction machine is not self-exciting, and must be operated in connection with synchronous apparatus from which it may receive its magnetizing current.

If an induction motor, the rotor of which is driven above synchronism by an independent prime mover, is connected to a system supplied by an alternating-current generator (synchronous alternator), the following effects are noted when the driving torque supplied to the alternator is reduced to zero:

(*a*) The alternator operates as a synchronous motor.

(*b*) If the field excitation of the synchronous machine is constant, the electromotive force of the system decreases as the load increases.

(*c*) With constant rotor speed, the frequency of the system decreases as the load increases.

(*d*) The frequency of the system is proportional to the speed of the synchronous machine.

(*e*) The electromotive force of the system is increased by increasing the field excitation of the synchronous machine.

(*f*) The load on the system is proportional to the speed of the rotor above synchronism, *i.e.*, above the speed of the synchronous machine.

21. Parallel operation of induction generators. — The relative speeds of two or more synchronous machines connected to the same system must remain constant, the phase relations of the generated electromotive forces determining the ratio of load division.* The division of the load between two induction generators operating in parallel is proportional to the ratio of the deviation of the actual speeds of the rotors from the synchronous speed.

Since the speeds of two induction machines operating in parallel need not have a constant ratio to each other, induction generators need not be synchronized but simply brought up to approximately synchronous speed, and the switch connecting the incoming machine to the system closed. After closing the switch, the load division is adjusted, as in the case of synchronous machines, by manipulating the speed governing apparatus of the prime mover.

22. Commercial applications. — The induction generator has not, up to the present time, come into any extensive use † and must be considered as still in the stage of development. Theoretically it offers decided advantages when driven by water wheels, gas engines or other prime movers, the speed regulations of which are not good.

CHAPTER XIII — PROBLEMS

1. The maximum torque of an induction motor occurs when the slip is 18 per cent. Find the ratio of the rotor resistance and the resistance that must be added to the rotor circuit so maximum torque is developed at starting.

Note. — The starting effort of an induction motor is expressed in "synchronous watts" or "synchronous horse power," and is equal to the power that would be developed in the rotor if it were operating at synchronous speed and developing a torque equal to that developed at starting.

2. Find the frequency of the rotor current in a 10-pole, 60-cycle induction motor when operated at a speed of: (*a*) 720 r.p.m., (*b*) 600 r.p.m., (*c*) 400 r.p.m., (*d*) 100 r.p.m., (*e*) 0 r.p.m.

3. Compare the efficiencies of two induction motors, the full-load slip of one being 4 per cent and that of the other 8 per cent.

4. A 400-horse-power, 440-volt, 60-cycle, 3-phase induction motor was tested and the following data obtained:

	Stator current per phase	Watts input per phase
Without load	160	4000
With rotor blocked	2250	225000

Stator resistance per phase = 0.018 ohm.

Construct the circle diagram.

* See Chapter 9, Section 12.

† The 59th Street power house of the Interborough Rapid Transit Co. (New York), has an installation of induction generators. So far as the writer has been able to learn, this installation has been entirely satisfactory.

5. From the circle diagram of Problem 4 plot the following curves: (*a*) speed-torque, (*b*) efficiency, (*c*) power factor.

6. From the circle diagram of Problem 4 determine the following when the output of the motor is 400 brake horse power: (*a*) current input, (*b*) torque developed in the rotor, (*c*) power factor, (*d*) speed, (*e*) "added" copper losses in stator, (*f*) rotor losses.

7. From the circle diagram in Problem 4 determine: (*a*) the stator current required to produce maximum starting torque, and the value of this starting effort in synchronous horse power, (*b*) the stator current and the torque when the power factor is maximum.

8. Determine the synchronous speed of a 60-cycle induction motor having: (*a*) 2 poles, (*b*) 4 poles, (*c*) 6 poles, (*d*) 10 poles, (*e*) 16 poles, (*f*) 24 poles.

9. Determine the synchronous speed of a 25-cycle induction motor having poles as indicated in Problem 8.

10. Determine the voltages induced in the rotor circuit of the motor in Problem 2 when the ratio of the stator turns to the rotor turns is one, and the applied voltage is 230.

11. A 6- and a 10-pole induction motor are operated in cascade, the 6-pole motor being connected to a 60-cycle alternating-current circuit. Find the synchronous speed of the combination.

Note. — When two induction motors are to be operated in cascade or concatenation, their rotors are connected to the same shaft; the stator of one motor is supplied directly from the line, and the stator of the other motor from the rotor of the first motor. The motors may be connected in direct or in differential cascade, and the synchronous speed of the combination is that of a single motor having the sum or difference of the number of poles on the two motors, and operated from a line of the same frequency as that to which the first motor in the cascade combination is connected.

12. Find the voltage supplied to the stator windings of the 10-pole motor in Problem 11.

13. Find the frequency of the current in the stator windings of the 10-pole motor in Problem 11.

14. A 60-cycle alternator is direct-connected to a 6-pole, 25-cycle induction motor. Find the number of poles on the alternator.

15. An induction motor develops a starting torque of 250 foot-pounds when the applied voltage is 60 per cent of the rated voltage. Determine the starting torque that would be developed by the motor if rated voltage were applied to the stator windings.

Note. — Since the fundamental equation of the induction motor is the same as that of the transformer ($E = 4.44 f\Phi N 10^{-8}$) with the addition of a winding constant, both the flux and the rotor current are proportional to the applied voltage. Therefore, the torque of an induction motor is proportional to the square of the applied voltage, the slip of the rotor (the frequency of the rotor currents) remaining constant.

CHAPTER XIV

SINGLE-PHASE COMMUTATING MOTORS

1. Action of the shunt motor. — When alternating current is supplied to a shunt motor, two very noticeable effects take place: (*a*) the motor develops very little torque, (*b*) excessive sparking occurs at the brushes.

(*a*) *Torque.* — It is evident that if the current in the armature of a shunt motor supplied from a single-phase alternating-current system and the flux set up by the field winding are in phase, *i.e.*, attain their maximum and minimum at the same instant, a torque varying from zero to maximum but always in the same direction is produced. The average torque of an alternating-current shunt motor is proportional to the average product of the armature current and the flux in the air gap, as in the continuous-current motor.

When the field winding of a shunt motor is excited from an alternating-current system, the current lags behind the electromotive force by a very considerable angle, the field circuit being highly inductive, and the flux has a corresponding lag behind the electromotive force. The power factor of the armature circuit being higher than that of the field circuit, the vector of armature current leads the vector of field flux, as shown in Fig. 183.

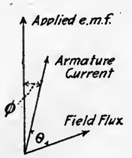


FIG. 183.

Because of the phase relations of the armature current and the field flux, their product is negative during part of the cycle and positive during the remainder of the cycle. The torque, therefore, tends to rotate the armature first in one direction and then in the other, and the net torque producing, or tending to produce, rotation is the algebraic sum of the average positive and negative torques during one complete cycle. If the angle θ equals 90 degrees, the sum of the instantaneous torques is zero, and the armature has no tendency to rotate.

The torque of a shunt motor may be improved by connecting the armature to one phase, and the field to the other phase of a two-

phase system. Such a machine is seldom used because of the complications involved, and the low power factor at which the phase supplying the field circuit must operate, and the fact that more satisfactory apparatus has been devised.

(*b*) *Sparking.* — Commutation in an alternating-current motor is more complicated than in a continuous-current dynamo. Let the position of the armature of an alternating-current motor be that shown in Fig. 184, the armature coil *c* being short-circuited by the brush. The changing value of flux in the armature core causes the coil *c* to act as the short-circuited secondary of a transformer in which the current may be many times the normal current flowing in the coil. This large current flowing in the local circuit, composed of the coil, the two commutator segments to which the coil is connected, and the brush, causes excessive heating of the armature as well as destructive sparking when the brush passes from one commutator segment to another.

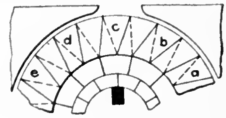


FIG. 184.

2. The series motor. — When a continuous-current series motor is operated with alternating current, the phase displacement between the armature current and the field flux largely disappears, since the same current flows in both windings and the inductance of a series field winding is much less than that of a shunt field winding, but commutation difficulties still exist. The commutation of a series motor operating with alternating current is improved by: (*a*) reducing the flux density of the magnetic circuit, (*b*) reducing the number of series turns per armature coil, (*c*) reducing the frequency of the supply circuit, (*c*) special devices.

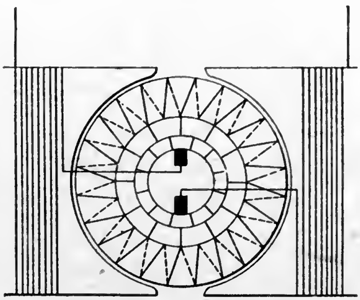


FIG. 185. The Series Motor.

(*a*) *Flux density.* — For a given frequency, the average rate at which the flux changes is proportional to its maximum value, and the electromotive force induced in the short-circuited armature coil is proportional to the rate of change.* The impedance of the

* See Chapter 11, Section 1.

short-circuit remaining constant, the current in the local circuit is proportional to the induced electromotive force.

(*b*) *Series turns.* — Since the electromotive force induced in the secondary of a transformer is proportional to the number of series turns,* the smaller the number of series turns per armature coil, the less is the induced voltage. It might appear that reducing the number of turns in the coil reduces the impedance of the circuit in the same ratio, and that the current, therefore, remains constant; but the resistance of the brush, brush contact, commutator bar, and connecting leads must be taken into account in calculating the impedance of the local circuit.

(*c*) *Frequency.* — The rate of flux change is proportional to the frequency, *i.e.*, the higher the frequency the shorter the time in which the flux must change from zero to maximum. The electromotive force induced in a short-circuited coil is, therefore, proportional to the frequency of the supply system, and low frequencies tend to reduce commutation troubles. Series alternating-current motors are seldom used on circuits having a frequency greater than twenty-five cycles per second.

(*d*) *Special devices.* — While the above considerations of design and operation materially reduce both heating and commutation troubles, special devices have been found necessary to bring them within practical operating limits. The simpler (and more satisfactory) of these devices are: (1) resistance leads connected between the terminals of the armature coils and the commutator segments, (2) balanced choke coils.

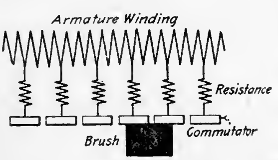


FIG. 186a. Resistance Leads A. C. Series Motor.

(1) *Resistance leads.* — If resistances are connected as indicated in Fig. 186a, the local current is reduced correspondingly since it must flow through the coil and two resistances connected in series. The load current must, also, flow through these resistances and this would, apparently, decrease the efficiency by increasing the resistance losses. It is an experimental fact that the introduction of resistance leads increases the efficiency, the increased loss due to the load current flowing in the added resistance being less than the decreased loss due to the smaller current flowing in the local circuit.

* See Chapter 11, Section 3.

(2) *Choke coils.* — The connections for the use of choke coils are shown in Fig. 186b. The windings on each core are so related that their inductance is cumulative to the short-circuit current, but differential to the load current. This arrangement is not entirely satisfactory because the coils balance for the load current only when the current is equally divided between two coils, and for certain positions of the armature, the coils neutralize each other, and become ineffective so far as the short-circuit current is concerned.

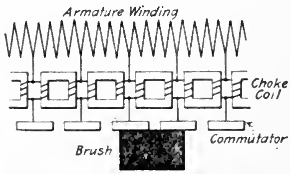


Fig. 186b. Balanced Choke Coils.

3. Compensation for armature inductance. — The inductance of the armature winding may be neutralized by means of a "compensating" winding connected as shown in Fig. 187. The compensating winding may be supplied with current: (*a*) inductively, (*b*) conductively.

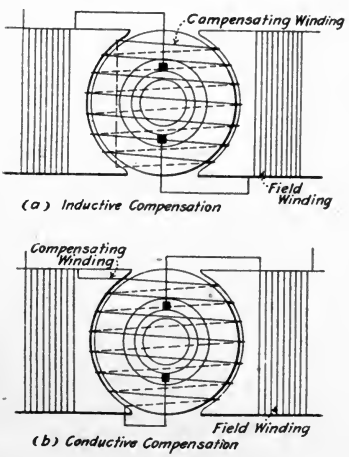


Fig. 187.

(*a*) *Current supplied inductively.* — If the compensating winding is closed on itself (short-circuited) as indicated in Fig. 187a, the alternating field flux induces in the winding an electromotive force, and the magnetic effect of the winding is approximately equal and opposite to that of the armature winding. The magnetic fields of the two windings, therefore, neutralize each other.

(*b*) *Current supplied conductively.* — If the compensating winding is connected in series with the armature winding as indicated in Fig. 187b, the same current flows in the windings. By properly proportioning the compensating winding, and connecting it so that its magnetic effect is opposite to that of the armature winding, the magnetic fields of the two windings neutralize each other.

4. Comparison of series motors. — The alternating-current and the continuous-current series motors differ in the following respects:

(*a*) The weight of the alternating-current motor is from one and one-half to two times that of the continuous-current motor developing the same torque.

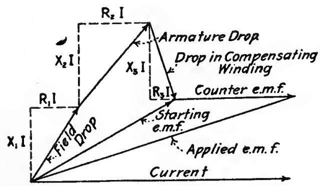


FIG. 188. Clock Diagram, Series A. C. Motor.

(*b*) The alternating-current motor has the larger number of armature coils.

(*c*) The alternating-current motor has the larger commutator, and the larger number of segments.

(*d*) The entire magnetic circuit of the alternating-current motor is laminated.

5. The repulsion motor. — When a continuous-current armature is placed in a magnetic field produced by an alternating current as indicated in Fig. 189, the field winding acts as the primary and the armature winding as the secondary of a transformer, and current flows between the short-circuited brushes for any position of the brushes except that shown in Fig. 189b, when the algebraic sum of the voltages induced in the coils between brushes is zero.

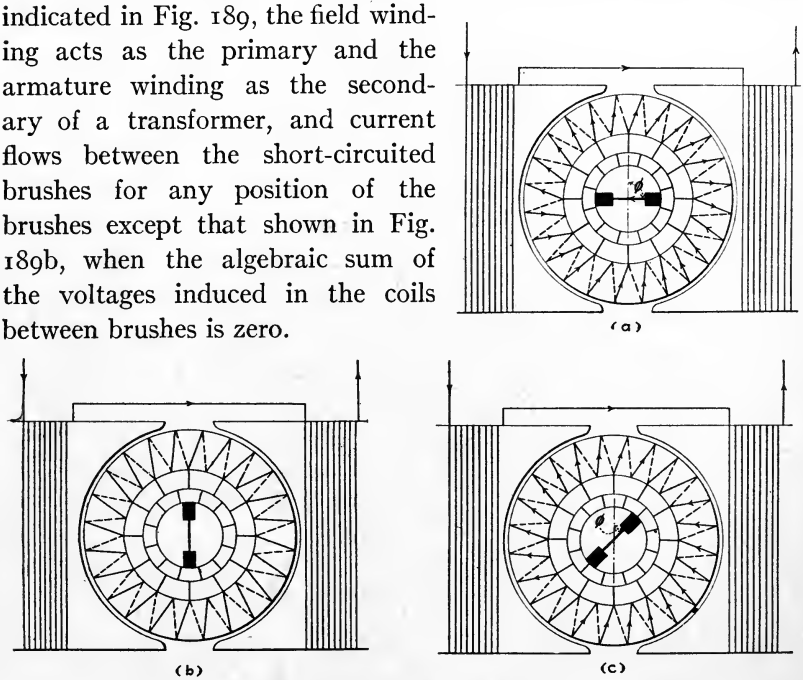


FIG. 189. Repulsion Motor.

For the position indicated in Fig. 189a, maximum current flows between the brushes. This current tends to set up a flux diametrically opposed to that set up by the field windings, and the magnetic effect of the field windings is largely neutralized. The torque produced by the reaction between any current-carrying conductor and the field flux is neutralized by an equal and oppo-

site torque produced by some other armature conductor and the same field flux. The armature has, therefore, no tendency to rotate.

This equality of opposite torques is destroyed, and the armature caused to rotate, by moving the brushes in either direction, the direction of rotation being the same as the movement of the brushes. It has been determined, experimentally, that maximum torque is developed when the angle θ equals approximately 45 degrees.

While the repulsion motor is little used commercially,* its good starting torque offers means by which the single-phase induction motor may be made self-starting. The Wagner Electric Mfg. Co. make a motor of this type in which the rotor structure, at starting, is essentially that described above. When the speed reaches a predetermined value, a centrifugal device removes the brushes from the commutator, and short-circuits the commutator bars so that the armature conductors form a squirrel-cage structure.

6. The compensated repulsion motor. — The so-called compensated repulsion motor is, mechanically, a series motor with the addition of short-circuited brushes placed in quadrature with the main brushes, as indicated in Fig. 190. While both series and repulsion motors have series characteristics, *i.e.*, the speed tends to rise to an infinite value as the load approaches zero, the compensated repulsion motor has shunt characteristics, and its principles of operation are materially different from either the series or the repulsion motor.

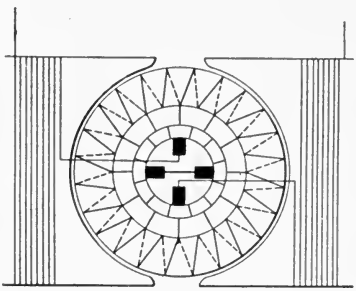


FIG. 190. Compensated Repulsion Motor.

The short-circuited brushes of this type of motor cause the armature winding to produce a magnetic field which opposes and largely neutralizes the flux set up by the field winding. The current which flows between the short-circuited brushes is produced by transformer action, as described for the repulsion motor when the brushes are in the position indicated in Fig. 189a.

The current flowing between the main (series) brushes sets up a

* Commutation is inherently poor.

field at right angles to the line joining the short-circuited brushes. This flux, reacting with the current flowing in the armature conductors by reason of the short-circuited brush connection, produces the greater part of the torque of the motor, although some torque is doubtless produced by a reaction between the series-field flux and the current between the main (series) brushes.

As the armature speed increases, the counter-electromotive force induced in the conductors by reason of their motion reduces the current flowing between the short-circuited brushes and the torque becomes less. Therefore, the flux is approximately constant, the current varies inversely with the speed of the armature and the motor has shunt characteristics.

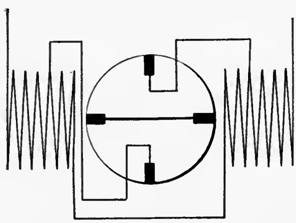


Fig. 191. Diagram of General Electric "RI" Motor.

7. The General Electric RI motor. — The connections of the General Electric Company's Type RI motor are shown in Fig. 191, and typical performance curves in Fig. 192. From the curve sheet, it will be

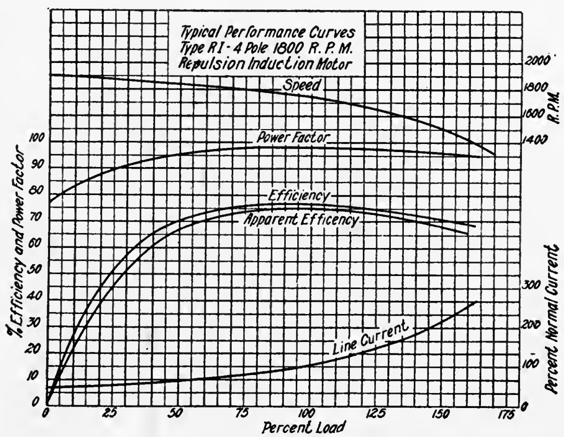


Fig. 192. Typical Performance Curves of Type RI Motor

observed that the power factor is high for loads above 50 per cent of the rated output.

8. The Wagner BK motor. — The Unity Power Factor (so-called by the manufacturers) single-phase motor of the Wagner Electric Mfg. Co. is a combination of the compensated repulsion

motor and the single-phase induction motor. The armature has a squirrel-cage, as well as a commutated winding, as indicated in Fig. 193. The electrical connections are indicated in Fig. 194.

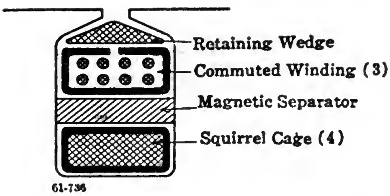


FIG. 193. Section Through Slot of Wagner "BK" Motor.

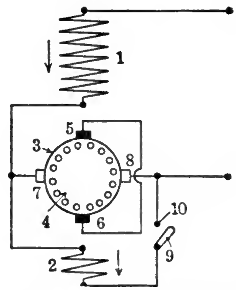


FIG. 194. Wiring Diagram for Wagner "BK" Motor.

At starting, the main field winding 1 induces, by transformer action, currents in the commutated winding 3, and these currents flow between the short-circuited brushes 5 and 6. Similarly, currents are induced in the squirrel-cage winding as described for the single-phase induction motor.* The series currents flowing in

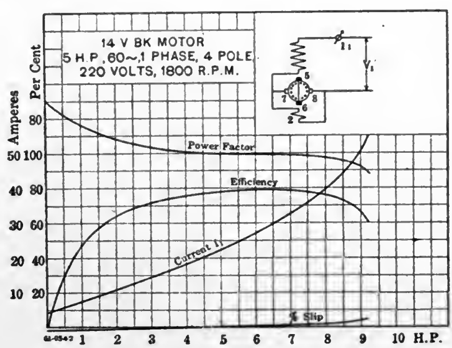


FIG. 195. Performance Curves of Wagner "BK" Motor.

the commutated winding 3 set up a flux, the direction of which is at right angles to the flux set up by winding 1. The currents in the commutated and in the squirrel-cage windings react with this quadrature flux to produce a torque, and start the motor. As the speed of the motor increases, the squirrel-cage winding sets up a quadrature flux of its own, and develops a corresponding torque.

* See Chapter 13, Section 14.

The torque of this motor is, then, a resultant of two torques, one of which is maximum at starting and decreases as the speed increases, while the other is zero at starting, increases to maximum as the speed increases, and then decreases as the speed increases still further.

The action of the auxiliary winding 2, is to improve the power factor of the motor. The switch 9 is open at starting, but is automatically closed by a centrifugal device at a predetermined speed.

The performance curves of this motor are shown in Fig. 195. The power factor will be observed to be 70 per cent leading at zero load, and approximately unity from 75 to 150 per cent of rated load. It will also be noted that the speed is very nearly synchronous at rated load, rises slightly above synchronism at no-load, and drops below synchronism as the load is increased above the rated capacity.

CHAPTER XV

ELECTRIC LAMPS *

1. **Arc lamps.** — Arc lamps are largely used for lighting streets and other areas where a few large units are preferable to a larger number of smaller units.

The basic principle of the arc lamp is the arc which is established when an electric circuit is interrupted. The heat of the arc causes one or both electrodes to be consumed, and the incandescent particles emit a very intense light. Arc lamps may be classified as: (*a*) carbon arcs, (*b*) magnetite arcs, (*c*) flaming arcs. Carbon and flaming arc lamps may be operated on either alternating or unidirectional current; magnetite lamps on unidirectional current only.

Arc lamps may be operated either in parallel or in series. In parallel operation, the voltage is constant and the regulating mechanism must maintain the proper value of current; in series operation, the current is automatically maintained at a constant value, and the lamp mechanism controls the voltage between the terminals of the lamp.

(*a*) *Carbon arc lamps.* — In this type of lamp the electrodes are made of finely ground carbon mixed with a suitable binder, pressed into the form of pencils and baked. In the early lamps, the arcs were "open," *i.e.*, air currents circulated freely about the arc. The light from such a lamp is unsteady and the electrodes are consumed very rapidly. By enclosing the arc in a glass globe to which a very limited quantity of air is admitted, the flickering of the arc is materially reduced, and the electrodes are consumed at a much slower rate. Because of these facts and the reduced fire hazard of the enclosed arc, open arc lamps are no longer manufactured.

Fig. 196 shows the circuits of a lamp designed for parallel operation. Regulation is accomplished by means of an electromagnet,

* Problems in illumination are beyond the scope of this volume. For information regarding methods of calculating light densities, etc., the reader is referred to "Illumination and Photometry," by W. E. Wickenden, and "Electrical Illuminating Engineering," by W. E. Barrows, Jr.

the windings of which are connected in series with the arc. When the circuit is open, the core of the magnet drops to its lowest position and the clutch releases the upper electrode, which then rests on the lower electrode as indicated. Closing the lamp circuit, causes the magnet to attract its core, raise the upper electrode, and "strike" the arc.

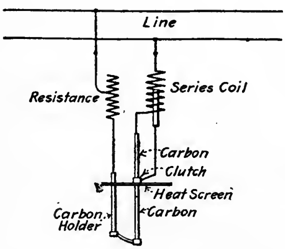


Fig. 196. Schematic Diagram of Connections for Parallel (Carbon) Arc Lamp.

The resistance of the circuit becomes greater as the distance between the electrodes increases, and the magnet is so proportioned that its pull balances that of a spring, when rated current flows in the circuit. The core, therefore, moves upward until this equilibrium, which corresponds to a fixed distance between the electrodes, is established. As the electrodes burn away, the length of the arc increases, the current in the circuit decreases, the pull of the magnet no longer balances that of the spring, and the core moves downward, increasing the current and decreasing the length of the arc, until equilibrium is restored. When the core has descended to its lower limit, the clutch releases the upper electrode which drops into contact with the lower electrode, the resistance of the circuit is reduced, and the increased current causes the magnet to separate the electrodes again. The result of this process is the periodical feeding of the upper electrode through the clutch by an amount equal, approximately, to the length of the arc.

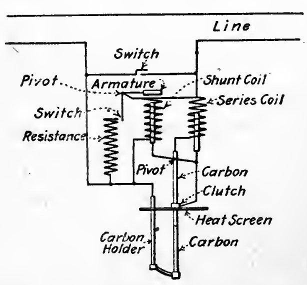


Fig. 197. Schematic Diagram of Connections for Series (Carbon) Arc Lamp. (Differential Control.)

Since multiple arc lamps are usually connected to 110-volt mains, and the arc requires only from seventy to eighty volts, the lamp is provided with a ballast by means of which the line voltage is reduced. In continuous-current lamps the ballast is resistance; in alternating-current lamps, reactance.

The series lamp (Fig. 197) has, in addition to the series magnet of the parallel lamp, a magnet connected in parallel with the arc and so arranged that its action opposes that of the series magnet. When

current flows through the lamp, the series magnet raises the upper electrode as in the parallel lamp, but as the length of the arc increases, the current in the windings of the shunt magnet increases, and equilibrium is established at the length of arc at which the pull of the series magnet (constant) minus the pull of the shunt magnet, balances the pull of a spring. As the electrodes burn away, the effect of the shunt magnet increases, and equilibrium is maintained by the changing position of the series magnet core. At the lower limit of the movement of the core, the clutch releases the upper electrode and it is fed downward as in the parallel lamp.

In the series lamp, a bypass must be provided for the passage of the current in case the carbons burn out, or for any other reason the arc circuit becomes opened. This may be provided by means of a shunt coil armature which closes the circuit through an auxiliary resistance whenever the voltage across the arc reaches a given value, which is always higher than that reached in normal operation. Should the arc circuit be re-established, the voltage between the terminals of the shunt winding is reduced, the armature of the shunt magnet is released and the auxiliary circuit broken.

(*b*) *Magnetite* (*or "luminous"*) *arc lamps.* — The magnetite arc lamp is one of the results of the researches of Dr. C. P. Steinmetz of the General Electric Co., and has a fixed upper electrode * of copper and a movable lower electrode composed of a mixture of magnetite (one of the oxides of iron) and titanium oxide encased in an iron tube. This lamp is economical in power consumption and gives a brilliant "white" light which is well distributed.

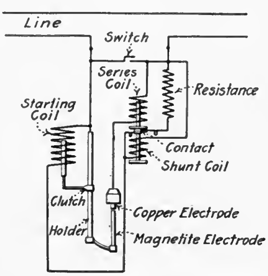


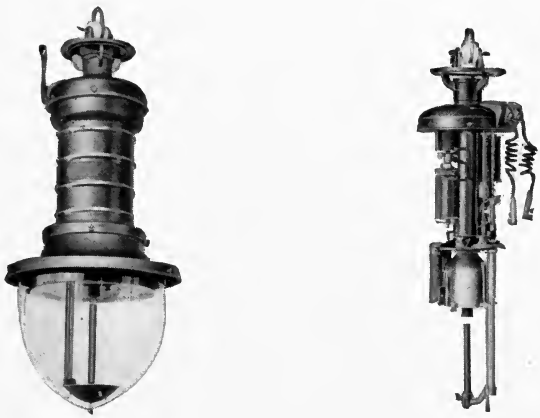
Fig. 198. Schematic Diagram of Connections for Series Magnetite (Luminous) Arc Lamp.

Fig. 198 shows the circuits of a series magnetite lamp, and its operating mechanism. Unlike the carbon lamp, the arc circuit is open when the lamp is not in operation.† When the current is turned on, the starting magnet closes the arc circuit by raising the lower electrode into contact with the copper rod. The series magnet then attracts its armature and opens the circuit of the starting

* In the Westinghouse magnetite lamp the lower electrode is fixed.

† If the arc circuit is not kept open the terminals weld or stick together, making starting difficult or impossible.

magnet, allowing the lower electrode to drop and "strike" the arc. As the electrode burns away, the voltage over the arc increases until the shunt magnet attracts its armature, closes the circuit of the starting magnet, and feeds the lower electrode upward by a definite step. In case the arc circuit becomes open, the shunt magnet closes the circuit of the starting magnet and the current flows through the starting magnet and its series resistance, so that the other lamps in



(*a*) Complete. (*b*) Casing Removed.

FIG. 199. G. E. Series Luminous (Magnetite) Arc Lamp.

the circuit are unaffected by the burning out of an electrode, or by failure to feed properly.

The mechanism of the magnetite lamp, when intended for parallel operation, is similar to that of the series lamp, with the omission of the shunt magnet. The starting magnet raises the lower electrode and the series magnet breaks the circuit of the starting magnet, which allows the lower electrode to fall and strike the arc as in the series lamp. When the electrode has burned away to such an extent that the series coil is no longer able to hold its armature, the circuit of the starting magnet is closed, and the electrode fed upward.

(*c*) *Flaming-arc lamps.* — The light of the flaming-arc lamp is due to the luminescent vapor of metallic salts, the salts of calcium (yellow light) and of titanium (white light) being those commonly used. The electrodes consist, essentially, of a mixture of carbon, the metallic salt, and an alkaline salt. The purpose of the alkaline salt is to steady the arc and to prevent the accumulation of slag.

High efficiency is obtained only by rapid consumption of the electrodes, the great length (14 to 24 inches) and small diameter ($\frac{1}{4}$ to $\frac{3}{8}$ inch) of which make vertical mounting impracticable. The electrodes are, therefore, usually placed as indicated in Fig. 200, and fed downward by gravity, the mechanism holding them being released by an electromagnet connected in parallel with the arc, when the voltage across the arc reaches a predetermined value. The electrical circuits of one of the simpler regulating mechanisms are shown in Fig. 200, and are similar to those of the differential carbon lamp described above. The electrodes are normally separated. When the circuit is closed, the shunt magnet pulls the points of the electrodes together (lateral movement of one electrode is provided for), closes the arc circuit, and energizes the series magnet. The series magnet separates the electrodes, thus striking the arc, and equilibrium is established when the arc is from $1\frac{1}{2}$ to 2 inches in length. As the electrodes burn away, the voltage over the shunt coil increases, while the current in the series coil tends to decrease (parallel operation), and equilibrium is maintained by an inward movement of the movable electrode. When the electrode reaches the limit of its movement, the shunt magnet causes the mechanism holding the electrodes to be released, and they are fed downward, reducing the length of the arc. The increased current in the windings of the series magnet causes the electrodes to be separated and the arc is restored to its normal length.

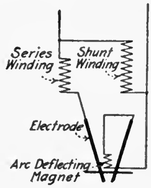


FIG. 200. Wiring Diagram Flaming Arc Lamp.

The short life (10 to 18 hours) of flaming-arc electrodes led to the development of the regenerative flaming-arc lamp, which is an adaptation of the flaming-arc principle to the enclosed lamp, and increases the life of the electrodes to seventy hours or more. Tubes through which the gases circulate conserve the heat, produce a more perfect combustion, and increase the efficiency of the lamp. The electrodes used in regenerative lamps, being shorter and of greater diameter than those used in the flaming-arc lamps, may be arranged vertically as in the carbon and magnetite lamps.

The flaming-arc is the most efficient of artificial illuminants, and gives approximately three candle power per watt.

Arc lamps are always provided with air dash pots or some similar device for damping the movements of the electrode, which would otherwise be jerky and irregular.

2. Instability of the electric arc. — The electric arc is inherently unstable when operated from constant potential mains, *i.e.*, when the arc is connected directly between the mains. Suppose an electric arc to be established between constant voltage mains. Any slight decrease in the value of the current flowing in the circuit causes the cross-sectional area of the arc to decrease, increases the resistance of the circuit, and still further decreases the current; any slight increase in the value of the current flowing in the circuit causes the cross-sectional area of the arc to increase, decreases the resistance of the circuit, and still further increases the current. The effects of any momentary change in the value of the current flowing in the circuit are, therefore, cumulative, and either cause the arc to "break," or the current to increase to an excessive value.

Stable operation of the multiple arc lamp is established by means of the ballast (resistance in the continuous-current lamp and reactance in the alternating-current lamp) referred to above, the operation of which is as follows: If the current decreases slightly, the drop in the ballast decreases, and the voltage between the terminals of the arc increases; if the current increases slightly, the drop in the ballast increases, and the voltage between the terminals of the arc decreases. The ballast thus produces a compensating change in the voltage between the terminals of the arc for any change in the resistance of the circuit (area of the arc).

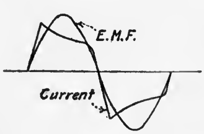


FIG. 201. Distorted Current Wave of the Electric Arc.

3. Power factor of the alternating-current arc. — It is an experimental fact that the power factor of the alternating-current arc is less than unity, its average being about 0.85. This low power factor is not due to a time lag between the current and the electromotive force, but to distortion of the current wave. As pointed out in Section 2, the resistance of the electric arc is a function of the current flowing in the circuit, and the resistance of an alternating-current arc depends on the instantaneous value of the current. Fig. 201 is a reproduction of oscillograms of the current and electromotive force waves in an alternating-current arc.

4. Incandescent lamps. — The incandescent lamp, which is universally used for interior lighting, consists of a hair-like filament enclosed in a highly exhausted and hermetically sealed glass globe. The light of the lamp is an indirect effect, and is due to the fact that

the electric current raises the temperature of the filament to a "white" heat. The purpose of the globe, in addition to the mechanical protection which it gives, is to retard oxidation of the filament and thus increase the life of the lamp. Connection between the filament and the line conductors is made through short wires embedded in the glass.

For more than twenty-five years after the incandescent lamp became a commercial article, carbon was practically the only substance of which filaments were made, but within a few years carbon filaments have been largely displaced by those of metallic tungsten. The light produced by the tungsten lamp is decidedly superior to that of the carbon lamp, and approximately three times as much light is available from a given expenditure of energy. The perfection of the tungsten lamp, which is sold under the trade name "Mazda," has greatly reduced the cost of electric light, and thereby largely increased its use.

While the efficiency of an incandescent lamp increases as the applied voltage is increased, the life of the filament is shortened. Lamps are rated at that voltage which has been found to give the most satisfactory results, taking into account both efficiency and length of life, and should be operated at this voltage.

The temperature, and therefore the efficiency, at which a tungsten filament may be operated is materially increased when it is surrounded by an atmosphere of nitrogen. The nitrogen-filled tungsten lamp is now a commercial article and has an efficiency, in large units, as low as $\frac{1}{2}$ watt per candle power.

The incandescent lamp is a constant potential lamp, and is used, primarily, for parallel connection between constant voltage mains, although series lamps are used to a considerable extent in street lighting.

5. The Nernst lamp. — The luminous element (glower) of the Nernst lamp is a hollow cylinder composed of oxides of some of the rarer elements (zirconia, yttria, etc.). The glower is a non-conductor at ordinary temperatures, and is heated to a conducting temperature by a coil of platinum wire connected in parallel with the glower. The circuit of the heating coil is automatically opened when current begins to flow in the glower circuit.

The negative temperature coefficient of the glower makes the operation of the lamp unstable unless a ballast is connected in series

with the glower. The ballast coil is made of iron, the temperature coefficient of which is positive.

The glowers of Nernst lamps are always enclosed, usually by alabaster globes which conserve the heat and prolong the life of the glowers. The color effects of this lamp are good and the efficiency is high.

6. The mercury vapor lamp. — The mercury vapor lamp consists of a highly exhausted glass tube containing a small quantity of mercury, which may be vaporized by an electric current. The light, which is due to the luminescence of the mercury vapor, is entirely devoid of red rays. This absence of red rays causes colors to be distorted, but the light is very acceptable in draughting rooms, packing rooms, warehouses and other places where its peculiar color effects are not objectionable. There is now on the market a phosphorescent reflector which adds red rays to the light emitted by the mercury lamp.

Like the mercury rectifier, to which it is analogous, the mercury vapor lamp, because of its high resistance when cold, must be started by an auxiliary device of some kind. Starting may be effected by tilting the tube until the terminals are connected by liquid mercury through which the current flows, heating the mercury and filling the tube with mercury vapor. The tube is tilted by hand, or by means of an electromagnet which is automatic in its action. Starting may also be accomplished by breaking an auxiliary circuit, the inductive kick from which breaks down the high initial resistance of the tube.

Because of its low power factor, the use of the alternating-current mercury vapor lamp is objectionable.

7. The quartz lamp. — The fundamental principle of the quartz lamp is the same as that of the mercury vapor lamp in that its light is due to luminescent mercury vapor. The pressure of the mercury vapor in the quartz lamp is materially higher than in the mercury vapor lamp, the temperature and the luminous intensity are correspondingly greater, and the light is not entirely devoid of red rays.

The quartz lamp emits a considerable quantity of ultra-violet rays which are decidedly harmful to living organisms, and should not be used unless it is enclosed by a protecting globe.

8. The Moore tube. — An application of the Geissler discharge to commercial lighting is made by means of the Moore tube, which

consists of a highly exhausted glass tube, of any length up to about 200 feet, the luminous properties of which are due to an electric discharge through rarified gases introduced into the tube. The quality of the light produced varies with the medium through which the electric discharge takes place; carbon dioxide gives a light approximating daylight, nitrogen an orange tinted light, air a pinkish light.

The operation of the tube depends essentially on the maintenance of the proper pressure of the rarified gas in the tube, the electric discharge forming a solid precipitate which reduces the pressure in the tube. As the pressure decreases the conductivity increases, and the increased current operates a valve which admits gas to the tube, and restores the required pressure.

CHAPTER XVI

CIRCUIT-INTERRUPTING APPARATUS

THE circuit-interrupting apparatus of an electric plant is the means by which the generators and the load are connected and disconnected, and includes not only the switches, but fuses and other circuit-breaking devices, and the auxiliary apparatus for their manipulation. The satisfactory operation of a power plant depends, in no small degree, on the proper selection, installation, and operation of the switching gear and the protective apparatus.

1. **Fuses.** — A fuse is a short piece of lead and tin alloy, of such cross section and so connected in the circuit that it is melted and the circuit opened when an excessive current flows. Fuses are either "open" or "enclosed." Enclosed fuses are to be preferred since the arc and the hot metal incident to the opening of the circuit are confined, thus reducing the fire hazard.

The current at which a fuse will open a circuit can be determined only approximately because of external conditions, such as the temperature of the air, area of contact, etc.

2. **Switches.** — Switches are for the specific purpose of connecting and disconnecting the generator and the load apparatus, and may be divided into: (*a*) air-break switches, (*b*) carbon-break switches, (*c*) oil-break switches.

(*a*) *Air-break switches.* — An air-break switch consists of one or more blades of copper hinged at one end and making contact at the other end with spring clips which form part of the circuit. As operating switches, *i.e.*, for opening current-carrying circuits, air-break switches are used on small apparatus only, because of the burning of the contacts when the switch is opened; for completely isolating other apparatus from a "live" line, they are universally used.

(*b*) *Carbon-break switches.* — To protect the copper terminals of an air break switch, a circuit having carbon terminals is connected in parallel with the one having copper terminals. In operating the switch, the copper terminals open first, without an arc, thus shunt-

ing the current through the carbon circuit and protecting the copper contacts from injury. The carbon terminals, which may be renewed, are not so easily burned as are copper terminals.

(*c*) *Oil-break switches.* — For the manipulation of high voltage circuits or those carrying large currents, switches having their contacts immersed in oil are always used. The arc formed when the contacts are separated is promptly smothered by the oil without appreciable damage to the contacts.

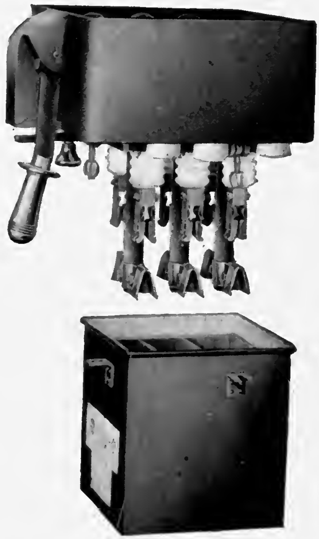


FIG. 202. Triple-pole, Single-throw Non-automatic Oil Switch. General Electric Co.

Since a fuse cannot be depended on to open a circuit promptly when the current exceeds a specified value, and must be renewed each time it operates, automatically opening switches have been devised for the protection of electrical apparatus. An automatic switch is closed against the pressure of a spring, and is held by a latch. When the current exceeds a predetermined value, an electromagnet trips the latch, and the spring opens the switch.

The coils of the tripping magnet are excited: (1) by the line current, the winding of the magnet being connected in series with the load, (2) from an auxiliary source, the circuit through the coils of the magnet being closed by a relay connected in the secondary circuit of a series transformer, as indicated in Fig. 205. The first method is applicable to either continuous- or alternating-current circuits; the second, to alternating-current circuits only. The current at which an automatic switch opens is adjusted by changing the length of the air gap in the magnetic circuit of the electromagnet, or the relay, or by changing the tension of a spring.

In many cases, particularly in motor operation, both automatic switches and fuses are placed in the circuit. The fuses are rated slightly higher than the current at which the switch is set to operate, and their purpose is to protect the motor in case the operating mechanism of the switch becomes deranged.

By the addition of an air dash-pot or other mechanical device,

the tripping mechanism of an automatic switch is made less sensitive to momentary overloads or surges, *i.e.*, the switch opens only after the overload has been maintained for a definite length of time, which may be made inversely proportional to the overload. Such devices are known as time limit relays.

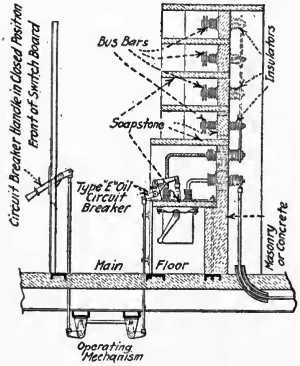


FIG. 203. Section through Typical Bus Bar and Switch Compartments. Westinghouse.

3. **Switch operation.** — Switching operations in small plants may be done manually, but in larger plants hand operation becomes unsatisfactory and often impossible, because of the size of the moving parts, as well as dangerous, because of high voltages. Electrical power is, naturally, used for the operation of large switches. The two methods of switch operation in general use are: (*a*) solenoid, (*b*) motor.

(*a*) *Solenoid-operated switches.* — Solenoid-operated switches require the use of two solenoids, one for closing the switch, the other

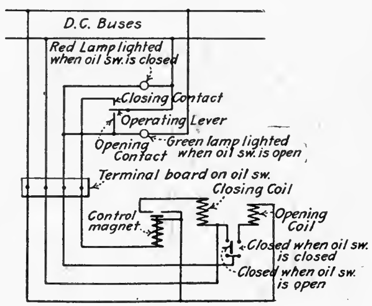


FIG. 204. Diagram of Connections for Non-automatic Solenoid-operated Oil Switch.

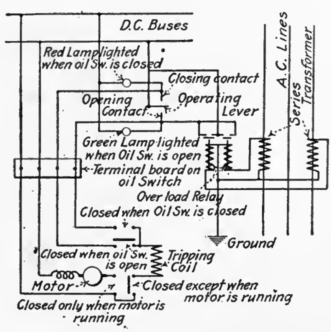


FIG. 205. Diagram of Connections for Automatic Motor-operated Oil Switch.

for opening it. In alternating-current plants the solenoids (Fig. 204) are usually supplied with continuous current from the exciter buses, and are energized only during the time the switch is opening or closing. When the switch is to open automatically, it may be closed against the pressure of a spring, and the opening coil replaced

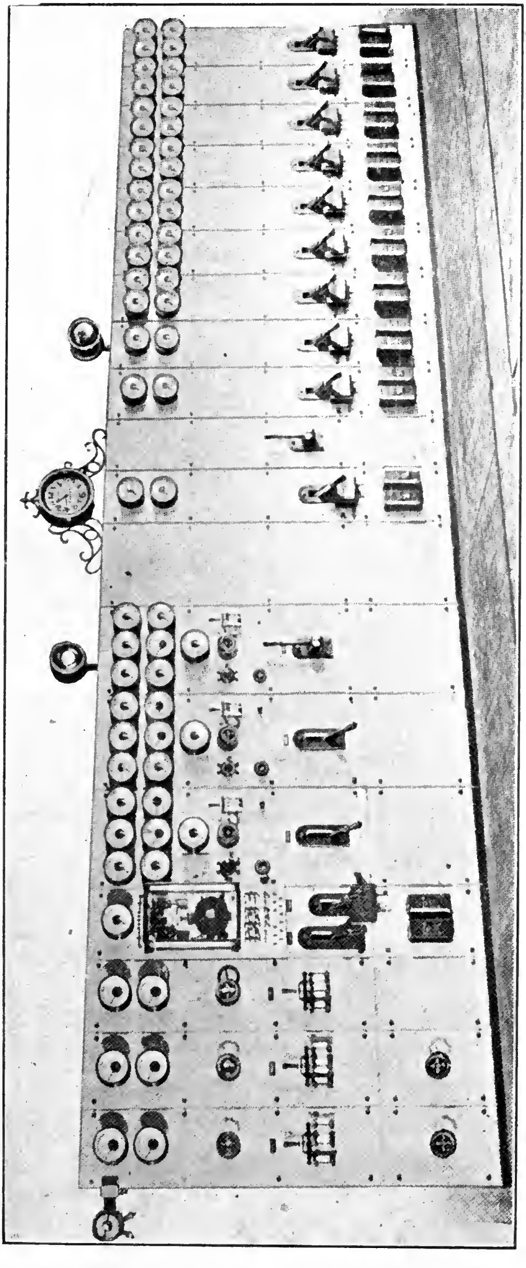


Fig. 206. Switchboard, Mare Island Navy Yard. Westinghouse.

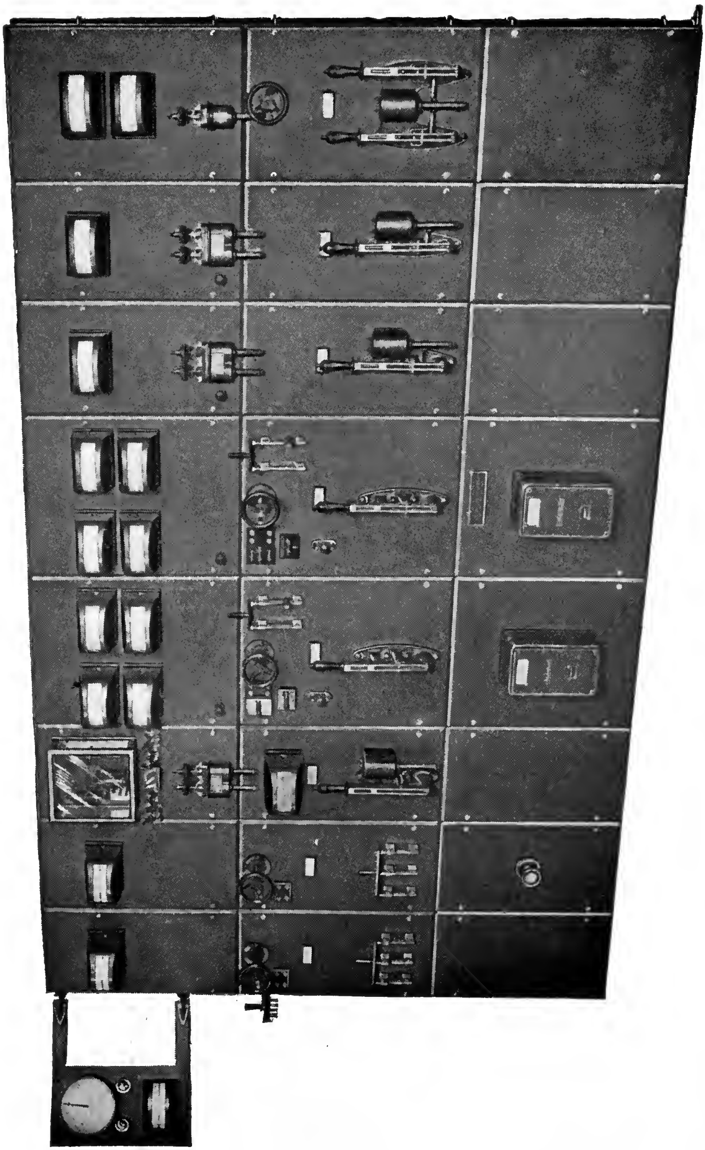


Fig. 207. Typical Alternating-current Switchboard. General Electric Co.

by a tripping coil which releases a latch, and allows the spring to open the switch.

(*b*) *Motor-operated switches.* — In motor-operated switches, the actual closing and opening of the switch is done by springs, the function of the motor being to wind up the springs after each operation. Closing the circuit of a tripping coil or control magnet, automatically or by hand, operates the switch, the movement of the switch parts closing the circuit of the motor so that it winds up the springs ready for the next operation. Fig. 205.

Bull's eye lamps, usually red and green, the circuits of which are opened and closed by the movement of the switch parts, indicate whether the switch is open or closed.

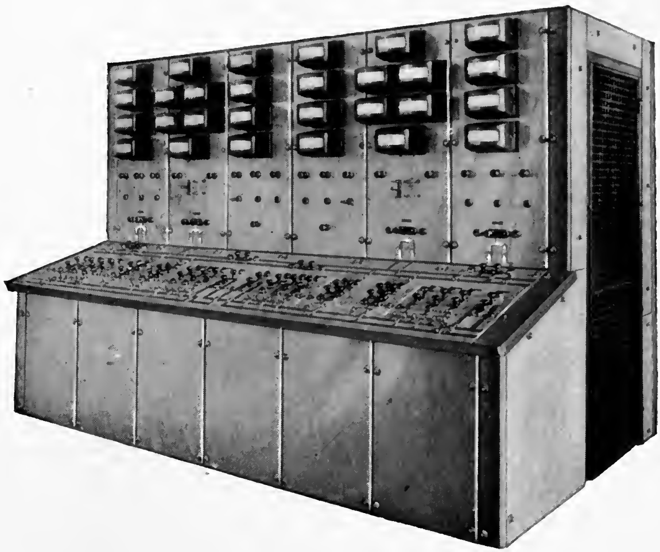


FIG. 208. Typical Benchboard. General Electric Co.

4. **Switchboards.*** — The switching apparatus and the indicating instruments of a plant are grouped and placed on slate or marble panels which are combined to form the switchboard, typical examples of which are shown in Figs. 206 and 207. The arrangement of the apparatus is a matter of convenience, indicating instruments being placed near the top of the board where they are easily read; switch and rheostat handles at such a height as to be easily

* For details of switchboard construction the reader is referred to publications of manufacturing companies.

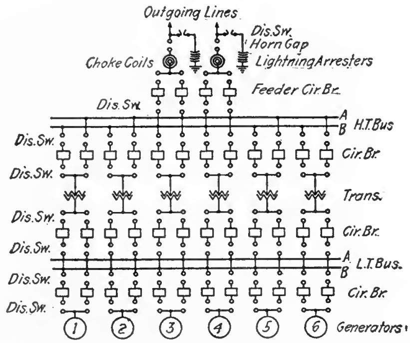
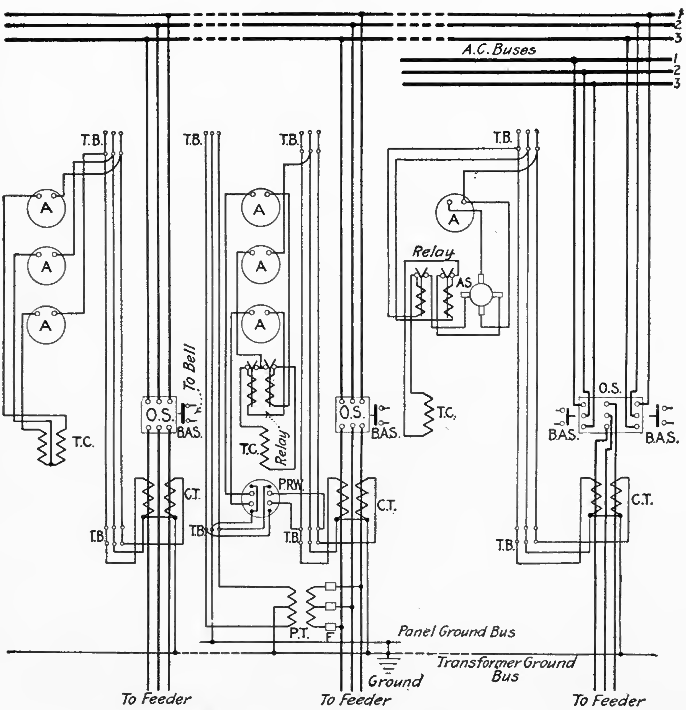


FIG. 209. Skeleton Wiring Diagram (Single Line).



KEY TO SYMBOLS

A	= Ammeter.	*P.T.*	= Potential Transformer.
A.S.	= Three-way ammeter switch.	*P.R.W.*	= Polyphase watthour meter.
B.A.S.	= Bell alarm switch.	*T.B.*	= Terminal board for secondary leads from current and potential transformers.
C.T.	= Current transformer.		
F	= Fuse.		
O.S.	= Oil switch.	*T.C.*	= Trip coil on oil switch.

FIG. 210. Typical Wiring Diagrams for Three-phase Feeder Panels. General Electric Co.

operated; recording meters and other apparatus requiring only periodic or infrequent attention, near the floor, and on either the front or the back of the panel.

Electrical operation of switches permits of maximum concentration of control apparatus. Control wiring involves only low voltage circuits of small ampere capacity. The wires and apparatus are, therefore, of small size and may be placed close together. Control levers, indicating lamps, instruments, relays, etc., may be placed on a "bench" or control board (Fig. 208) the location of which is entirely independent of the location of the switches.

CHAPTER XVII

METERS *

1. Ammeters and voltmeters. — Commercial ammeters and voltmeters operate, in general, on the same essential principles. The current-carrying coils of an ammeter consist of a few turns of heavy wire connected in series with the load apparatus; the current-carrying coils of a voltmeter consist of a large number of turns of fine wire connected in parallel with the load apparatus. Voltmeter and ammeter connections are shown in Fig. 211.

Ammeters and voltmeters may be classified as: (*a*) hot wire instruments, (*b*) permanent magnet instruments, (*c*) electrodynamometers, (*d*) soft iron instruments, (*e*) induction instruments.

(*a*) *Hot wire instruments.* — The fact that an electric current heats the conductor through which it flows, and that the length of the conductor varies with the temperature, are made use of in the design of hot wire instruments. The operation of such an instrument will become clear from a study of Fig. 212. Since the current-carrying wire is always under tension, it tends to assume a permanent "set" which destroys its calibration.

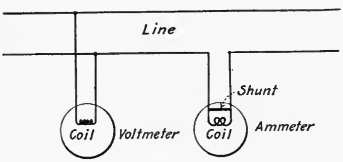


FIG. 211. Voltmeter and Ammeter Connections.

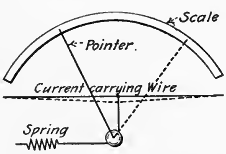


FIG. 212. Diagram of Hot Wire Instrument.

(*b*) *Permanent magnet instruments.* — In this type of instrument a current-carrying coil is pivoted between the poles of a permanent magnet. When current flows in the coil, the reaction between the magnetic flux and the coil produces a torque,† and the coil tends

* This discussion of meters is necessarily very brief, but the writer believes that even this short study will be found both interesting and profitable. For a detailed discussion of the design, construction and operation of electrical meters and their auxiliary apparatus, the reader is referred to "Electrical Meters," by Cyril M. Jansky.

† See Chapter 2, Section 14.

to rotate, but its movement is opposed by springs and the coil assumes that position where the torque, which is proportional to the current flowing in the coil, is balanced by the tension of the springs. Because of its permanent magnetic field, this type of instrument is applicable to continuous-current circuits only.

The movable coil is often wound over an aluminum frame (Fig. 213) which acts as a supporting structure, and also tends to make the instrument "dead beat," *i.e.*, to cause the coil to come to rest without a prolonged series of oscillations, the movement of the aluminum frame in the magnetic field inducing currents in the frame, which oppose its movement.*

(*c*) *Electrodynamometer.* — The electrodynamometer is similar in operation to permanent magnet instruments, the magnetic field being produced by a stationary coil connected in series with a movable coil. Because of the varying strength of the magnetic field, the deflection of the movable coil is proportional to the square of the current, and the divisions of the scale are not uniform. The electrodynamometer is applicable to either continuous or alternating-current measurements.

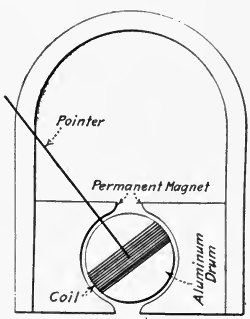


FIG. 213. Operating Principle of Permanent Magnet Ammeters and Voltmeters.

(*d*) *Soft iron instruments.* — Soft iron instruments include: (1) the plunger type, in which a fixed current-carrying coil and a movable soft iron plunger react to move a pointer over a scale as indicated in Fig. 214, (2) the magnetic vane type, in which the movable coil of the electrodynamometer is replaced by a vane of soft iron to which the pointer is attached. Soft iron instruments are cheap, and applicable to either continuous- or alternating-current circuits, but are less accurate than other types.

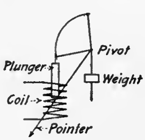


FIG. 214. Schematic Diagram of Plunger Ammeter (or Voltmeter).

(*e*) *Induction instruments.* — When a disc or drum of copper, aluminum or other conducting material is placed between the poles of an alternating-current magnet, currents are induced in the metal by the changing flux.† By the addition of a "shading" coil,‡ as

* See Chapter 2, Section 13. † See Chapter 2, Section 13. ‡ See Chapter 13, Section 17.

shown in Fig. 217a, or of a transformer coil and an auxiliary winding, as indicated in Fig. 217b, a shifting flux is produced, and the reaction between this shifting flux and the currents

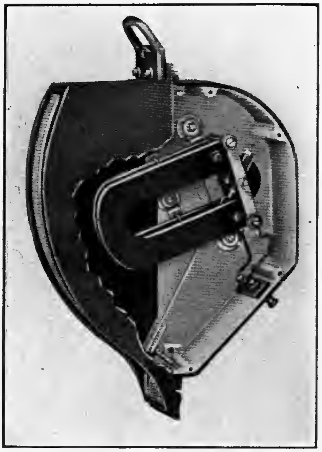


FIG. 215. Edgewise Permanent Magnet Voltmeter. Westinghouse Electric & Mfg. Co.

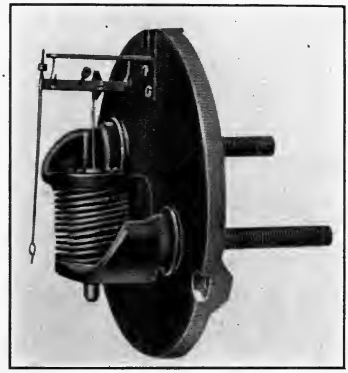


FIG. 216. Soft Iron Core Ammeter. (Without Case.) Westinghouse Electric & Mfg. Co.

induced in the disc or drum, causes the deflection of the latter against the tension of a spring.* Induction instruments are applicable to alternating-current circuits only.

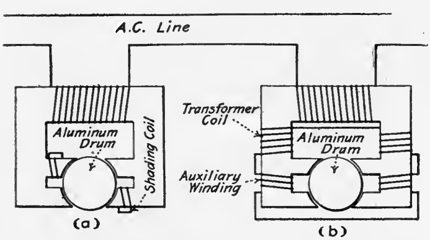


FIG. 217. Diagram of Connections for Induction Ammeter.

2. Ammeter shunts. — If the entire current output of a large plant were to flow in the coils of an ammeter, the meter would be both expensive and cumbersome. If the meter coil is connected in parallel with a resistance, as indicated in Fig. 218a, the currents in the two branches are inversely proportional to the resistances of the branches, and the ammeter indication is proportional to the total current in the circuit.

Such a resistance, known as an ammeter or current shunt, is used with practically all commercial ammeters intended for use

* See Chapter 13, Section 17.

on continuous-current circuits. On instruments indicating twenty-five amperes or less, the shunts are enclosed in the cases of the instruments; for instruments of greater capacity, external shunts are usually used. Fig. 219.

3. Series transformers. — On alternating-current circuits, the ammeter shunt is replaced by a series or "current" transformer. If the primary winding of a transformer is connected in series with the load, as indicated in Fig. 218b, a certain fall of potential takes place when current flows in the windings. This voltage drop is due to the impedance of the windings, and is directly proportional to the current, the impedance of the windings being constant. Since there is a constant ratio* between the primary and the secondary voltages of a transformer, the secondary voltage is proportional to the current flowing in the primary circuit.† Therefore, if the impedance of the secondary circuit is constant, the secondary current is proportional to the primary current.

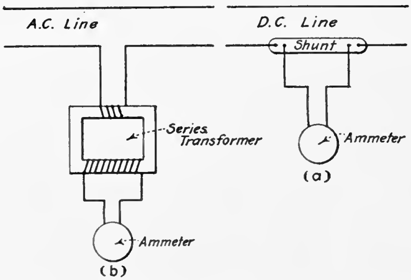


FIG. 218. Series Transformer and Ammeter Shunt Connections.

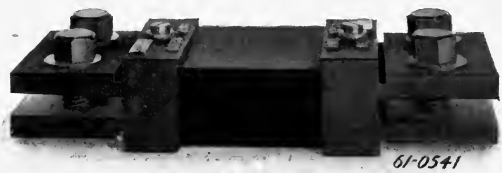


FIG. 219. Ammeter (Current) Shunt. Wagner Electric Mfg. Co.

Series transformers, when designed for use with ammeters or wattmeters, are usually so proportioned that the full-load secondary current is five amperes. If the instrument is not calibrated to read direct, the instrument indication must be multiplied by the ratio of current transformation.

* See Chapter 12, Section 3.

† *Warning.* — If the primary current remains constant, the secondary voltage of a series transformer increases with the impedance of the circuit and this type of transformer should *never* be operated with the secondary circuit open. *Before opening any circuit supplied from the secondary winding of a series transformer, short-circuit the secondary terminals of the transformer.*

4. High voltage measurements. — The above types of instruments are not adapted to the direct measurement of voltages above seven hundred volts. For measuring voltages higher than this there must be used: (*a*) a multiplier, (*b*) a potential transformer, (*c*) a special voltmeter.

(*a*) *The multiplier.* — A multiplier is a resistance connected in series with the current-carrying coil of a voltmeter, or the voltmeter element of a wattmeter, and serves simply to reduce the voltage that would otherwise be applied to the terminals of the coil. The resistance of the multiplier should bear a simple ratio to the resistance between the terminals of the voltmeter with which it is to be used, so that the voltage of the circuit is some multiple of the meter indication. The multiplier is applicable to continuous- or to alternating-current circuits. Fig. 220a.

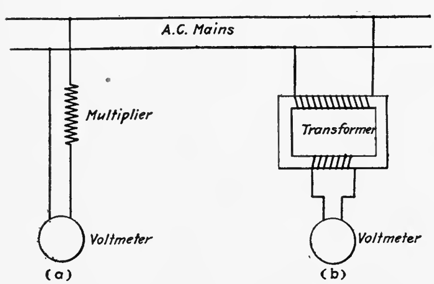


FIG. 220. Voltmeter Connections. (*a*) Multiplier. (*b*) Transformer.

(*b*) *The potential transformer.* — The voltage of an alternating-current circuit may be stepped down by means of a small potential transformer, the voltage of the primary circuit being the indication of the voltmeter multiplied by the ratio of transformation. Fig. 220b.

(*c*) *Special voltmeters.* — The voltage of a high potential system is determined by means of: (1) the electrostatic voltmeter, (2) the spark gap.

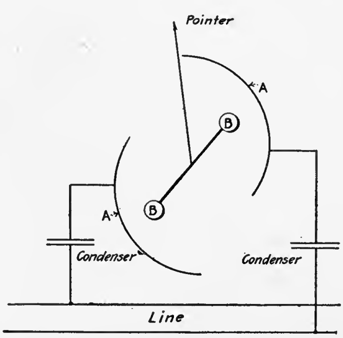


FIG. 221. Schematic Diagram of Connections for Westinghouse Static Voltmeter.

(1) *The electrostatic voltmeter.* — The principle on which the electrostatic voltmeter operates is the attraction between two oppositely charged bodies.* The essential parts of the Westinghouse electrostatic voltmeter are shown schematically in Fig. 221. *AA*

* See Appendix C, Section 2.

are curved metallic plates connected, through condensers, to the conductors, the difference of potential between which it is desired to measure; and *BB* are hollow cylinders to which a pointer is attached. The position of the hollow cylinders changes as the voltage between lines changes, and the scale is calibrated to read volts.

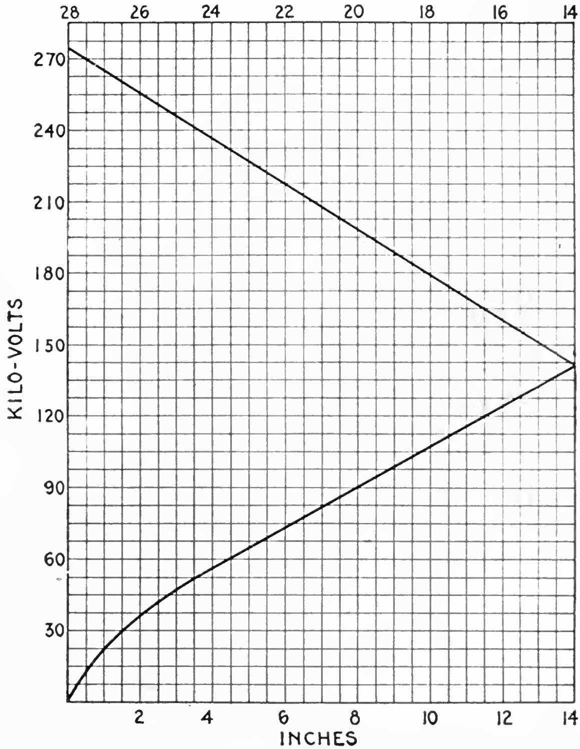


FIG. 222. Sparking Distances.

Standard Conditions: 00 sewing needles.
25° Centigrade.
760 mm. barometer.
80% humidity.
Sinusoidal voltage wave form.

(2) *The spark gap.* — The voltage required to break down the resistance of the air between two needle points is approximately constant. If the distance between points at which the voltage of a given line breaks down the intervening air is determined, the voltage of the line may be determined by reference to a table or a curve. Fig. 222.

5. **Wattmeters.** — A wattmeter is a combination of a voltmeter and an ammeter operating on the same movable element, so that the deflection of a pointer is proportional to the product of the current in the circuit and the voltage between the terminals of the voltage coil. Two types of wattmeters have been developed: (*a*) electrodynamometer, (*b*) induction.

(*a*) *Electrodynamometer wattmeter.* — The electrodynamometer wattmeter consists of a fixed coil of large wire (ammeter element) connected in series with the load, and a movable coil of fine wire (voltmeter element) connected in parallel with the load. The ammeter element sets up a flux with which the voltmeter element reacts to cause a deflection of the fine wire coil,* and the deflection is proportional to the product of the currents in the coils, *i.e.*, to the power in the circuit. This type of wattmeter indicates the power in either continuous- or alternating-current circuits. Fig. 223.

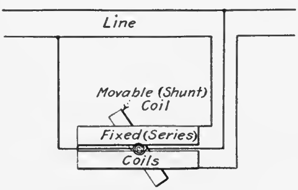


FIG. 223. Schematic Diagram of Dynamometer Wattmeter.

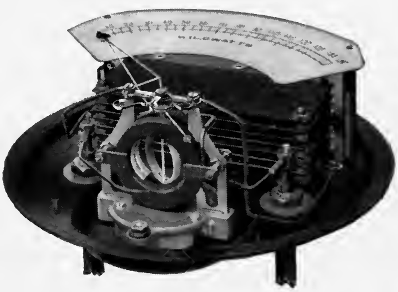


FIG. 224. Weston Wattmeter (Cover Removed).

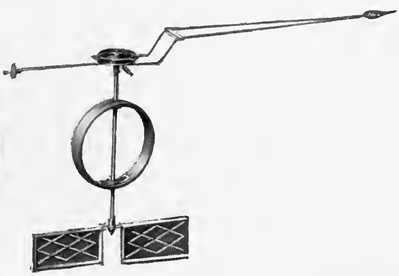


FIG. 225. Movable Element of Weston Wattmeter.

(*b*) *Induction wattmeter.* — The induction wattmeter is identical in principle with the induction ammeter and voltmeter described above, *i.e.*, deflection of a pointer is caused by the reaction between a shifting flux and the currents induced in a disc or drum. The ammeter element, which is wound with a few turns of heavy wire, has negligible inductance; the voltmeter element, which is wound with many turns of fine wire, has large inductance. The relations

* See Chapter 2, Section 14.

of the fluxes are as represented in Fig. 226a, *i.e.*, the flux due to the parallel winding lags behind that due to the series winding by the angle θ.

The angle θ in Fig. 226a is increased to 90 degrees by means of an auxiliary or compensating winding which is a short-circuited winding similar to a "shading coil," * except that it surrounds the entire pole. The compensating coil acts as the short-circuited secondary of a transformer, and sets up a flux the magnitude and phase relations of which are represented by OC in Fig. 226b. The flux in the magnetic circuit of the parallel winding is, then, the geometric difference of that due to the shunt coil and that set up by the compensating winding. By properly proportioning the compensating winding, a flux in exact quadrature with that set up by the series winding is produced. The resultant of these quadrature fluxes is a shifting or rotating flux, and a torque which is proportional to the product of the voltage of the circuit and the load amperes. Like other induction apparatus, the induction wattmeter is applicable to alternating-current circuits only.

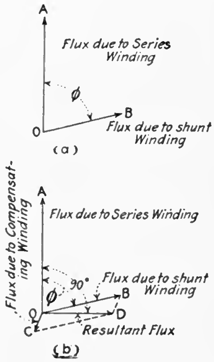


FIG. 226. Vector Diagrams of Fluxes in the Induction Wattmeter.

That a properly designed wattmeter indicates the power in an alternating-current circuit when the load is either non-inductive or inductive is evident when the torque relations during a cycle are considered. In a non-inductive single-phase circuit, the torque is constant in direction but varies in value from zero to maximum, and the wattmeter indication is proportional to the average torque. In an inductive circuit, the direction of the torque is not constant, and the wattmeter indication is proportional to the algebraic sum of the average positive and negative torques. The current and voltage remaining constant, it may be proved both mathematically and experimentally, that the average net torque is proportional to the cosine of the phase angle.† The deflection of the movable system of a wattmeter is, therefore, proportional to the product of the current, the electromotive force, and the cosine of the angle by

* See Chapter 13, Section 17.

† See Chapter 1, Section 28.

which the current leads or lags behind the electromotive force, *i.e.*, to the power in the circuit.

6. Polyphase wattmeters. — For the measurement of power in polyphase alternating-current circuits, polyphase wattmeters have been designed. The polyphase wattmeter is a combination of two or more single wattmeter elements acting on the same movable part, and each pair of coils is connected as if it formed an independent instrument.

7. Watt-hour meters. — A watt-hour meter is a small motor, the speed of the rotating parts of which is proportional to the power in the circuit to which it is connected. The movable parts of such motors are made very light, and friction is reduced to a minimum by the use of jewel bearings.

Watt-hour meters are divided into two classes: (*a*) commutator meters, (*b*) induction meters.

(*a*) *Commutator meters.* — Commutator watt-hour meters are similar, in principle and in action, to the continuous-current shunt motor, and are applicable to either direct- or alternating-current circuits. The armature is wound of many turns of fine wire and is connected in series with a resistance, between the supply lines. The field, which is produced by the series windings, is proportional to the load current, the magnetic circuit being in air.

With constant electromotive force between the terminals of the armature winding, the torque of the motor is proportional to the current flowing in the field windings. Therefore, to make the speed proportional to the power in the circuit, the load on the motor (counter torque) must be proportional to the speed. This is accomplished by means of an eddy-current brake, consisting of a disc of copper or aluminum attached to the shaft of the motor, and rotated between the poles of a permanent magnet. When current flows in the coils of the meter, a torque is produced, and the movable parts of the meter increase in speed until the driving torque is balanced by the retarding torque of the permanent magnet and the eddy currents induced in the rotating disc. Since the torque producing motion is proportional to the power in the circuit to which the meter is connected, and the retarding torque is proportional to the speed of the rotating parts, the speed of the rotating parts is proportional to the power in the circuit. By means of a train of gears, the total number of revolutions made by the arma-

ture is recorded, the calibration being such that the meter reads in either watt-hours or kilowatt-hours. For a given load, the speed of the rotating parts of a meter is changed by changing the position of the permanent magnet, which is made adjustable.

Friction between the moving parts of the meter and the supporting bearing, which would disturb the torque-speed relations at light loads to a very considerable extent, is compensated for by the addition of an auxiliary field winding connected in series with the armature winding, and so adjusted that when zero current flows in the series windings a slight jar causes the armature to rotate. If the field set up by this auxiliary winding is too strong, the meter "creeps," *i.e.*, the armature rotates when the load circuit is disconnected; if the field set up by the auxiliary winding is too weak, the meter does not register on light loads.

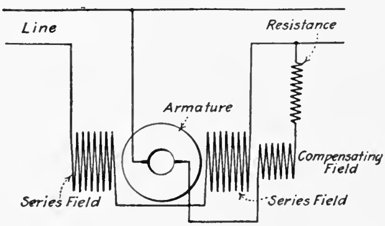


Fig. 227. Schematic Diagram of Connections for Commutating Watt-hour Meter.

The electrical circuits of a commutator watt-hour meter are shown in Fig. 227.

(*b*) *Induction meters.* — The induction watt-hour meter is, in principle, a two-phase squirrel-cage induction motor, the quadrature fluxes being produced in the same manner as described for the induction watt-meter. The squirrel-cage element of the induction meter is an aluminum disc or cup which also serves as the movable part of the eddy current brake, thus making the movable system of an induction meter lighter than that of the commutator type. The torque per unit weight is also greater. The necessity for friction compensation is, therefore, reduced, but is provided by means of a "shading coil"* on the shunt winding.

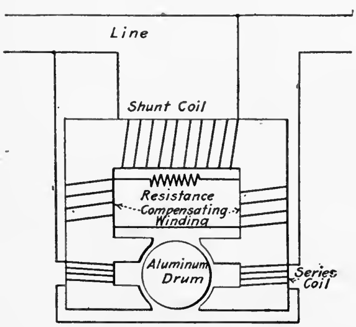


Fig. 228. Schematic Diagram of Connections for Induction Watt-meter and Watt-hour Meter.

An induction watt-hour meter is shown schematically in Fig. 228.

* See Chapter 13, Section 17.

Because of the principle on which it operates, the induction watt-hour meter is not applicable to continuous-current circuits.

8. Recording or graphic meters. — It is often desirable to have a continuous record of the momentary fluctuations in the values of such electrical quantities as voltage, current, power, etc. Such records are obtained by means of a pen or pencil, the position of

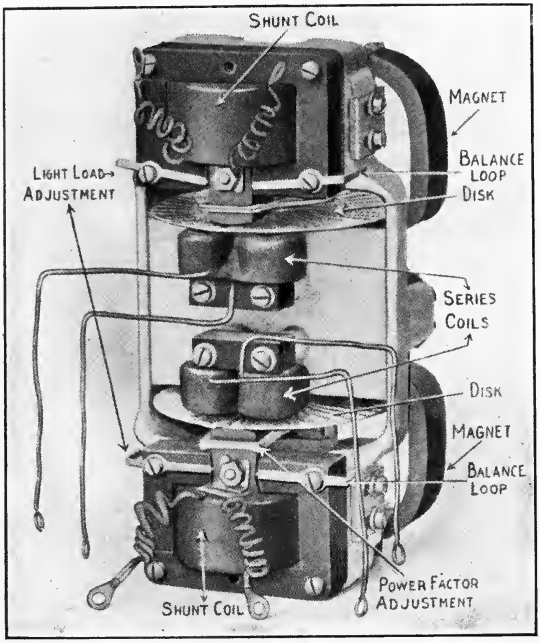


FIG. 229. Elements of Polyphase Induction Watt-hour Meter. Westinghouse Elec. & Mfg. Co.

which is controlled by the magnitude of the quantity to be recorded, and a uniformly moving paper on which the pen or pencil traces a record. A detailed description of the mechanism of such instruments is beyond the scope of this work.

9. The synchroscope. — The synchroscope or synchronism indicator is a device by means of which the phase relations of two electromotive forces and their relative frequencies are indicated.

The synchronism indicator used by the General Electric Co. is, structurally, a small synchronous motor the field winding of which is connected to the bus bars. Fig. 230. The movable coils are connected to the incoming machine through a phase-

splitting device. There is, then, superimposed on the field set up by the movable coils, an alternating flux due to the stationary winding, and the movable parts assume a fixed position which is dependent on the phase relation between the electromotive force of the incoming machine and that of the bus bars, the frequencies of the two electromotive forces being the same. Correct phase relations for parallel connection of alternators (phase opposition) exist when the stationary (" shadow ") and movable pointers coincide.

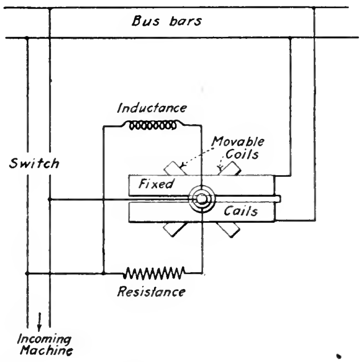


FIG. 230. Schematic Diagram of Connections for General Electric Synchronism Indicator.

If the frequencies of the electromotive forces are not the same, a torque which causes the armature to rotate is produced. The direction of rotation is clockwise or counter clockwise according as the speed of the incoming machine is too fast or too slow, the rate at which the pointer rotates indicating the difference in the frequencies of the electromotive forces.

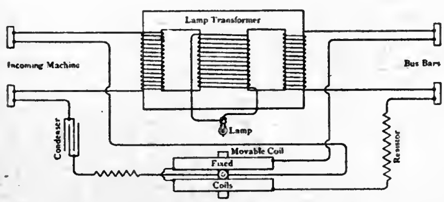


FIG. 231. Schematic Diagram for Weston Synchroscope.

The Weston synchroscope operates on the electrodynamometer principle, the movable element vibrating instead of rotating. By reference to Fig. 231, it will be seen that the fixed coils are connected to the bus bars in series with a slightly inductive resistance, and the movable coil to the incoming machine in series with a condenser. The relative values of the capacitance, the resistance and the inductance are such that the currents are in exact quadrature when the voltage of the incoming machine is in phase, or in phase opposition, with that of the bus bars. Under this condition no torque is exerted between the coils, and the pointer is at rest in the middle of the scale.

When the electromotive forces are not in phase or in phase opposition, a torque, proportional to the phase displacement, causes the

movable coil to be deflected, and the pointer is moved to the right or to the left as the electromotive force of the incoming machine leads or lags behind that of the bus bars.

If the frequencies are not the same, the phase displacement, and consequently the position of equilibrium, changes momentarily, and the pointer swings back and forth across the scale. The lamp, which illuminates the dial of the instrument, is so connected that it is dark when the voltages are in phase, and light when they are in phase opposition. Consequently, the pointer seems to rotate either clockwise or counter clockwise, the direction of apparent rotation indicating whether the incoming machine is too fast or too slow, and the rate of apparent rotation, the amount by which the frequencies differ.

10. Power-factor meters. — If the stationary coils of a synchroscope are connected in series with the load apparatus and the movable coils between the supply mains, the position of the movable element depends on the phase relations of the currents in the two coils as explained in Section 9. Since the phase relations of the currents in the coils are dependent on the power factor of the load circuit, the scale may be calibrated to read power factors.

The Weston power-factor meter. — The movable element of the Weston power-factor meter differs from that in the synchroscope in that it has two coils, the planes of which are at right angles. The stationary coils are connected in series with the load, or to the secondary terminals of a series transformer, and the movable coils across different phases of a polyphase system or are provided with a phase-splitting device. Fig. 232. If the current in the stationary coils is in phase with that in either of the movable coils, the plane of one movable coil coincides with the axis of the fixed coils; if the current in the fixed coil is not in phase with that in either of the movable coils, the movable system is deflected, and the angle of deflection is equal to the angle between the current in the stationary coil and that in one of the movable coils, *i.e.*, to the power-factor angle.

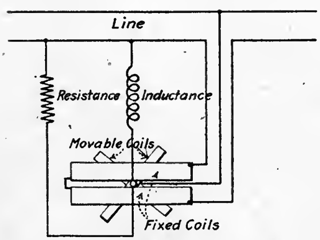


FIG. 232. Schematic Diagram of Connections for Weston Power-factor Meter.

Westinghouse power-factor meter. — The essential parts of a Westinghouse power-factor meter are: (*a*) a soft iron vane or armature of the shape indicated in Fig. 233, (*b*) two or more angularly displaced stator coils which are connected, through series transformers, with a polyphase system, (*c*) a stationary potential coil, the axis of which coincides with that of the iron vane.

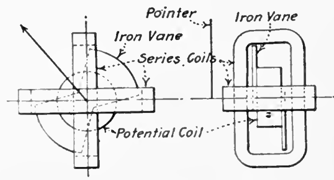


Fig. 233. Schematic Diagram of Westinghouse Power-factor Meter.

The stator coils set up a rotating flux which, when unaffected by other forces, causes the armature to rotate. The potential coil polarizes the armature and, at unity power factor, it assumes a position at right angles to the stator coil, the current in which is in phase with the current in the potential coil. When the power factor of the system is not unity, the current in the potential coil is not in phase with that in the stator coil, and the armature is deflected through an angle equal to the angle of phase displacement.

Polyphase power-factor meter. — The indications of a split-phase meter are, obviously, affected by the frequency of the circuit to which it is connected. This difficulty is overcome by connecting the movable coils to a polyphase system, thus utilizing the inherent voltage displacement of such a system. The operation of such an instrument is in no other way different from that of the split-phase meter.

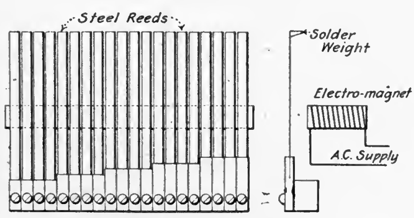


Fig. 234. Principle of Operation of Vibrating Reed Frequency Meter.

11. Frequency meters. — Frequency meters indicate the frequency at which a system is operating, and are of two types: (*a*) vibrating-reed meters, (*b*) split-phase meters.

(*a*) *Vibrating-reed meters.* — If a series of steel strips, differing in length and in inertia, are arranged in front of an alternating-current magnet, as indicated in Fig. 234, those strips whose natural period of vibration corresponds to the frequency of the alternating-current system are thrown into violent vibration, while the others are affected only slightly or not at all.

(*b*) *Split-phase meters.* — Split-phase frequency meters are essentially differential induction voltmeters. One coil is connected to the mains in series with a non-inductive resistance and its current is, therefore, practically independent of frequency; the other coil is connected to the same mains through an inductive resistance which causes the current to vary as the frequency changes. At normal frequency, the torque of one coil is equal and opposite to that of the other; as the frequency of the system increases or decreases the torques are no longer equal, and equilibrium is restored by a deflection of the movable element. Fig. 236.

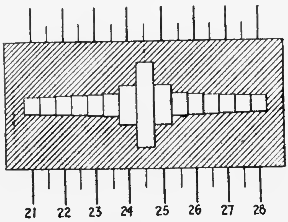


Fig. 235. Section of Vibrating-Reed Frequency Meter Scale Showing Method of Indication.

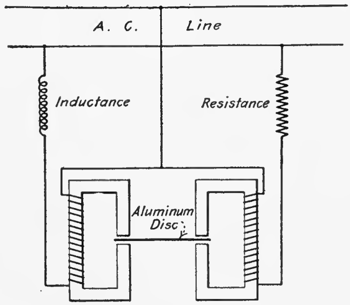


Fig. 236. Schematic Diagram of Connections for Induction (Split-phase) Frequency Meter.

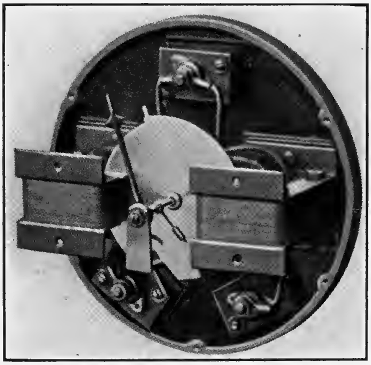


Fig. 237. Westinghouse (Split-phase) Frequency Meter.

In the Weston frequency meter two stationary coils are interconnected with resistances and reactances so as to form a Wheatstone bridge which, at normal frequency, is balanced. As the frequency increases or decreases, the bridge is unbalanced, and a movable element of soft iron caused to deflect. The electrical connections and the movable element of this meter are shown in Fig. 238.

12. Ground detectors. — A "ground" is a connection between a current-carrying conductor and the earth, which materially reduces the normal insulation resistance of the line. A ground

cannot be stated to exist when the resistance between the conductor and the earth falls below a certain fixed minimum, because of the different insulation requirements of different systems — what is good insulation under one condition may be a very serious ground under other circumstances.

A ground on one line of a system does not, in itself, cause any trouble, but if two lines become grounded, a serious leakage occurs,

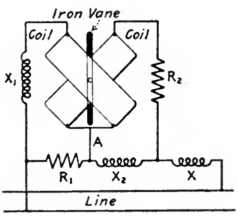


FIG. 238. Schematic Diagram for Weston Frequency Meter.

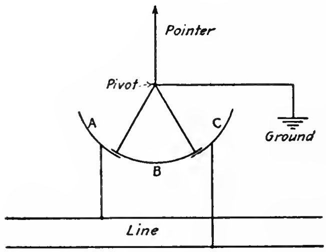


FIG. 239. Schematic Diagram of Static Ground Detector.

or a short-circuit is established. Since a line may accidentally become grounded at any time, it is good practice to have on the switchboard, instruments which indicate the fact whenever the resistance between any line and the earth becomes seriously reduced. Such instruments are known as "ground detectors."

The static ground detector operates on the same principle as the electrostatic voltmeter, *i.e.*, the attraction between two oppositely charged bodies. Let *A* and *C* in Fig. 239 be curved plates connected to current-carrying conductors, as indicated; *B* a plate which is free to move about the pivot and which is connected, through a negligible resistance, to the earth. If the lines are equally insulated, equal and opposite forces tend to deflect plate *B* * from its position midway between plates *A* and *C*. If the lines are not equally insulated, the equilibrium of forces acting on *B* is destroyed, and *B* moves toward the plate (*A* or *C*) which is connected to the line having the higher (better) insulation; and the deflection is proportional to the relative resistances between the lines and the earth.

* See Appendix C, Section 2.

CHAPTER XVIII

POWER TRANSMISSION AND DISTRIBUTION *

The advantages of power transmission by means of the electric current are such that this method has practically superseded all other methods except for very limited distances. Within the last few years a number of large hydroelectric plants have been completed, the power from which is transmitted one hundred miles or more at voltages up to 140,000. The transmission line is the connecting link between the generators and the distributing network, and is usually run over a private right of way, the current-carrying conductors being supported by wooden poles or steel towers. Fig. 240. Conductors of distributing systems are carried on poles or are placed in conduits underground.

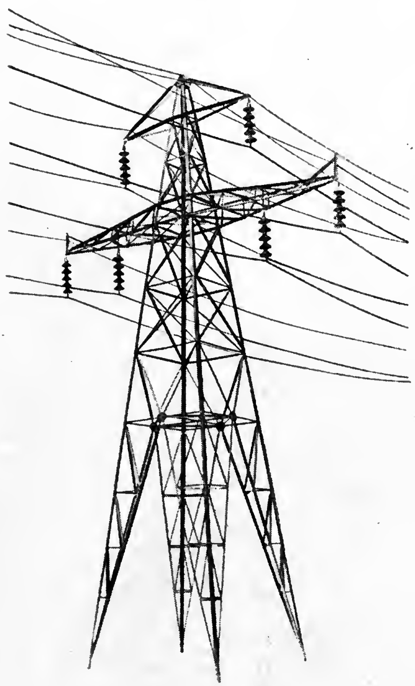


FIG. 240.

1. Conductors. — Copper wires or cables are almost universally used for the transmission and the distribution of electric power, although aluminum is used to some extent because its light weight makes the number of supporting structures required a minimum. The sizes of commercial conductors are expressed by their areas in circular mils, or by gauge

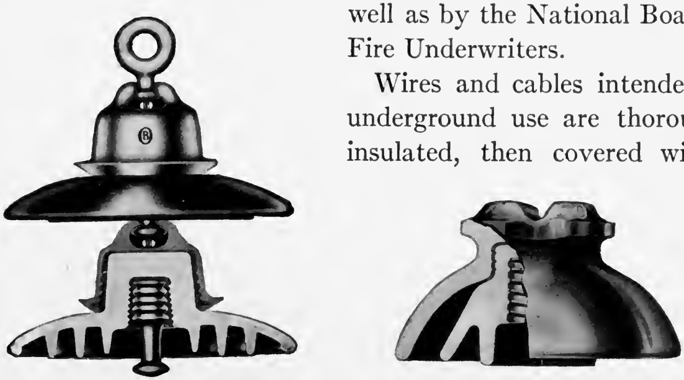
* For an extended discussion of transmission problems, the details of pole line design, etc., the reader is referred to "Overhead Electric Power Transmission" by Alfred Still, and to "Elements of Electrical Transmission" by O. J. Ferguson.

numbers. The American Wire Gauge (A. W. G.), often termed Brown & Sharpe (B. & S.), is universally used in the United States.

A characteristic of the A. W. G. which makes it easy to approximate the area corresponding to any given gauge number without the use of a wire table, is the fact that No. 10 is approximately 0.1 inch in diameter and has an area of approximately 10,000 circular mils, and that the area halves or doubles, approximately, for each three gauge numbers, *i.e.*, No. 7 has an approximate area of 20,000 circular mils while No. 13 has an approximate area of 5000 circular mils. Gauge numbers and areas of wires, with their weights and resistances, are given in Table VI.

2. **Insulation.** — For many purposes it is required that current-carrying wires be covered with some insulating material. The amount and the quality of this insulation depends on the use for which the conductor is intended, and may consist of: (*a*) a cotton braid, (*b*) a cotton braid impregnated with a liquid compound which dries and becomes hard, (*c*) a coating of vulcanized rubber over which, as a mechanical protection for the rubber, is wound a braid having a polished surface. Rubber insulation is required for all interior wiring by a rule of the National Electric Code (N. E. C.), which has been adopted by the national engineering societies as well as by the National Board of Fire Underwriters.

Wires and cables intended for underground use are thoroughly insulated, then covered with a continuous moisture-proof lead sheath over which may be wound a braid, or other mechanical protection.



(a) Suspension Insulator. (b) Pin Insulator.

FIG. 241. Line Insulators. The Ohio Brass Co.

For insulating aërial conductors from their supporting structures glass or porcelain insulators are used. Two types of line insulators are in general use, illustrations of which are shown in Fig. 241.

Pin-type insulators are generally used for voltages up to 66,000; suspension type for voltages greater than this.

3. **Carrying capacity of conductors.** — The current which an insulated conductor may safely carry is limited by the temperature at which the insulation softens, or by reason of which its insulating properties deteriorate. The maximum current allowed by the N. E. C. is given in Table VII for rubber and for other insulations.

4. **Inductance.** — A magnetic field which opposes* any change in the value of the current is set up around any current-carrying conductor.† Alternating-current circuits are, therefore, subject to an inductance which varies with the distance between the conductors and with their size. Inductances for different sizes of wires and for different spacings are given in Table VIII, the values being those obtained by the following formula: ‡

$$L = 0.03028 + 0.282 \log_{10} \frac{2D}{d}, \qquad (1)$$

when L = the inductance in millihenries per 1000 feet of two-wire circuit (2000 feet of conductor),
D = the distance between wires in inches,
d = the diameter of the wires in inches.

The inductance of each wire in a three-phase system is one-half that of the loop formed by any two of the three conductors.

5. **Capacitance.** — Two or more metallic conductors § separated by air or other insulating material, form a condenser, the effect of which is to cause a leading current to flow in an alternating-current circuit. The value of this leading current, termed the charging current, is proportional to the applied voltage, to the capacitance of the circuit, and to the frequency.

$$I_c \text{ (single phase)} = 2\pi f E C\, 10^{-6}, \qquad (2)$$

$$I_c \text{ (three phase)} = \frac{4\pi f E C\, 10^{-6}}{\sqrt{3}}, \qquad (3)$$

when E = the line-to-line voltage,
C = the capacitance in microfarads,
f = the frequency of the supply circuit,
I_c = the charging current.

* Because of the electromotive force induced in the conductor when the magnetic field changes.

† See Chapter 2, Section 3. ‡ See Appendix B, Section 1. § See Appendix C, Section 9.

The capacitance of a transmission line varies with the distance between the wires, with their diameter, and with the nature of the insulating material between the conductors. The capacitances given in Table IX were calculated by means of the following formula: *

$$C = \frac{0.003677}{\log_{10} \frac{2D}{d}}, \qquad (4)$$

when C = the capacitance in microfarads per 1000 feet of two-wire circuit (2000 feet of conductor),
D = distance between wires in inches,
d = diameter of conductors in inches.

6. Skin effect. — When an alternating current flows in a conductor, the current density over the cross-sectional area is not uniform, but increases as the distance from the center of the conductor increases. This is the "skin" effect, and increases the resistance losses in the conductor. The increase in the resistance of a conductor to alternating currents as compared with its resistance to continuous currents is a function of the product of the area of the conductor and the frequency of the alternating current. The skin effect is negligible when the product of the area of the conductor in circular mils and the frequency does not exceed 30,000,000, or, for commercial frequencies, when the diameter of a conductor is not greater than three-quarters of an inch.

7. Line calculations. — The inductance and the capacitance of a transmission line are made up of an infinite number of inductances and capacitances uniformly distributed over the system. An exact solution of such a system involves equations too complicated for practical use, but an approximate solution giving fairly accurate results is easily made when the capacitance is assumed to be concentrated, *e.g.*, one-half at each end of the line. Let

E_l = the voltage at the terminals of the load,
E_d = the voltage drop in the line,
E = the voltage at the terminals of the generator,
I_l = the load current,
I_c = the charging current due to the capacitance at the load end of the line,

* See Appendix C, Section 9.

I = the line current,
L = the inductance of the line,
R = the resistance of the line,
$\cos\phi$ = the power factor of the load circuit,
ϕ' = the angle between E_l and I,

$$\theta = \tan^{-1}\frac{\omega L}{R}.$$

The relations between E_l, I_l and I_c are known, and their vectors may be plotted as shown in Fig. 242, and the value of I and that of the angle ϕ' determined.

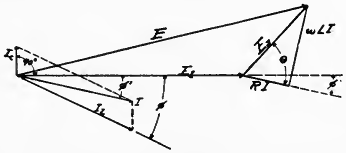


FIG. 242. Vector Diagram for Transmission Line.

$$I = \sqrt{I_l^2\cos^2\phi + (I_l\sin\phi - I_c)^2}, \quad (5)$$

$$\phi' = \tan^{-1}\frac{I_l\sin\phi - I_c}{I_l\cos\phi}, \quad (6)$$

also,

$$E_d = I\sqrt{R^2 + \omega^2 L^2}. \quad (7)$$

Adding, vectorially, the line drop and the load voltage, the voltage at the generator terminals is obtained.

$$E = \sqrt{[E_l + E_d\cos(\theta - \phi')]^2 + E^2\sin^2(\theta - \phi')}. \quad (8)$$

Problem — Single phase. — 500 kw. are to be transmitted a distance of 20 miles, the voltage at the load end (primary terminals of step-down transformers) is 25,000, the frequency is 25, and the power factor of the load circuit is 86.6 per cent. Find: (*a*) the drop in the line, (*b*) the voltage at the generator terminals.

$$C = 0.00155 \times 5.28 \times 20 = 0.1637 \text{ microfarad.}$$
$$I_c = 2\pi \times 25{,}000 \times 0.1637 \times 25 \times 10 = 0.64 \text{ ampere.}$$
$$L = 0.694 \times 5.28 \times 20 \times 10 = 0.073 \text{ henry.}$$
$$\omega L = 0.073 \times 157 = 11.26 \text{ ohms.}$$
$$R = 0.3258 \times 20 \times 2 = 13.03 \text{ ohms.}$$
$$I_l = \frac{500{,}000}{25{,}000 \times 0.866} = 23.1 \text{ amperes.}$$
$$I = \sqrt{(23.1 \times 0.866)^2 + (23.1 \times 0.5 - 0.64)^2} = 22.8 \text{ amperes.}$$
$$E_d = 22.8\sqrt{(13.03)^2 + (11.26)^2} = 392 \text{ volts.}$$
$$\theta = \tan^{-1} 0.864 = 40^\circ\, 50'.$$
$$\phi' = \tan^{-1}\frac{11.6 - 0.64}{20} = 0.548 = 28^\circ\, 45'.$$
$$\theta - \phi' = 12^\circ\, 5'.$$
$$E = \sqrt{(25{,}000 + 392 \times 0.977)^2 + (392 \times 0.209)^2} = 25{,}338 \text{ volts.}$$

If greater accuracy is required, the charging current may be taken as that due to the mean voltage on the line, and a second approximation made. This is, however, seldom necessary as it is only at very high voltages, or over long distances that the capacitance of transmission lines becomes very marked, and, in many cases, it may be disregarded entirely as having no appreciable effect on the voltage regulation of the line.

Problem — Three phase. — 2000 kw. are to be transmitted over a distance of 20 miles, the conductors used are 000 spaced 48 inches apart, the voltage at the load terminals (line to line) is 25,000, the frequency is 25, and the power factor of the load circuit is 86.6 per cent. Assume the capacitance to be concentrated at the ends of the line. Find: (*a*) the drop in the line, (*b*) the voltage between the generator terminals.

$$C = 0.00155 \times 5.28 \times 20 = 0.1637 \text{ microfarad.}$$

$$I_c = \frac{4\pi \times 25 \times 25{,}000 \times 0.1637 \times 10^{-6}}{2\sqrt{3}} = 0.37 \text{ amperes.}$$

$$L = \frac{0.694 \times 5.28 \times 20 \times 10^{-3}}{2} = 0.037 \text{ henry.}$$

$$\omega L = 0.037 \times 157 = 5.75 \text{ ohms (per line).}$$

$$R = 0.3258 \times 20 = 6.5 \text{ ohms (per line).}$$

$$I_l = \frac{2{,}000{,}000}{25{,}000 \times 0.866 \times \sqrt{3}} = 53.5 \text{ amperes.}$$

$$I = \sqrt{(53.5 \times 0.866)^2 + (53.5 \times 0.5 - 0.37)^2} = 53.3 \text{ amperes.}$$

$$E_d = 53.3\sqrt{(6.5)^2 + (5.75)^2} = 454 \text{ volts.}$$

$$\theta = \tan^{-1} 0.8847 = 41^\circ 30'$$

$$\phi' = \tan^{-1} 0.569 = 29^\circ 40'.$$

$$\theta - \phi' = 11^\circ 50'.$$

$$E = \sqrt{\left(\frac{25{,}000}{\sqrt{3}} + 454 \times 0.978\right)^2 + (454 \times 0.205)^2}$$

$$= 14{,}895 \text{ volts from line to neutral}$$

$$= 25{,}768 \text{ volts from line to line.}$$

8. Distributing systems. — The distributing systems in common use at the present time are: (*a*) series, (*b*) parallel, (*c*) series-parallel, (*d*) multiple wire.

(*a*) *The series system.* — In the series system the entire current flows successively in each piece of apparatus, the voltage varying as the load changes, and the current remaining approximately constant. It is largely used in alternating current street lighting

installations in connection with a constant-current transformer, either with or without a mercury arc rectifier.

(*b*) *The parallel system.* — The parallel system is the one commonly used in supplying motors and incandescent lamps. The current divides before reaching the lamps or the motors, so that the current flowing in one piece of apparatus may be greater or less than that flowing in another piece.

(*c*) *Series-parallel system.* — It is undesirable to increase the voltage of a distributing system above that required to operate about one hundred arc lamps connected in series. Therefore, in large installations, the lamps are connected into series groups, and these groups are connected in parallel, each group, if the supply is from alternating-current mains, being supplied through a constant-current transformer.

(*d*) *Multiple-wire systems.* — When the power in a system is constant, the current decreases as the voltage increases. If the line loss (RI^2) is constant, the resistance of the conductors is four times as great when the voltage of the system is doubled, *e.g.*, the cross section (or weight) of wire used in a 220-volt system is only one-quarter that used in a 110-volt system, the line loss and the total power delivered being the same in each case. This fact has led to the development of multiple-wire systems which effect a large saving in copper without increasing the voltage between terminals of the load apparatus. The only one of these multiple-wire systems in extensive use is that using three wires, a greater number of wires adding undesirable complications in both the generating and the distributing system.

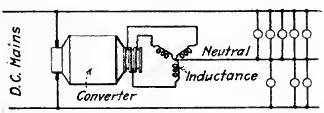


FIG. 243. The Rotary Converter as a Three-wire Balancer.

The simplest application of the three-wire system is in connection with an alternating-current system where the third or neutral wire is connected to the middle point of the secondary winding of a transformer, and the load is connected between each of the main wires and the neutral.*

The application of the three-wire principle to continuous-current distribution requires the use of: (1) a rotary converter and inductance coils, (2) a special generator, (3) a motor-generator balancing set.

* See Chapter 12, Section 2.

(1) *Rotary converter.* — If the rings of a rotary converter are connected through suitable inductance coils, as indicated in Fig. 243, the converter armature acts as a balance, and the voltage between either main and a neutral connected to the junction of the inductance coils is approximately equal to one-half that between the mains, whatever may be the relative value of the currents in the mains.

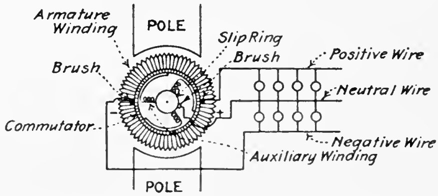


FIG. 244. Schematic Diagram of Crocker-Wheeler Three-wire Generator.

(2) *Three-wire generators.* — Three-wire generators are essentially rotary converters, the neutral connection to the armature being made through an auxiliary winding or through inductance coils.

The armature of the Crocker-Wheeler three-wire generator carries three auxiliary windings which are star-connected to the main winding and to a single collector ring, as shown in Fig. 244. The Triumph Electric Company's three-wire generator makes use of star-connected external inductance coils.

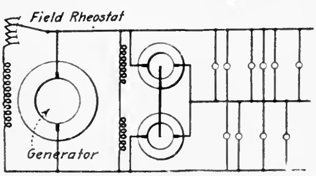


FIG. 245. Motor-generator Balancer Set.

(3) *The motor-generator balancer.* — If two similar shunt dynamos are mounted on the same shaft and connected as indicated in Fig. 245, a three-wire system is produced. The action of a balancer set is as follows: As long as the load is balanced, *i.e.*, equal currents flow in the main wires, zero current flows in the neutral, and the currents in the armatures of the balancer set are equal. When the currents in the mains are unequal, their difference flows in the neutral, and the current in the armature connected between the neutral and the main carrying the smaller current increases. This increased armature current causes the speed of the armature to increase, and the other dynamo is driven as a generator. The speed of the balancer set increases until equilibrium is reëstablished, and the voltage between each main and the neutral is automatically maintained at a value equal, approximately, to one-half that between the mains. Neglecting the losses, the armatures of the balancer set are required

to carry the current flowing in the neutral wire, and this current divides inversely as the voltages between the neutral and the main wires.

The voltage regulation of a three-wire system is improved by the use of compound balancers, the series field windings being so connected that the motor is differentially wound and the generator cumulatively wound.

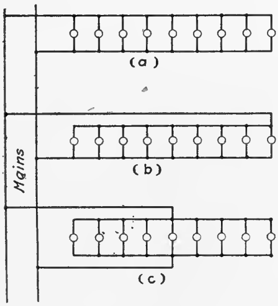


FIG. 246. Parallel Distribution.

9. Voltage regulation of parallel systems. — For the satisfactory operation of incandescent lamps the electromotive force between the terminals of the lamps must be maintained at an approximately constant value. It is not difficult to design a wiring system for a group of lamps that is operated as a unit, *i.e.*, all the lamps in the group are either lighted or not lighted, but to design a wiring system for the same group of lamps, any part of which may be in use at a given time, may be a rather complicated problem.

The general problem is to determine the center of distribution (the center of gravity) of the group, at which the voltage should be maintained constant by means of feeder regulation, and to proportion the wiring between this center and the individual lamps, so that the variation of voltage at the most disadvantageously situated lamp never exceeds the allowable limits.

To meet the requirements of a distributed load, various wiring schemes have been devised, their object being to minimize the difference in voltage between the terminals of different lamps in the group. Some of these schemes are indicated in Fig. 246.

10. General wiring formulæ. — The following expressions will be found useful in the solution of wiring problems:

$$I = \frac{P}{E \times \cos \phi} \text{ for single-phase or continuous-current circuits.} \tag{9}$$

$$I = \frac{P}{2\,E \times \cos \phi} \text{ for two-phase circuits.} \tag{10}$$

$$I = \frac{P}{\sqrt{3}\,E \times \cos \phi} \text{ for three-phase circuits.} \tag{11}$$

$$E_d = \frac{21.6 \times D \times I \times k}{\text{C.M.}}$$ for continuous, single-phase or two-phase circuits. (12)

$$E_d = \frac{18.7 \times D \times I \times k}{\text{C.M.}}$$ for three-phase circuits. (13)

when C.M. = the circular mil area of the conductor used,
D = the distance of transmission in feet,
E = the line voltage at terminals of the load,
E_d = the volts lost in the line.
I = the line current,
k = the impedance factor $\left(\frac{\text{impedance}}{\text{resistance}}\right)$. (Tables XV or XVI.)
Cos ϕ = the power factor of the load circuit,
P = the total power (watts) delivered to the load circuit.

Example. — 30 k.w. are to be transmitted over a distance of 500 feet, the voltage at the terminals of the load is 440, and the frequency is 60, single-phase. Find the drop in the line when No. 4 wires spaced 18 inches apart are used, and the power factor is 0.85.

$$I = \frac{30{,}000}{440 \times 0.85} = 80.2 \text{ amperes.}$$

$$E_a' = \frac{21.6 \times 500 \times 80.2 \times 1.12}{41{,}740} = 23.2 \text{ volts.}$$

Example. — 100 k.w. are to be transmitted a distance of 300 feet at a voltage of 440, a frequency of 60, a power factor of 0.85, three-phase. Find the size of wire required, the line drop not to exceed 4 per cent.

$$I = \frac{100{,}000}{\sqrt{3} \times 440 \times 0.85} = 155 \text{ amperes.}$$

From Table VIII the minimum size wire allowable is No. 1 the area of which is 83,690 c.m.

Substituting in equation (13)

$$E_d = \frac{18.7 \times 390 \times 155 \times 1.37}{83{,}690} = 14.3 \text{ volts,}$$

which is within the allowable limits and should be used.

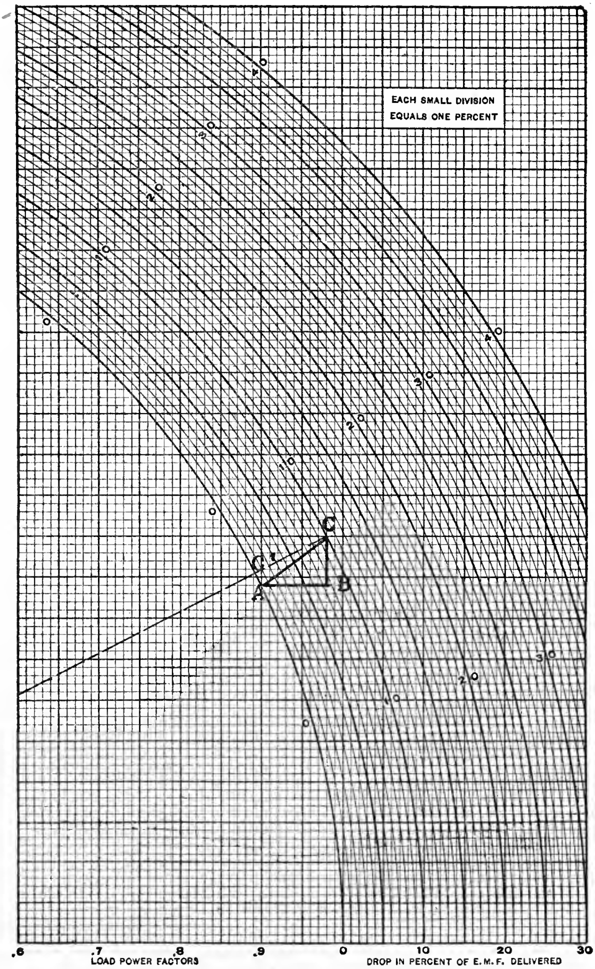


FIG. 247. Mershon's Diagram.

11. Mershon's diagram. — Graphical solution of alternating-current distribution problems may be made by means of Mershon's diagram.* Referring to Fig. 247 let

the radius of the smallest arc represent the voltage at the end of the feeder, or at the center of distribution;

AB represent the resistance drop in the line, expressed as a percentage of the delivered voltage;

BC represent the reactance drop in the line, expressed as a percentage of the delivered voltage.

* Originated by Ralph D. Mershon, Past-President of the American Institute of Electrical Engineers.

Lay off AB beginning at a point A where the arc cuts the vertical representing the power factor of the load circuit; lay off BC perpendicular to AB, as indicated in the diagram. The generator voltage is indicated by the position of the point C and may be read directly from the diagram. The power factor of the feeder circuit is determined by the point C' where a line drawn from C to the origin cuts the arc.

12. Feeder regulation. — When several feeders take current from the same bus bars, it is desirable to be able to regulate, independently, the feeder voltages. Feeder regulation may be accomplished by the use of: (*a*) boosters, (*b*) auto-transformers, (*c*) induction regulators. Auto-transformers and induction regulators are applicable to alternating-current circuits only; boosters are used on continuous-current circuits.

(*a*) *Boosters.* — A line booster consists of a series generator driven by a shunt motor, or other constant-speed engine. The armature

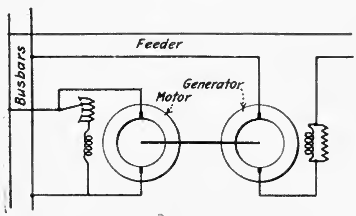


FIG. 248. Diagram of Booster Connections.

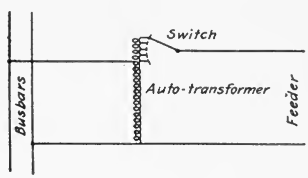


FIG. 249. Diagram of Connections for Auto-transformer Feeder Regulator.

and the field of the generator are connected in series with the load, as indicated in Fig. 248. The voltage of the generator is proportional to the current flowing in the feeder and is added to the bus-bar voltage, and any desired increase in the voltage may be obtained by properly proportioning the booster field.

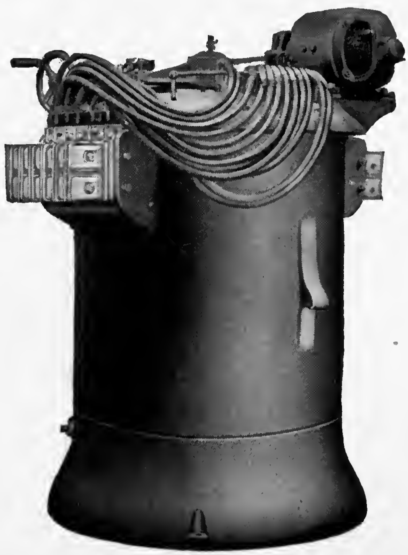
Because of the addition of two rotating machines, the efficiency of the system is reduced and the operating complications greatly increased.

(*b*) *Auto-transformer.* — If an auto-transformer is connected as indicated in Fig. 249, the number of effective turns in the secondary depends on the position of the switch. The Stillwell regulator operates on this principle, and is provided with a reversing switch so that the secondary voltage may either "boost" or "buck" that of the bus bars. This type of regulator changes the voltage by definite steps.

(*c*) *Induction regulators.* — The induction regulator is, essentially, an auto-transformer in which the angular relations of the primary and secondary coils may be changed.

(*a*) Movable Core.

(*b*) Stationary Core.



(*c*) Complete.

FIG. 250. Six-phase, Motor-operated Induction Regulator. General Electric Co.

Structurally, the polyphase induction regulator resembles the polyphase induction motor with a wound rotor. The primary windings, equal in number to the number of phases in the system

on which it is to operate, are symmetrically placed in slots on the surface of a movable laminated iron drum (Fig. 250a), and are connected between the lines of a polyphase system. The secondary windings are symmetrically placed in the slots of a stationary core (Fig. 250b), and are connected in series with the load apparatus.

The primary windings set up a rotating flux which induces a constant electromotive force in the secondary windings. The feeder voltage is the vector sum, or difference, of the bus-bar voltage and the voltage induced in the secondary winding of the regulator. Since the phase relation of the bus-bar voltage and the induced secondary electromotive force depends on the angular position of the movable coils, the bus-bar voltage may be raised or lowered by an amount equal to the secondary voltage of the regulator. Fig. 251.

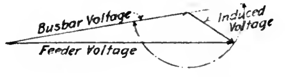


Fig. 251. Clock Diagram of Polyphase Induction Regulator Voltages.

The single-phase induction regulator differs, both in principle and in structure, from the polyphase regulator. A schematic diagram of the single-phase regulator is shown in Fig. 252. The voltage induced in the secondary coil is proportional to the cosine of the angle β between the axis of the primary coil and that of the secondary winding. The induced electromotive force is, therefore, zero when the coils are at right angles. Since the flux set up by the primary coil passes through the secondary coil in one direction when the angle β is less than 90 degrees, and in the opposite direction when the angle β is greater than 90 degrees, the induced electromotive force is in phase with, or in phase opposition to, the bus-bar voltage as the angle β is greater or less than 90 degrees.

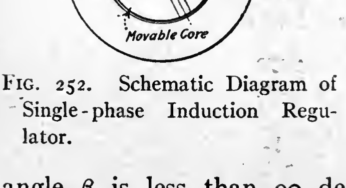


Fig. 252. Schematic Diagram of Single-phase Induction Regulator.

The short-circuited winding which is placed at right angles to the primary coil reduces the reactance of the secondary winding, and thus improves the power factor of the feeder circuit. By reason of their angular relations, the effect of the short-circuited coil increases as that of the primary winding decreases.

Feeder regulators are operated either by hand or by motor.

When motor operated, the regulation may be made automatic by means of a contact-making voltmeter which causes the motor circuit to be closed when the voltage becomes either too high or too low.

13. Regulating effect of a constantly-excited synchronous motor. — It was shown in Chapter 9 that the power factor of a synchronous motor depends on the relative values of the applied and the counter-electromotive force. Let the field excitation of a synchronous motor at the end of a feeder be such that the power factor of the feeder circuit is unity. If the load increases, the voltage at the terminals of the motor drops, the current leads the electromotive force, and the leading current tends to increase the voltage to its former value; if the load decreases, the voltage at the terminals of the motor increases, the current lags behind the electromotive force, and the lagging current tends to decrease the voltage to its former value. The tendency, therefore, of a constantly-excited synchronous motor, when located at the end of a feeder, is to maintain constant voltage at the center of distribution.

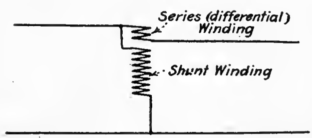


FIG. 253. Voltmeter Compensation (C. C. Circuits).

14. Voltmeter compensation. — When feeder regulators are used it is required that the voltage at the end of the feeder, or at the center of distribution, be indicated by an instrument at the station. For continuous-current feeders, it is only necessary that a properly proportioned series winding be added to the voltmeter. Fig. 253. The series winding is so connected that it opposes the effect of the shunt coil, and the voltmeter indication is reduced by an amount equal to the drop in the line.

Since the drop in an alternating-current feeder is not a simple ohmic drop, but the combined effect of the resistance and the reactance of the conductors, the voltage at the end of an alternating-current feeder is indicated by a voltmeter only when the voltage and current relations in the instrument circuit are identical with those in the feeder circuit. This condition is effected by properly proportioning the resistance and the reactance of the secondary circuit of a series transformer, and connecting the voltmeter as indicated in Fig. 254. Commercial compensators are made so that their resistance and reactance may be varied, thus making the same apparatus applicable to different circuits.

15. Lightning arresters. — The effect of a lightning discharge on electrical apparatus may be either direct or indirect. The direct effect is the destruction of the insulation; the indirect effect is the establishment of a low-resistance circuit which may be maintained by the normal voltage of the system. A satisfactory lightning arrester must, then, divert or dissipate the energy of the discharge, and promptly interrupt any low-resistance circuit that may be established for the purpose of diverting or dissipating this energy.

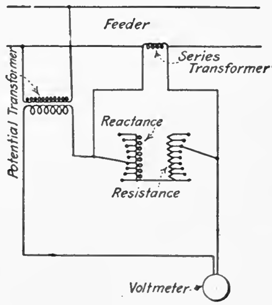


FIG. 254. Schematic Diagram of Connections for Voltmeter Compensator (A. C. Circuits).

The fundamental operation of lightning arresters will be explained by reference to Fig. 255 in which *G* is an air gap between the conductor and a low-resistance ground connection and *A* is an inductive coil connected in series with the line. Under normal operating conditions the impedance of *A* to the flow of the load current, either continuous or alternating, is small, while the resistance of the air gap *G* is so large that the leakage is negligible. Since the impedance of *A* is directly proportional to the frequency, the extremely high-frequency lightning (oscillatory) discharge is "choked" back, breaks down the resistance of the air gap and discharges to earth.

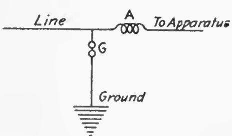


FIG. 255. Elementary Lightning Arrester.

It is a well-known fact that an electric arc, when once established, is maintained by a much smaller voltage than is required to establish it. Consequently the arc established by the lightning discharge may be maintained by the normal voltage of the system unless means are taken to suppress it. This is done by: (*a*) the use of "non-arcing" metals, (*b*) the horn gap.

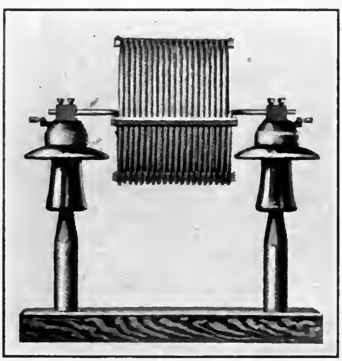


FIG. 256. Westinghouse Choke Coil (Air Cooled).

(a) *Non-arcing metal arresters.* — When the air-gap terminals are made of zinc or cadmium or of their alloys, it is found that the arc is not maintained after the passage of the lightning discharge. Arresters are, therefore, made of short cylinders of brass separated by a small air gap.

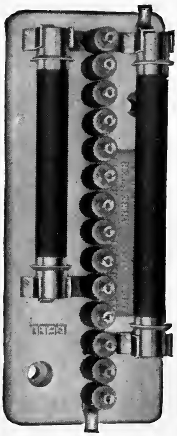


FIG. 257. G. E. Lightning Arrester.

Arresters having part of the gaps shunted by resistance have come into extensive use. Their construction is based on the theory that, for a given voltage, high-frequency discharges require a larger number of gaps than do discharges of low frequency. Fig. 257 shows a 2300-volt General Electric arrester having a high resistance and a low-resistance shunt. Low-frequency discharges pass through the high resistance (long carbon rod) and two air gaps; medium frequency discharges through the low resistance (short carbon rod) and four air gaps; while high-frequency discharges pass through the entire series of air gaps.

Westinghouse practice differs from the above in that there is a shunt and a series resistance, as indicated in Fig. 258.

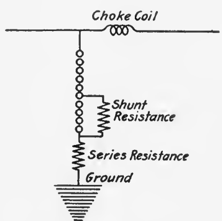


FIG. 258. Schematic Diagram of Westinghouse Lightning Arrester.

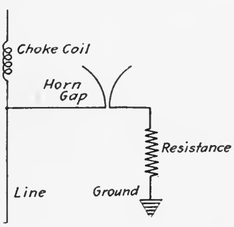


FIG. 259. Schematic Diagram of Horn Gap Arrester.

(b) *Horn gap arrester.* — The horn gap arrester has the appearance shown in Fig. 259, and is connected to ground in series with resistance. The lightning discharge breaks down the air gap and forms an arc between the horns. Air currents formed by the heat of the arc carry the arc itself upward, increasing the length of the arc until it can no longer be maintained by the normal voltage of the system.

During the past few years there has been developed an electrolytic cell (Fig. 260), which replaces the resistance used in connection with

the horn gap. This cell consists of a series of cone-shaped aluminum elements, the spaces between which are partly filled with electrolyte, and the whole is immersed in oil. The purpose of the oil is to increase the insulation, prevent evaporation of the electrolyte, and dissipate the heat liberated during discharge.

The action of the cell is valve-like in that, at a definite critical voltage, the resistance breaks down, and a small increase in voltage above this critical value causes a large current to flow. When the voltage drops below the critical value, which is about 40 per cent above the normal voltage of the system, the high-resistance property of the cell is restored and the horn gap promptly quenches the arc formed by the discharge.

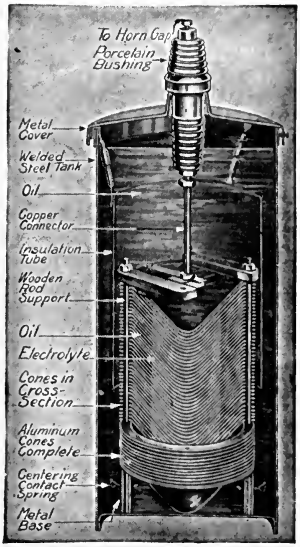


FIG. 260. Cross-section of General Electric Electrolytic Cell.

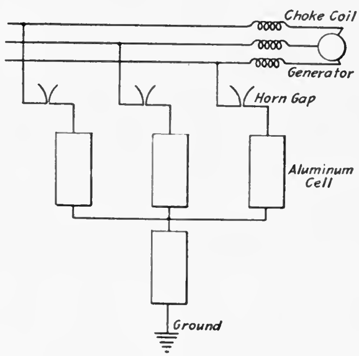


FIG. 261. An Approved Connection for Three-phase Electrolytic Lightning Arrester.

The high resistance of the aluminum cell is due to a film which is deposited on the aluminum elements when a current flows through the cell. Since this film tends to dissolve, the cell is maintained in operating condition by periodically bridging the horn gaps and allowing a momentary discharge through the cell.

The electrolytic cell is, by construction, a condenser in which, for a constant applied voltage, the current is directly proportional to the frequency ($I = 2\pi fCE$). A high-frequency lightning discharge is, therefore, diverted through the cell to the ground. The air gap between the horns prevents the small leakage current that would flow if the cell were connected directly to the line.

TABLE VI

Dimensions, Weights and Resistances of Copper Wires

$$\text{Diameter} = \frac{0.3249}{1.123^n} \text{ when } n = \text{gauge number.}$$

$$\text{Weight (pounds) per 1000 feet} = \frac{\text{circular mils}}{330}.$$

$$\text{Resistance (ohms) per 1000 feet} = \frac{10,354}{\text{circular mils}}.$$

No. A.W.G. (B. & S.)	Diameter, inches	Area, C.M.	Weight-pounds		Resistance, 20° C.	
			1000 feet	Mile	1000 feet	Mile
0000	0.460	211,600	640.5	3381	0.04893	0.2583
000	0.407	167,800	508.0	2682	0.06170	0.3258
00	0.365	133,100	402.8	2127	0.07780	0.4108
0	0.325	105,500	319.5	1687	0.09811	0.5180
1	0.289	83,690	253.3	1337	0.1237	0.6531
2	0.258	66,370	200.9	1062	0.1560	0.8237
3	0.229	52,630	159.3	841.1	0.1967	1.0386
4	0.204	41,740	126.4	667.4	0.2480	1.3094
5	0.182	33,100	100.2	529.0	0.3126	1.6516
6	0.162	26,250	79.46	419.5	0.3944	2.0824
7	0.144	20,820	63.02	332.7	0.4973	2.6257
8	0.129	16,510	49.98	263.9	0.6271	3.3111
9	0.114	13,090	39.63	209.2	0.7908	4.1754
10	0.102	10,380	31.43	166.0	0.9972	5.2652
11	0.091	8,234	24.93	131.6	1.257	6.637
12	0.081	6,530	19.77	104.4	1.586	8.374

Resistance of aluminum wire = 160 per cent of copper wire.
Weight of aluminum wire = 30 per cent of copper wire.

TABLE VII

Current Carrying-capacity of Copper Wires

National Electric Code

A.W.G., (B. & S.)	Sectional area, circular mils	Rubber insulation, amperes	Other insulation, amperes
18	1,624	3	5
16	2,583	6	8
14	4,107	12	16
12	6,530	17	23
10	10,380	24	32
8	16,510	33	46
6	26,250	46	65
5	33,100	54	77
4	41,740	65	92
3	52,630	76	110
2	66,370	90	131
1	83,690	107	156
0	105,500	127	185
00	133,100	150	220
000	167,800	177	262
0000	211,600	210	312
....	200,000	200	300
....	300,000	270	400
....	400,000	330	500
....	500,000	390	590
....	600,000	450	680
....	700,000	500	760
....	800,000	550	840
....	900,000	600	920
....	1,000,000	650	1000

Current-carrying capacity of aluminum wire = 75 per cent of copper wire.

TABLE VIII

Inductance

Millihenries per 1000 feet of single-phase circuit (2000 feet of conductor)

Size, A.W.G. (B. & S.)	Distance between wires in inches											
	6	12	18	24	36	48	60	72	84	96	108	120
0000	0.4270	0.5112	0.5606	0.5956	0.6448	0.6798	0.7070	0.7292	0.7480	0.7644	0.7786	0.7914
000	0.4412	0.5254	0.5748	0.6098	0.6590	0.6940	0.7212	0.7434	0.7622	0.7784	0.7928	0.8056
00	0.4352	0.5394	0.5888	0.6238	0.6730	0.7080	0.7352	0.7574	0.7762	0.7924	0.8068	0.8196
0	0.4694	0.5536	0.6030	0.6380	0.6872	0.7222	0.7494	0.7716	0.7904	0.8066	0.8210	0.8338
1	0.4836	0.5678	0.6172	0.6522	0.7014	0.7364	0.7636	0.7858	0.8046	0.8208	0.8352	0.8480
2	0.4976	0.5818	0.6312	0.6662	0.7154	0.7504	0.7776	0.7998	0.8186	0.8348	0.8492	0.8620
3	0.5118	0.5960	0.6454	0.6804	0.7296	0.7646	0.7918	0.8140	0.8328	0.8490	0.8634	0.8762
4	0.5254	0.6100	0.6594	0.6944	0.7436	0.7786	0.8054	0.8280	0.8468	0.8630	0.8774	0.8902
5	0.5400	0.6242	0.6736	0.7086	0.7578	0.7928	0.8200	0.8422	0.8610	0.8772	0.8916	0.9044
6	0.5540	0.6382	0.6876	0.7226	0.7718	0.8068	0.8340	0.8562	0.8750	0.8912	0.9056	0.9184
7	0.5682	0.6524	0.7018	0.7368	0.7860	0.8210	0.8482	0.8704	0.8892	0.9054	0.9198	0.9326
8	0.5822	0.6664	0.7158	0.7508	0.8000	0.8350	0.8622	0.8844	0.9032	0.9184	0.9338	0.9466
9	0.5964	0.6806	0.7300	0.7650	0.8142	0.8492	0.8764	0.8986	0.9174	0.9328	0.9480	0.9608
10	0.6104	0.6946	0.7440	0.7790	0.8282	0.8632	0.8904	0.9126	0.9314	0.9476	0.9620	0.9748
11	0.6246	0.7088	0.7582	0.7932	0.8424	0.8774	0.9046	0.9268	0.9456	0.9618	0.9762	0.9890
12	0.6386	0.7228	0.7722	0.8072	0.8564	0.8914	0.9186	0.9408	0.9596	0.9758	0.9902	1.0030

$$L = 0.03028 + 0.282 \log_{19} \frac{2D}{d}.$$

D = distance between wires (center to center).
d = diameter of wires.

TABLE IX

Capacitance

Microfarads per 1000 feet of single-phase circuit (2000 feet of conductor)

Size, A.W.G. (B. & S.)	Distance between wires in inches: 6	12	18	24	36	48	60	72	84	96	108	120
0000	0.00263	0.00215	0.00195	0.00182	0.00168	0.00159	0.00152	0.00147	0.00143	0.00140	0.00138	0.00135
000	0.00253	0.00209	0.00190	0.00178	0.00164	0.00155	0.00149	0.00144	0.00141	0.00138	0.00135	0.00133
00	0.00244	0.00203	0.00185	0.00174	0.00160	0.00152	0.00146	0.00142	0.00138	0.00135	0.00133	0.00130
0	0.00236	0.00197	0.00180	0.00170	0.00157	0.00149	0.00143	0.00139	0.00136	0.00133	0.00130	0.00128
1	0.00229	0.00192	0.00176	0.00166	0.00153	0.00146	0.00140	0.00136	0.00133	0.00130	0.00128	0.00126
2	0.00222	0.00187	0.00172	0.00162	0.00150	0.00143	0.00138	0.00134	0.00131	0.00128	0.00126	0.00124
3	0.00215	0.00182	0.00168	0.00159	0.00147	0.00140	0.00135	0.00131	0.00128	0.00126	0.00124	0.00122
4	0.00208	0.00178	0.00164	0.00155	0.00144	0.00138	0.00133	0.00129	0.00126	0.00124	0.00122	0.00120
5	0.00203	0.00174	0.00160	0.00152	0.00142	0.00135	0.00130	0.00127	0.00124	0.00122	0.00120	0.00118
6	0.00197	0.00170	0.00157	0.00149	0.00139	0.00133	0.00128	0.00125	0.00122	0.00120	0.00118	0.00116
7	0.00192	0.00166	0.00153	0.00146	0.00136	0.00130	0.00126	0.00123	0.00120	0.00118	0.00116	0.00114
8	0.00187	0.00162	0.00150	0.00143	0.00134	0.00128	0.00124	0.00121	0.00118	0.00116	0.00114	0.00112
9	0.00181	0.00158	0.00147	0.00140	0.00131	0.00125	0.00121	0.00118	0.00116	0.00114	0.00112	0.00110
10	0.00178	0.00155	0.00144	0.00138	0.00129	0.00123	0.00120	0.00117	0.00114	0.00112	0.00110	0.00109
11	0.00173	0.00152	0.00142	0.00135	0.00127	0.00122	0.00118	0.00115	0.00112	0.00110	0.00109	0.00107
12	0.00169	0.00149	0.00139	0.00132	0.00125	0.00120	0.00116	0.00113	0.00110	0.00109	0.00107	0.00106

$$C = \frac{0.003677}{\log_{10} \frac{2D}{d}}.$$

D = distance between wires (center to center).
d = diameter of wires.

TABLE X

Inductive Reactance

25 cycles, single phase. Ohms per 1000 feet of conductor

Size, A.W.G. (B. & S.)	Distance between wires in inches											
	6	12	18	24	36	48	60	72	84	96	108	120
0000	0.0335	0.0402	0.0440	0.0468	0.0506	0.0534	0.0555	0.0573	0.0588	0.0600	0.0612	0.0622
000	0.0346	0.0413	0.0451	0.0479	0.0518	0.0545	0.0566	0.0584	0.0599	0.0611	0.0623	0.0633
00	0.0358	0.0424	0.0463	0.0490	0.0529	0.0556	0.0578	0.0595	0.0610	0.0622	0.0634	0.0644
0	0.0369	0.0435	0.0474	0.0501	0.0540	0.0567	0.0589	0.0606	0.0621	0.0634	0.0645	0.0655
1	0.0380	0.0446	0.0485	0.0512	0.0551	0.0578	0.0600	0.0617	0.0632	0.0645	0.0656	0.0666
2	0.0391	0.0457	0.0496	0.0523	0.0562	0.0589	0.0611	0.0628	0.0643	0.0656	0.0667	0.0677
3	0.0402	0.0468	0.0507	0.0534	0.0573	0.0601	0.0622	0.0639	0.0654	0.0667	0.0678	0.0688
4	0.0413	0.0479	0.0518	0.0546	0.0584	0.0612	0.0633	0.0650	0.0665	0.0678	0.0689	0.0699
5	0.0424	0.0490	0.0529	0.0557	0.0595	0.0623	0.0644	0.0662	0.0676	0.0689	0.0700	0.0710
6	0.0435	0.0501	0.0540	0.0568	0.0606	0.0634	0.0656	0.0673	0.0687	0.0700	0.0711	0.0722
7	0.0446	0.0513	0.0551	0.0579	0.0617	0.0645	0.0666	0.0684	0.0699	0.0711	0.0723	0.0733
8	0.0457	0.0524	0.0562	0.0590	0.0629	0.0656	0.0677	0.0695	0.0710	0.0722	0.0734	0.0744
9	0.0469	0.0535	0.0574	0.0601	0.0640	0.0667	0.0689	0.0706	0.0721	0.0733	0.0745	0.0755
10	0.0480	0.0546	0.0585	0.0612	0.0651	0.0678	0.0700	0.0717	0.0732	0.0745	0.0756	0.0766
11	0.0491	0.0557	0.0596	0.0623	0.0662	0.0689	0.0711	0.0728	0.0743	0.0756	0.0767	0.0777
12	0.0502	0.0568	0.0607	0.0634	0.0673	0.0700	0.0722	0.0739	0.0754	0.0767	0.0778	0.0788

$X = 2\pi f L.$

f = frequency of the alternating-current supply.

L = inductance in henries = 1000 millihenries.

TABLE XI

Inductive Reactance

60 cycles, single phase. Ohms per 1000 feet of conductor

Size, A.W.G. (B. & S.)	Distance between wires in inches											
	6	12	18	24	36	48	60	72	84	96	108	120
0000	0.0805	0.0964	0.1057	0.1123	0.1215	0.1281	0.1333	0.1375	0.1410	0.1441	0.1468	0.1492
000	0.0831	0.0990	0.1083	0.1149	0.1242	0.1308	0.1359	0.1401	0.1437	0.1467	0.1494	0.1518
00	0.0858	0.1017	0.1110	0.1176	0.1269	0.1335	0.1386	0.1428	0.1463	0.1494	0.1521	0.1545
0	0.0885	0.1043	0.1137	0.1203	0.1295	0.1361	0.1413	0.1454	0.1490	0.1520	0.1547	0.1572
1	0.0911	0.1070	0.1163	0.1229	0.1322	0.1388	0.1439	0.1481	0.1516	0.1547	0.1574	0.1598
2	0.0938	0.1097	0.1190	0.1256	0.1348	0.1414	0.1466	0.1508	0.1543	0.1574	0.1601	0.1625
3	0.0964	0.1123	0.1216	0.1282	0.1375	0.1441	0.1492	0.1534	0.1570	0.1600	0.1627	0.1651
4	0.0991	0.1150	0.1243	0.1309	0.1402	0.1468	0.1519	0.1561	0.1596	0.1627	0.1654	0.1678
5	0.1018	0.1176	0.1270	0.1336	0.1428	0.1494	0.1546	0.1587	0.1623	0.1653	0.1680	0.1705
6	0.1044	0.1203	0.1296	0.1362	0.1455	0.1521	0.1572	0.1614	0.1649	0.1680	0.1707	0.1731
7	0.1071	0.1230	0.1323	0.1389	0.1481	0.1547	0.1599	0.1641	0.1676	0.1707	0.1734	0.1758
8	0.1097	0.1256	0.1349	0.1415	0.1508	0.1574	0.1625	0.1667	0.1703	0.1733	0.1760	0.1784
9	0.1124	0.1283	0.1376	0.1442	0.1535	0.1601	0.1652	0.1694	0.1729	0.1760	0.1787	0.1811
10	0.1151	0.1309	0.1403	0.1469	0.1561	0.1627	0.1679	0.1720	0.1756	0.1786	0.1813	0.1838
11	0.1177	0.1336	0.1429	0.1495	0.1588	0.1654	0.1705	0.1747	0.1782	0.1813	0.1840	0.1864
12	0.1204	0.1363	0.1456	0.1522	0.1614	0.1680	0.1732	0.1774	0.1809	0.1840	0.1867	0.1891

$X = 2\pi f L$

f = frequency of the alternating-current supply.

L = inductance in henries = 1000 millihenries.

TABLE XII

IMPEDANCE AT 25 CYCLES

Ohms per 1000 feet of copper conductor

Size, A.W.G. (B. & S.)	Distance between wires in inches											
	6	12	18	24	36	48	60	72	84	96	108	120
0000	0.0592	0.0632	0.0657	0.0676	0.0703	0.0724	0.0739	0.0753	0.0764	0.0774	0.0783	0.0791
000	0.0707	0.0742	0.0764	0.0781	0.0805	0.0823	0.0837	0.0849	0.0860	0.0868	0.0876	0.0884
00	0.0856	0.0886	0.0905	0.0919	0.0941	0.0956	0.0969	0.0979	0.0988	0.0996	0.1003	0.1010
0	0.1048	0.1073	0.1089	0.1101	0.1119	0.1133	0.1144	0.1153	0.1161	0.1168	0.1174	0.1179
1	0.1294	0.1315	0.1328	0.1339	0.1354	0.1365	0.1374	0.1382	0.1389	0.1395	0.1400	0.1405
2	0.1608	0.1625	0.1637	0.1645	0.1658	0.1667	0.1675	0.1681	0.1687	0.1692	0.1696	0.1700
3	0.2007	0.2022	0.2031	0.2038	0.2048	0.2056	0.2063	0.2068	0.2072	0.2077	0.2081	0.2084
4	0.2514	0.2525	0.2533	0.2539	0.2548	0.2554	0.2559	0.2563	0.2567	0.2571	0.2574	0.2576
5	0.3156	0.3166	0.3172	0.3177	0.3184	0.3189	0.3193	0.3197	0.3200	0.3203	0.3205	0.3207
6	0.3967	0.3975	0.3980	0.3984	0.3990	0.3994	0.3998	0.4001	0.4003	0.4005	0.4007	0.4009
7	0.4992	0.4999	0.5003	0.5006	0.5011	0.5014	0.5017	0.5019	0.5021	0.5023	0.5035	0.5026
8	0.6287	0.6292	0.6296	0.6298	0.6302	0.6305	0.6307	0.6309	0.6311	0.6312	0.6314	0.6315
9	0.7922	0.7926	0.7929	0.7931	0.7934	0.7936	0.7938	0.7940	0.7941	0.7942	0.7943	0.7944
10	0.9983	0.9986	0.9989	0.9991	0.9993	0.9995	0.9996	0.9997	0.9998	0.9999	1.0000	1.0001
11	1.2579	1.2582	1.2584	1.2585	1.2587	1.2589	1.2590	1.2591	1.2592	1.2593	1.2594	1.2594
12	1.5868	1.5870	1.5872	1.5873	1.5874	1.5875	1.5876	1.5877	1.5877	1.5878	1.5879	1.5879

$$Z = \sqrt{R^2 + X^2}.$$

TABLE XIII

Impedance at 60 Cycles

Ohms per 1000 feet of copper conductor

Size, A.W.G. (B. & S.)	Distance between wires in inches											
	6	12	18	24	36	48	60	72	84	96	108	120
0000	0.0942	0.1081	0.1164	0.1225	0.1309	0.1371	0.1420	0.1459	0.1492	0.1521	0.1547	0.1570
000	0.1035	0.1166	0.1246	0.1304	0.1386	0.1446	0.1492	0.1530	0.1563	0.1591	0.1616	0.1638
00	0.1158	0.1280	0.1355	0.1410	0.1488	0.1545	0.1589	0.1626	0.1657	0.1684	0.1708	0.1729
0	0.1321	0.1432	0.1501	0.1552	0.1624	0.1677	0.1720	0.1754	0.1784	0.1809	0.1831	0.1853
1	0.1536	0.1635	0.1697	0.1743	0.1810	0.1859	0.1897	0.1929	0.1956	0.1980	0.2002	0.2020
2	0.1820	0.1907	0.1962	0.2002	0.2061	0.2105	0.2140	0.2169	0.2194	0.2216	0.2235	0.2252
3	0.2190	0.2265	0.2312	0.2347	0.2400	0.2438	0.2469	0.2494	0.2516	0.2535	0.2552	0.2568
4	0.2670	0.2733	0.2774	0.2804	0.2848	0.2882	0.2908	0.2930	0.2949	0.2966	0.2981	0.2994
5	0.3289	0.3341	0.3376	0.3401	0.3438	0.3466	0.3489	0.3507	0.3524	0.3537	0.3550	0.3562
6	0.4079	0.4123	0.4151	0.4172	0.4203	0.4227	0.4245	0.4261	0.4275	0.4286	0.4297	0.4307
7	0.5087	0.5122	0.5146	0.5163	0.5188	0.5208	0.5223	0.5236	0.5247	0.5257	0.5266	0.5274
8	0.6366	0.6395	0.6414	0.6428	0.6449	0.6465	0.6478	0.6488	0.6498	0.6506	0.6513	0.6519
9	0.7987	0.8011	0.8027	0.8038	0.8055	0.8068	0.8078	0.8087	0.8094	0.8101	0.8107	0.8112
10	1.0038	1.0058	1.0070	1.0079	1.0093	1.0104	1.0112	1.0119	1.0125	1.0131	1.0135	1.0139
11	1.2625	1.2641	1.2651	1.2658	1.2669	1.2678	1.2685	1.2691	1.2696	1.2700	1.2704	1.2707
12	1.5905	1.5915	1.5924	1.5932	1.5942	1.5948	1.5954	1.5959	1.5963	1.6966	1.5969	1.5972

$$Z = \sqrt{R^2 + X^2}$$

TABLE XIV

Impedance Factors (k)

Single phase, 25 cycles

Size, A.W.G. (B. & S.)	Distance between wires in inches											
	6	12	18	24	36	48	60	72	84	96	108	120
0000	1.2107	1.2924	1.3435	1.3824	1.4376	1.4805	1.5110	1.5398	1.5623	1.5828	1.6012	1.6176
000	1.1458	1.2025	1.2382	1.2658	1.3047	1.3338	1.3565	1.3760	1.3938	1.4068	1.4198	1.4343
00	1.1002	1.1388	1.1632	1.1812	1.2095	1.2288	1.2455	1.2596	1.2699	1.2802	1.2892	1.2982
0	1.0683	1.0937	1.1101	1.1223	1.1406	1.1549	1.1666	1.1753	1.1845	1.1906	1.1967	1.2018
1	1.0461	1.0630	1.0735	1.0824	1.0946	1.1034	1.1107	1.1172	1.1227	1.1277	1.1317	1.1358
2	1.0311	1.0417	1.0494	1.0545	1.0630	1.0686	1.0736	1.0776	1.0814	1.0846	1.0872	1.0904
3	1.0203	1.0279	1.0325	1.0361	1.0412	1.0452	1.0448	1.0513	1.0534	1.0559	1.0579	1.0595
4	1.0137	1.0181	1.0214	1.0238	1.0274	1.0294	1.0314	1.0335	1.0351	1.0367	1.0379	1.0387
5	1.0089	1.0121	1.0141	1.0156	1.0179	1.0195	1.0208	1.0221	1.0230	1.0239	1.0246	1.0252
6	1.0058	1.0079	1.0091	1.0101	1.0116	1.0127	1.0139	1.0144	1.0149	1.0154	1.0159	1.0164
7	1.0038	1.0052	1.0060	1.0066	1.0076	1.0082	1.0089	1.0093	1.0097	1.0101	1.0104	1.0106
8	1.0025	1.0033	1.0039	1.0044	1.0049	1.0054	1.0057	1.0061	1.0064	1.0066	1.0068	1.0070
9	1.0017	1.0023	1.0026	1.0029	1.0032	1.0035	1.0038	1.0040	1.0042	1.0043	1.0044	1.0045
10	1.0011	1.0014	1.0017	1.0019	1.0021	1.0023	1.0025	1.0026	1.0027	1.0028	1.0029	1.0029
11	1.0007	1.0009	1.0011	1.0012	1.0013	1.0014	1.0015	1.0016	1.0017	1.0018	1.0019	1.0019
12	1.0005	1.0006	1.0007	1.0008	1.0008	1.0009	1.0009	1.0010	1.0011	1.0010	1.0012	1.0012

$$k = \frac{\text{Impedance per unit length}}{\text{Resistance per unit length}}.$$

TABLE XV

IMPEDANCE FACTORS (k)

Single phase, 60 cycles

Size, A.W.G. (B. & S.)	Distance between wires in inches											
	6	12	18	24	36	48	60	72	84	96	108	120
0000	1.9266	2.2106	2.3803	2.5051	2.6768	2.8037	2.9039	2.9836	3.0511	3.1104	3.1636	3.2106
000	1.6775	1.8898	2.0194	2.1134	2.2496	2.3436	2.4181	2.4799	2.5332	2.5818	2.6191	2.6548
00	1.4884	1.6452	1.7416	1.8111	1.9126	1.9858	2.0424	2.0899	2.1298	2.1646	2.1953	2.2224
0	1.3466	1.4597	1.5300	1.5821	1.6554	1.7095	1.7533	1.7879	1.8186	1.8440	1.8675	1.8888
1	1.2417	1.3217	1.3718	1.4090	1.4632	1.5028	1.5335	1.5594	1.5812	1.6006	1.6184	1.6329
2	1.1666	1.2160	1.2577	1.2833	1.3211	1.3493	1.3711	1.3910	1.4064	1.4205	1.8327	1.4436
3	1.1133	1.1515	1.1754	1.1932	1.2201	1.2394	1.2552	1.2679	1.2791	2.2888	1.2974	1.3055
4	1.0766	1.1020	1.1185	1.1306	1.1484	1.1621	1.1726	1.1814	1.1891	1.1959	1.2020	1.2072
5	1.0511	1.0681	1.0793	1.0841	1.0991	1.1081	1.1153	1.1211	1.1266	1.1307	1.1349	1.1387
6	1.0342	1.0451	1.0525	1.0578	1.0656	1.0718	1.0763	1.0803	1.0838	1.0867	1.0895	1.0920
7	1.0209	1.0299	1.0348	1.0382	1.0432	1.0473	1.0503	1.0529	1.0551	1.0571	1.0589	1.0611
8	1.0103	1.0198	1.0228	1.0252	1.0283	1.0309	1.0330	1.0346	1.0362	1.0375	1.0388	1.0395
9	1.0099	1.0130	1.0150	1.0162	1.0183	1.0202	1.0215	1.0226	1.0238	1.0244	1.0252	1.0258
10	1.0066	1.0086	1.0098	1.0107	1.0121	1.0133	1.0140	1.0148	1.0153	1.0159	1.0163	1.0167
11	1.0044	1.0056	1.0064	1.0070	1.0079	1.0086	1.0091	1.0096	1.0100	1.0103	1.0107	1.0109
12	1.0022	1.0035	1.0042	1.0045	1.0052	1.0056	1.0061	1.0063	1.0065	1.0067	1.0069	1.0070

$$k = \frac{\text{Impedance per unit length}}{\text{Resistance per unit length}}.$$

TABLE XVI

Line Voltage and Distance Between Conductors

Line voltage	Length of transmission line, miles	Distance between wires, inches
2,300	1– 4	12– 24
6,600	4– 8	30– 36
13,200	8– 12	36– 42
22,000	12– 18	48– 60
44,000	18– 30	60– 72
66,000	30– 50	72– 84
88.000	50– 80	96–108
110,000	80–125	108–120

While the above voltages have become standard for transmission lines, the distance of transmission and the distance between wires for a given voltage vary over wide limits, and the above table is intended to give only an approximation to average values.

TABLE XVII

Relative weight of copper required for different systems of electrical power distribution, the total power, the distance of transmission, the line loss and the voltage at the load terminals remaining constant.

No. of wires	System	Relative copper	Load connected between
2	Continuous-current or single-phase alternating current	100	Line wires
3	Three-wire continuous-current or single-phase alternating current, neutral full size	37.5	Line and neutral
3	Three-wire continuous-current or single-phase alternating current, neutral half size	31.25	Line and neutral
3	Three phase	75.0	Any two lines
4	Three phase with neutral full size	33.3	Line and neutral
4	Three phase with neutral half size	29.2	Line and neutral
4	Two phase	100.0	Phase wires
3	Two phase with common return	72.9	Line and common return
5	Two phase with neutral full size	31.25	Line and neutral

CHAPTER XVIII — PROBLEMS

1. The conductors of a 100-mile,* 3-phase, 25-cycle transmission system are No. 0 copper wire spaced 10 feet. The voltage at the load end (between lines) is 104,000 when 10,000 kw. are being delivered to a receiving circuit, the power factor of which is 0.85. Calculate the voltage at the line terminals of the step-up transformers supplying the transmission line, assuming that one-half the capacitance of the line is concentrated at each end of the line.

2. Same as Problem 1 except the capacitance is assumed to be concentrated mid-way between the generator and the load.

3. Same as Problem 1 except the capacitance is assumed to be concentrated as follows: one-sixth at the generator, one-sixth at the load and two-thirds mid-way between the generator and the load.

4. Same as Problem 1 except the capacitance of the line is neglected.

5. The conductors of a 40-mile, 3-phase, 60-cycle transmission line are ½-inch aluminum† wire spaced 7 feet. The phase voltage at the load is 38,500 when 200 amperes flow in each line. The power factor of the load circuit is 0.85. Find: (*a*) the load current, (*b*) the voltage at the generator end of the line, assuming the capacitance to be concentrated as in Problem 1.

6. Same as Problem 5 except the capacitance is neglected.

7. The allowable voltage drop in a single-phase, 60-cycle feeder is 3 per cent, and the distance to the center of distribution is 2000 feet. Determine the size wire required when the voltage at the load is 2300, the power delivered is 500 kw., and the power factor of the load circuit is 0.82. Assume spacing of wires.

8. The voltage at the center of distribution of a single-phase, 60-cycle feeder is automatically maintained at 6600. The length of the feeder is 2 miles; No. 4 copper wires are used and spaced 3 feet. The load on the feeder is 400 kw. and the power factor of the load circuit is 0.75. Determine: (*a*) the volts drop in the line, (*b*) the voltage at the generator.

9. A 500-horse-power, 25-cycle, 3-phase induction motor is 300 feet from the generator supplying it with current. Efficiency of motor = 90 per cent. Power factor of motor = 0.85. Determine the size of wire required so that the voltage drop between no load and full load shall not exceed 10 per cent.

10. 100 alternating-current series arcl amps are operated on a certain circuit of No. 6 copper wire (diameter = 0.162 inch). The first and the last lamp in the circuit are each 300 feet from the power house, and the lamps are spaced 100 feet apart. Each lamp takes 40 volts and 6.6 amperes. The lamps operate at a power factor of 0.85. Find: (*a*) the drop due to the resistance of the line, (*b*) the power lost in the line, (*c*) the power input to the system, (*d*) the power factor of the system.

* Because of the "sag" and the contraction and expansion due to changes in temperature, the length of wire required is approximately 10 per cent greater than the distance.

† Aluminum conductors are always stranded, and their nominal diameter is the solid equivalent of the actual diameter.

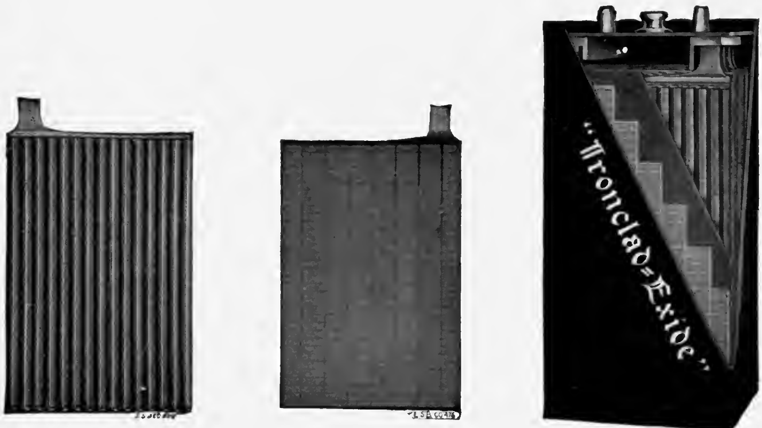
11. The input to an induction motor installation is 400 kw. at a power factor of 0.80. Find the rating of a synchronous condenser, the power factor of which is 0.10, to make the power factor of the feeder circuit unity.

12. Find the power factor of the feeder circuit in Problem 11 when the induction motor load is increased to 500 kw.

CHAPTER XIX

THE STORAGE BATTERY *

By means of a storage battery, energy may be stored and made available at some future time. Structurally the storage battery consists of a set of positive plates, a set of negative plates, and a chemical solution (electrolyte) in which the plates are immersed.



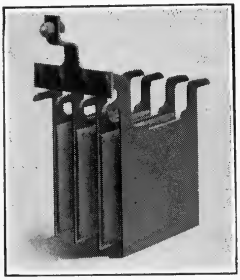
Positive Plate Negative Plate

FIG. 262. "Ironclad-Exide" Storage Battery. The Electric Storage Battery Company.

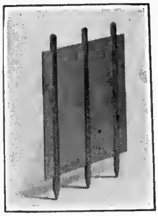
The internal actions of a storage battery are chemical and not mechanical as the name might imply. When an electric current passes through the *electrolyte* from the positive to the negative plates, it produces a chemical reaction and a structural change in the "active material" of the plates. When the positive and the negative plates are connected by means of a metallic wire or other electrical conductor, an inverse chemical action takes place, the plates are restored to their original condition, and an electric current flows in the *wire* from the positive to the negative plates.

* For details of storage battery construction, operation and maintenance, the student is referred to "Secondary Batteries" by E. J. Wade.

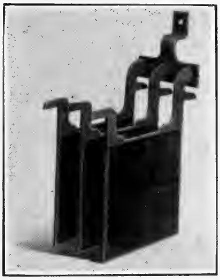
1. Lead batteries. — In the completely discharged condition, the plates of lead storage batteries are lead sulphate ($PbSO_4$), and the electrolyte is water. When a unidirectional current passes through this cell from the positive to the negative plates, the chemical reaction produces peroxide of lead at the positive plate,



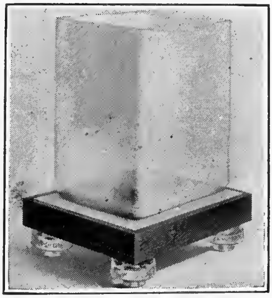
Negative Group



Wood Separator



Positive Group



Glass Jar, Oak Sand Tray with Sand Glass Insulators

FIG. 263. Storage Battery Parts. Gould Storage Battery Co.

"sponge" lead at the negative plate, and converts the electrolyte into sulphuric acid. This action may be represented as follows:

	Positive plate	Electrolyte	Negative plate
Discharged..............	$PbSO_4$	$2\,H_2O$	$PbSO_4$
Charging current........	$\longrightarrow$		
Charged.................	PbO_2	$2\,H_2SO_4$	Pb
Discharging current......	$\longleftarrow$		

While other reactions may, and probably do, take place in the cell, the scientific world generally accepts the above as representing the fundamental reactions.

The plates used in lead storage batteries are of two general types, differentiated by the process of their manufacture: (*a*) Planté type

in which the active material of the plates is formed by chemical processes, (*b*) Fauré type in which the active material is prepared and applied mechanically (pasted) to a supporting frame or grid through which the electric current flows. The plates in commercial batteries are often a combination of the above types.

The open-circuit voltage of a fully charged lead cell is about 2.2. Because of internal resistance, which is not constant, this voltage is not realized when the cell is discharging, and the voltage gradually

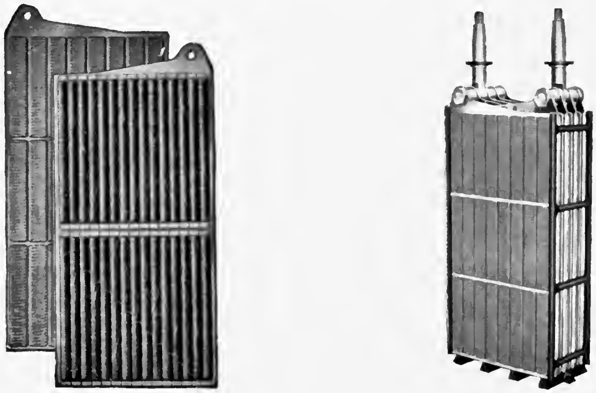


FIG. 264. Edison Storage Battery.

decreases as the cell discharges, as indicated in Fig. 265. The practical limit of discharge is reached when the voltage is 1.75, and the current is that at which the cell is rated. Further discharge causes an excess of lead sulphate to form on the plates, increase the internal resistance* of the cell, and sets up stresses† which cause the plates to crack and fall away from the supporting grid.

2. **The Edison battery.** — The Edison storage battery consists of a positive plate of nickel hydrate and metallic nickel, and a negative plate of the oxides of iron and mercury, immersed in a 20 per cent solution of caustic potash. The purpose of the metallic nickel and the mercury oxide is to increase the conductivity of the active materials of the plates. The electrolyte undergoes no chemical change during either charge or discharge, acting simply as a medium through which oxygen is transferred from one plate to the other. The charging current reduces the oxide of iron (negative plate) to a mass of "sponge" iron, and produces peroxide of nickel at the positive plate.

* Lead sulphate is a non-conductor of electricity.

† Because of the change in volume.

As shown in Fig. 265, the voltage of the Edison cell is less than that of the lead cell, and decreases more rapidly as the cell discharges. The cell is not injured by abnormal or complete discharge. Its capacity per unit weight is greater than that of the

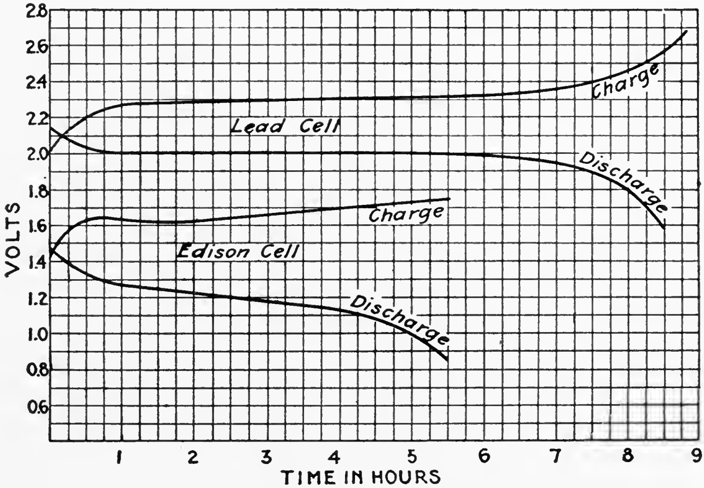


FIG. 265. Charge and Discharge Curves for Storage Cells.

lead cell, but its energy efficiency is only 60% as compared with 80% in the lead cell.

3. Storage battery ratings. — Storage batteries are rated in ampere-hours, and the normal discharge period of the lead is eight hours, *i.e.*, when the discharge current is constant and normal, the voltage of the lead cell drops to 1.75 in eight hours. The capacity of the cell is decreased when the rate of discharge is increased, as shown by Table XVIII. As shown in Fig. 265, the normal period for the charge or discharge of an Edison cell is five hours.

TABLE XVIII

Time of discharge, hours	Relative capacity	Time of discharge, hours	Relative capacity
8	100	4	83
7	97	3	75
6	93	2	65
5	89	1	50

4. Applications of the storage battery. — The following are a few of the more important applications of the storage battery:

(*a*) To furnish energy during periods of light load when the generating apparatus is shut down.

(*b*) To aid in carrying peak loads which would otherwise overload the generating apparatus.

(*c*) To maintain a more nearly constant voltage with varying load.

(*d*) To maintain a more nearly uniform load on the generating apparatus, the battery charging during periods of light load and discharging during periods of heavy load.

From the nature of its internal actions, it is evident that a storage battery cannot be charged from an alternating-current system without using a rectifying device, such as a rotary converter or a motor-generator.

5. **Storage-battery connections.**—In Fig. 266, the battery is connected in parallel with the load, and is preferably placed at the end of a feeder. As the load on the feeder increases, the line drop reduces the voltage at the terminals of the battery, and the battery discharges; as the load on the feeder decreases, the voltage at the terminals of the battery increases, and the battery charges.

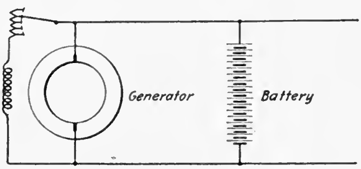


Fig. 266. "Floating" Storage Battery.

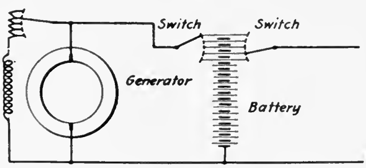


Fig. 267. Storage Battery with End-cell Regulation.

In Fig. 267, end-cell switches are provided so that either the charging or the discharging voltage may be regulated. With this connection the load voltage is independent of the generator voltage.

In Fig. 268, the generator of the motor-generator set is differentially wound and has its armature connected in series with the battery. The field windings are so proportioned that the voltage of the generator is zero when the load on the system is at its average value, and the battery neither charges nor discharges. If the load increases, the differential series field reduces the generator voltage, and the battery discharges; if the load decreases, the generator voltage increases and the battery charges.

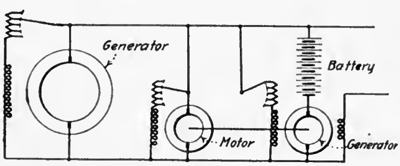


Fig. 268. Storage Battery with Differential Booster.

6. Limitations of the storage battery. — The commercial use of the storage battery is limited because of:

(*a*) Its high first cost and rapid depreciation.

(*b*) The additional mechanical complications introduced by reason of the regulating apparatus required.

(*c*) The cost of maintenance.

7. Care of Storage Batteries. — Explicit instructions for the care of storage batteries cannot be given because service conditions, as well as the ideas of different manufacturers, differ very radically. It is reasonable to assume that a manufacturer knows the conditions under which his apparatus will give the best results, and storage batteries should be operated in strict accordance with the instructions of the manufacturer.

APPENDIX A

HARMONIC QUANTITIES

1. **Definition.** — A harmonic quantity is a quantity, the instantaneous value of which is proportional to the sine or the cosine of a uniformly increasing angle, and the constant of the equation represents the maximum value of the quantity. Common examples of harmonic quantities are: (*a*) the velocity of a pendulum, (*b*) the vibration of a tuning fork, (*c*) the velocity of the cross-head of a reciprocating engine* when the flywheel rotates at a uniform angular velocity.

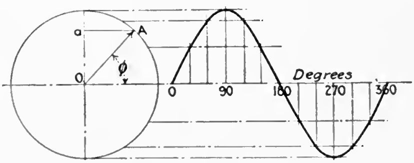


FIG. 269. Harmonic or Sine Curve.

Let OA (Fig. 269) represent the maximum value of any harmonically varying quantity. The value of the quantity at any given instant or position is proportional to the projection of OA on the vertical axis, *i.e.*, to the sine of the angle ϕ.

$$Oa = OA \sin \phi. \tag{1}$$

If the angular velocity of the vector OA is expressed in circular measure (radians),

$$\phi = \omega t \tag{2}$$

and

$$Oa = OA \sin \omega t, \tag{3}$$

when ω = the angular velocity (radians per unit of time) at which the vector rotates,

t = the time during which the vector has rotated, zero time coinciding with zero value of the angle ϕ.

2. **Rectangular representation.** — A sine wave may be plotted to rectangular coördinates as shown in Fig. 269, the curve there plotted showing a succession of values for one complete revolution (360 degrees). A representation of greater values of ωt would be a repetition of those values already plotted.

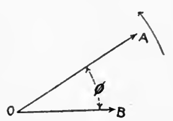


FIG. 270. Vector or Clock Diagram.

3. **Polar representation.** — It is usually unnecessary to draw rectangular representations of harmonic quantities, the magnitudes of the quantities and their relative positions being sufficient. Such a diagram (Fig. 270) is called a vector or clock diagram, and the vector rotates in a counter-clockwise direction.

4. **Combination of harmonic quantities.** — Two or more harmonic quantities, having the same angular velocities, may be replaced by a single harmonic

* This velocity is only approximately harmonic when the connecting rod is of finite length.

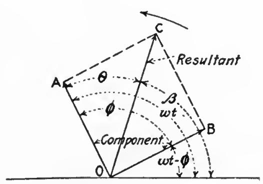
quantity, the vector of which is the geometric sum of the vectors of the two quantities.

Let OA (Fig. 271) represent the maximum value of one harmonic quantity and OB the maximum value of another harmonic quantity having the same angular velocity. Then

$$Oa = OA \sin \omega t \tag{4}$$

and

$$Ob = OB \sin (\omega t - \phi). \tag{5}$$



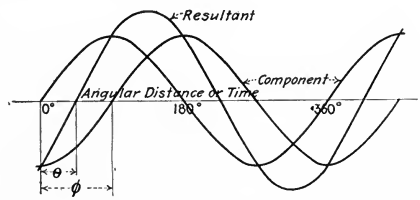


FIG. 271a. Combination of Vectors. FIG. 271b. Combination of Sine Waves.

Combining these two vectors graphically, the vector OC is obtained, for which the mathematical expression is

$$Oc = OC \sin (\omega t - \theta). \tag{6}$$

The mathematical proof that the resultant of two harmonic quantities is a harmonic quantity having the same angular velocity is as follows: Let

$$a = A \sin \omega t, \tag{7}$$

$$b = B \sin (\omega t - \phi). \tag{8}$$

Then

$$a + b = A \sin \omega t + B \sin (\omega t - \phi). \tag{9}$$

Expanding equation (9)

$$a + b = A \sin \omega t + B \sin \omega t \cos \phi - B \cos \omega t \sin \phi \tag{10}$$

$$= (A + B \cos \phi) \sin \omega t - B \cos \omega t \sin \phi. \tag{11}$$

But

$$A + B \cos \phi = C \cos \theta \tag{12}$$

and

$$B \sin \phi = C \sin \theta \tag{13}$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{B \sin \phi}{A + B \cos \phi}. \tag{14}$$

Substituting in equation (11)

$$a + b = \sqrt{(A^2 + B^2 - 2\,AB \cos \phi)} \sin \left(\omega t - \tan^{-1} \frac{B \sin \phi}{A + B \cos \phi} \right). \tag{15}$$

From equation (15) the maximum value of the resultant is

$$C = \sqrt{A^2 + B^2 - 2\,AB \cos \phi} \tag{16}$$

and the angle through which the vector has rotated is

$$\beta = \omega t - \tan^{-1} \frac{B \sin \phi}{A + B \cos \phi}. \tag{17}$$

5. Phase difference. — The phase (time) difference between two harmonic quantities having the same angular velocities, is the time required for the vector of either to pass through an angle equal to that between the vectors of the two quantities. The phase angle, or angle of phase difference, is the angle AOB in Fig. 271a.

6. Resolution of harmonic quantities. — If two harmonic quantities may be combined into a single harmonic quantity having the same angular velocity, it follows that any harmonic quantity may be resolved into harmonic components having any desired phase difference. In alternating-current problems, the resolution of harmonic electromotive forces and currents into components having a phase difference of 90 degrees, is a common and a useful expedient.

7. Rate of change in the instantaneous value of a harmonic quantity. — An inspection of a sine curve such as that shown in Fig. 269, shows that the rate at which its instantaneous value changes is not uniform, and that the rate of change is greatest when the quantity is passing through its zero value.

This may also be shown by differentiating the expression for the instantaneous value

$$a = A \sin \omega t \tag{18}$$

with respect to t. The rate at which a changes is

$$\frac{da}{dt} = \omega A \cos \omega t. \tag{19}$$

But $\cos \omega t$ is maximum when $\sin \omega t$ is zero.

8. Average value of the sin ωt. — The average value of the $\sin \omega t$ for a complete cycle is zero, since for each positive value there is an equal negative value.

For any half cycle beginning with $\omega t = 0$ or $\omega t = \pi$, all the values are either positive or negative and the average may be determined by adding together the values given in Table XIX, and dividing by the number of values. This gives

$$\text{av.} \sin \omega t = \pm 0.637. \tag{20}$$

The same result is obtained by integrating $\sin \omega t$ between the limits $\omega t = \pi$ and $\omega t = 0$, and dividing by π.

$$\text{av.} \sin \omega t = \frac{1}{\pi} \int_{\omega t = 0}^{\omega t = \pi} \sin \omega t \, d(\omega t) \tag{21}$$

$$= \frac{2}{\pi}. \tag{22}$$

9. Value of average $\sin^2 \omega t$. — Let the ordinates of a curve be equal to the squares of the instantaneous values of a harmonic quantity.

$$y = a^2 \tag{23}$$

$$= A^2 \sin^2 \omega t. \tag{24}$$

TABLE XIX

Natural Sines, Cosines, Tangents and Cotangents

A	Sin	Cos	Tan	Cot	
0	0.00000	1.00000	0.00000	Infinity	90
1	0.01745	0.9998	0.01745	57.2900	89
2	0.03490	0.9994	0.03492	28.6363	88
3	0.05234	0.9986	0.05241	19.0811	87
4	0.06976	0.9976	0.06993	14.3007	86
5	0.08716	0.9962	0.08749	11.4301	85
6	0.10453	0.9945	0.10510	9.5144	84
7	0.12187	0.9925	0.12278	8.1443	83
8	0.1392	0.9903	0.1405	7.1154	82
9	0.1564	0.9877	0.1584	6.3138	81
10	0.1736	0.9848	0.1763	5.6713	80
11	0.1908	0.9816	0.1944	5.1446	79
12	0.2079	0.9781	0.2126	4.7046	78
13	0.2250	0.9744	0.2309	4.3315	77
14	0.2419	0.9703	0.2493	4.0108	76
15	0.2588	0.9659	0.2679	3.7321	75
16	0.2756	0.9613	0.2867	3.4874	74
17	0.2924	0.9563	0.3057	3.2709	73
18	0.3090	0.9511	0.3249	3.0777	72
19	0.3256	0.9455	0.3443	2.9042	71
20	0.3420	0.9397	0.3640	2.7475	70
21	0.3584	0.9336	0.2839	2.6051	69
22	0.3746	0.9272	0.4040	2.4751	68
23	0.3907	0.9205	0.4245	2.3559	67
24	0.4067	0.9135	0.4452	2.2460	66
25	0.4226	0.9063	0.4663	2.1445	65
26	0.4384	0.8988	0.4877	2.0503	64
27	0.4540	0.8910	0.5095	1.9626	63
28	0.4695	0.8829	0.5317	1.8807	62
29	0.4848	0.8746	0.5543	1.8040	61
30	0.5000	0.8660	0.5774	1.7321	60
31	0.5150	0.8572	0.6009	1.6643	59
32	0.5299	0.8480	0.6249	1.6003	58
33	0.5446	0.8387	0.6404	1.5399	57
34	0.5592	0.8290	0.6745	1.4826	56
35	0.5736	0.8192	0.7002	1.4281	55
36	0.5878	0.8090	0.7265	1.3764	54
37	0.6018	0.7986	0.7536	1.3270	53
38	0.6157	0.7880	0.7813	1.2799	52
39	0.6293	0.7771	0.8098	1.2349	51
40	0.6428	0.7660	0.8391	1.1918	50
41	0.6561	0.7547	0.8693	1.1504	49
42	0.6691	0.7431	0.9004	1.1106	48
43	0.6820	0.7314	0.9325	1.0724	47
44	0.6947	0.7193	0.9657	1.0355	46
45	0.7071	0.7071	1.0000	1.0000	45
	Cos	Sin	Cot	Tan	*A*

For a more complete table of functions see any standard text on trigonometry.

Integrating equation (24) between the limits $\omega t = \pi$ and $\omega t = 0$, and dividing by π,

$$\text{av. } y = \frac{A^2}{\pi}\int_{\omega t = 0}^{\omega t = \pi} \sin^2 \omega t \, d(\omega t) \tag{25}$$

$$= \frac{A^2}{2}; \tag{26}$$

i.e., the average value of $\sin^2 \omega t$ equals $\frac{1}{2}$.

The same result may be deduced without recourse to calculus. From trigonometry

$$\sin^2 \omega t + \cos^2 \omega t = 1, \tag{27}$$

and

$$\text{av. } \sin^2 \omega t + \text{av. } \cos^2 \omega t = 1. \tag{28}$$

As ωt varies from 0 to 90 degrees, the sine passes through all values from 0 to 1 and the cosine passes through all values from 1 to 0. Therefore,

$$\text{av. } \sin^2 \omega t = \text{av. } \cos^2 \omega t \tag{29}$$

and

$$\text{av. } \sin^2 \omega t = \tfrac{1}{2}. \tag{30}$$

10. Frequency. — The frequency of a harmonic quantity is the number of complete revolutions made by its vector per unit of time. During one cycle a harmonic quantity passes through all possible instantaneous values, both positive and negative, *i.e.*, starting at zero the quantity increases to maximum, decreases to zero, increases to maximum in the opposite direction and again decreases to zero.

11. Value of ω. — Since the distance passed through by a rotating vector is expressed in radians, the angular velocity (radians per second) is equal to the frequency (number of revolutions the vector makes in one second) multiplied by 2π.

$$\omega = 2\pi f. \tag{31}$$

APPENDIX A — PROBLEMS

1. Find the resultant of two harmonic quantities, each of which has a maximum value of 1000, when the angle between their vectors is: (*a*) 30 degrees, (*b*) 45 degrees, (*c*) 60 degrees, (*d*) 90 degrees, (*e*) 120 degrees.

2. The maximum value of a harmonic quantity is 600, and the angle between its (two) components is 90 degrees. Find the maximum values of the components when: (*a*) the angle between the quantity and one component is 30 degrees, (*b*) the maximum values of the components are equal, (*c*) the maximum value of one is twice the maximum value of the other.

3. Find the angles of phase difference between the quantity in Problem 2 and its components.

4. Find the angular velocity of a rotating vector when the frequency is: (*a*) 20, (*b*) 25, (*c*) 30, (*d*) 40, (*e*) 50, (*f*) 60, (*g*) 100.

5. The maximum value of a harmonic quantity is 100. Find the rate at which the quantity is changing when ωt is equal to: (*a*) 0, (*b*) 30 degrees, (*c*) 45 degrees, (*d*) 60 degrees, (*e*) 75 degrees, (*f*) 90 degrees.

6. Find the average value of a harmonic quantity, the maximum value of which is: (*a*) 40, (*b*) 75, (*c*) 100, (*d*) 250, (*e*) 800.

7. Find the average square of a harmonic quantity, the maximum value of which is: (*a*) 40, (*b*) 75, (*c*) 100, (*d*) 250, (*e*) 800.

8. Find the frequency when ω is equal to: (*a*) 125.6, (*b*) 157, (*c*) 188.4, (*d*) 251.2, (*e*) 314, (*f*) 376.8, (*g*) 400.

APPENDIX B

INDUCTANCE

In Chapter 1, Section 11, inductance is defined as the proportionality factor between the electromotive force set up in a circuit by reason of a change in the value of the current flowing in the circuit, and the rate at which the current changes.

$$e = L\frac{di}{dt}. \tag{1}$$

From Chapter 2, Section 13

$$e = \frac{d\phi}{dt}. \tag{2}$$

Therefore,

$$L\frac{di}{dt} = \frac{d\phi}{dt} \tag{3}$$

and

$$L = \frac{\phi}{i}, \tag{4}$$

when ϕ = the total flux linking with the circuit (if the conductor has more than one turn or loop, ϕ is the product of the flux linking with one turn and the number of turns),

i = the current flowing in the circuit, in c.g.s. units,

e = the electromotive force of self-induction, in c.g.s. units,

L = the inductance of the circuit, in c.g.s. units.

1. Inductance of each of two parallel wires. — Let A and B be two parallel cylindrical conductors in which the currents are equal but flow in opposite directions. The current in each conductor, if uninfluenced by that in other conductors, would set up concentric circles of flux around the axis of the wire (Chapter 2, Section 3). The mutual effect of the currents is to produce the flux distribution indicated in Fig. 272.

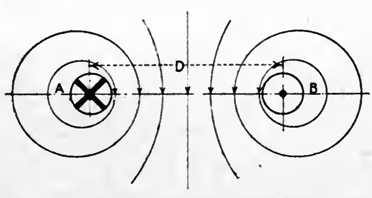


Fig. 272. Magnetic Field Between Two Current Carrying Conductors.

The inductance of the conductors is due to the flux which passes between the axes of the wires, the flux which encircles A at a greater distance than D being neutralized by an equal and opposite flux set up by B. The total flux encircling the axis of each wire may be divided into two parts: (a) that in the body of the wire, (b) that in the insulating material between the wires.

(*a*) *The flux in the body of the wire.* — The flux in any elemental zone dx (Fig. 273) within the wire is due to the current inside of zone dx, and this current is

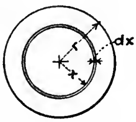


FIG. 273.

$$i' = i\frac{x^2}{r^2}\text{*}. \tag{5}$$

$$B\dagger = \frac{4\pi i'}{2\pi x} \tag{6}$$

$$= \frac{2xi}{r^2}. \tag{7}$$

Therefore, the flux in zone dx is

$$\phi' = \frac{2xi}{r^2}\,dx \text{ c.g.s. units.} \tag{8}$$

Since this flux surrounds only that part of the current which flows in the area of the conductor bounded by the zone dx, it is equivalent to

$$\phi' = \frac{x^2}{r^2} \times \frac{2xi}{r^2}\,dx \tag{9}$$

$$= \frac{2x^3 i}{r^4}\,dx \text{ c.g.s. units,} \tag{10}$$

linking with the entire current flowing in the conductor. The total flux per centimeter length of conductor within the area of the conductor is found by integrating equation (10) between the limits $x = r$ and $x = 0$.

$$\phi_1 = \frac{2i}{r^4}\int_0^r x^3\,dx \tag{11}$$

$$= \frac{i}{2} \text{ c.g.s. units.} \tag{12}$$

(*b*) *The flux in the insulating material between the wires.* — The flux passing between the conductors is due to the current flowing in the wires.

$$B\ddagger = \frac{4\pi i}{2\pi x} \tag{13}$$

$$= \frac{2i}{x}. \tag{14}$$

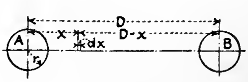


FIG. 274.

Therefore, the flux in zone dx (Fig. 274) due to the current in A is

$$\phi'' = \frac{2i}{x}\,dx \text{ c.g.s. units,} \tag{15}$$

* If the current density in the conductor is not uniform this expression becomes an approximation only.

† The wires are here assumed to be copper or aluminum, the permeability of which is unity. If the conductors are of magnetic material (iron) $B = \mu H$ (Chapter 2, Section 9).

‡ Insulating material assumed to be air, the permeability of which is unity.

and the flux per centimeter length of A due to the current in A is found by integrating equation (15) between the limits $x = D - r$ and $x = r$.

$$\phi_2 = 2\,i \int_r^{D-r} \frac{dx}{x} \tag{16}$$

$$= 2\,i \log_e \frac{D-r}{r} \text{ c.g.s. units.} \tag{17}$$

The total flux linking with the current in conductor A is, therefore,

$$\phi = \phi_1 + \phi_2 \tag{18}$$

$$= \frac{i}{2} + 2\,i \log_e \frac{D-r}{r} \text{ c.g.s. units.} \tag{19}$$

From equation (4),

$$L = \frac{\phi}{i} \tag{20}$$

$$= \frac{1}{2} + 2 \log_e \frac{D-r}{r} \text{ c.g.s. units * per centimeter length of conductor.} \tag{21}$$

2. Inductance of each conductor in a three-phase system. — The inductance of a conductor in a three-phase system depends on its position relatively to the other conductors of the system. There will be considered here only the two commonly used arrangements: (*a*) when the conductors are placed at the vertices of an equilateral triangle, (*b*) when the conductors are in the same plane.

(*a*) *When the conductors are placed at the vertices of an equilateral triangle.* — In a balanced three-phase system, the currents in A, B and C (Fig. 275a) are 120 degrees out of phase, the algebraic sum of the currents is zero (Chapter 7, Section 3), and the instantaneous flux set up around any conductor is the algebraic sum of the instantaneous fluxes set up around the other two conductors. Therefore, the inductance of each conductor in a three-phase system, when the conductors are placed at the vertices of an equilateral triangle, is equal to one half the inductance of the loop formed by any two of the conductors, and is calculated by means of equation (21).

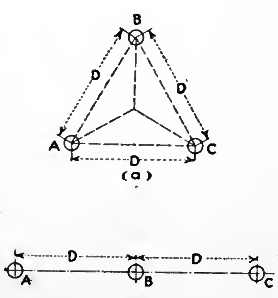


Fig. 275.

(*b*) *When the conductors are in the same plane.* — From equation (21) the inductance of either A or C (Fig. 275b) with regard to B is

$$L_1 = \frac{1}{2} + 2 \log_e \frac{D-r}{r}, \tag{22}$$

* To reduce c.g.s. units of inductance to henries, divide by 10^9. The expression $\log_e \frac{D-r}{r}$ may be replaced by $\log_e \frac{D}{r}$ without appreciable error when the ratio $\frac{D}{r}$ is large.

the inductance of A with regard to C or of C with regard to A is

$$L_2 = \frac{1}{2} + 2 \log_e \frac{2D - r}{r}, \tag{23}$$

and the inductance of B with regard to either A or C is

$$L_3 = \frac{1}{2} + 2 \log_e \frac{D - r}{r}. \tag{24}$$

It is common practice to transpose the conductors of polyphase alternating-current systems, thus making the inductances of the lines sensibly equal. Under this condition the inductance of each conductor in a three-phase system, when the conductors are in the same plane, is

$$L = \frac{1}{2} + \frac{4 \log_e \frac{D - r}{r} + 2 \log_e \frac{2D - r}{r}}{3} \tag{25}$$

$$= \frac{1}{2} + 2 \log_e \frac{1.26(D-r)}{r} \text{ c.g.s. units per centimeter length of conductor.*} \tag{26}$$

3. **Inductance of a conductor with earth return.**— A consideration of Fig. 272 shows that the flux linking with the current in each of two parallel wires separated D centimeters from each other, passes between the axis of the wire and a neutral plane distant $\frac{D}{2}$ centimeters from the axis of the wire. When the return current of a circuit flows through the earth, the surface of the earth is the neutral plane, and the inductance of a conductor placed h centimeters above, and parallel to, the surface of the earth is

$$L = \frac{1}{2} + 2 \log_e \frac{2h - r}{r} \text{ c.g.s. units per centimeter length of conductor.} \tag{27}$$

Note. — Equation (27) is strictly true only when the resistance of the earth return is zero, but a considerable resistance in the return circuit does not greatly increase the inductance.

4. **Inductance of a solenoid.** — The inductance of a solenoid, either with or without an iron core, is easily calculated when it is assumed that the flux set up by the coil is confined to the interior of the coil.† Let

N = the number of turns in the coil,
i = the current flowing in the coil, in c.g.s. units,
A = the internal cross-sectional area of the coil, in square centimeters,
l = the length of the coil, in centimeters,
μ = the permeability of the magnetic circuit.

The total flux set up by the coil (Chapter 2, Section 10) is

$$\phi = \frac{4\pi i N A \mu}{l} \tag{28}$$

* Equation (26) gives values only slightly greater than those given by equation (21).
† This condition is only approximated in commercial apparatus.

and the inductance of the helix is

$$L = \frac{N\phi}{i} \tag{29}$$

$$= \frac{4\pi N^2 A\mu}{l} \text{ c.g.s. units.} \tag{30}$$

Since the permeability of iron decreases as the flux density increases, the inductance of a coil wound on an iron core is not constant, but decreases as the current in the coil increases.

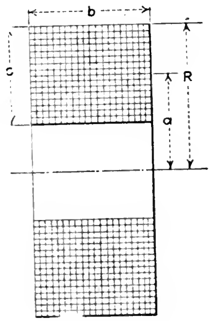


FIG. 276.

5. Inductance of a coil. — Professors Brooks and Turner * have developed an empirical formula by means of which the inductance of any closely wound cylindrical coil (Fig. 276) may be calculated. The error involved in the use of this formula seldom exceeds 4 per cent for a coil of any dimensions, and becomes less as the relative length of the coil increases.

$$L = \frac{4\pi^2 a^2 N^2 F' F''}{b + c + R} \text{ milhenries,} \tag{31}$$

when a = the mean radius of the winding, in centimeters,
b = the axial length of the coil, in centimeters,
c = the thickness of the winding, in centimeters,
R = the outer radius of the winding, in centimeters,
N = the total number of turns in the winding,

$$F' = \frac{10b + 12c + 2R}{10b + 10c + 1.4R},$$

$$F'' = 0.5 \log_{10}\left(100 + \frac{14R}{2b + 3c}\right).$$

6. Energy stored in a magnetic field. — In any circuit of inductance L,

$$e_L = L\frac{di}{dt}, \tag{32}$$

$$p_L = ei \tag{33}$$

$$= Li\frac{di}{dt} \tag{34}$$

and the energy required to increase or decrease the intensity of the magnetic field is

$$W_L = \int p\,dt \tag{35}$$

$$= L\int i\,di \tag{36}$$

$$= L\frac{i^2}{2}, \tag{37}$$

when the current varies between i and zero.

* University of Illinois Bulletin, Vol. IX, No. 10.

Substituting the value of L found in equation (30)

$$W_L = \frac{4\pi N^2 A\mu}{l} \times \frac{i^2}{2}. \tag{38}$$

But the intensity of the magnetic field (Chapter 2, Section 9) is

$$H = \frac{4\pi Ni}{l} \tag{39}$$

and the volume of the magnetic field is

$$V = Al \text{ cubic centimeters.} \tag{40}$$

Substituting the values of H and V in equation (38)

$$W_L = \frac{\mu H^2}{8\pi} \times V \text{ ergs} \tag{41}$$

$$= \frac{BH}{8\pi} \times \text{V ergs} \tag{42}$$

$$= \frac{BH}{8\pi} \text{ ergs per cubic centimeter} \tag{43}$$

$$= \frac{B^2}{8\pi\mu} \text{ ergs per cubic centimeter.} \tag{44}$$

7. Loss of energy due to hysteresis.—When the magnetic circuit is composed wholly or partly of iron, it has been shown that: (*a*) the energy delivered to the magnetic field is not all returned to the system when the magnetizing force is withdrawn, (*b*) that the flux density produced by a given magnetizing force is greater for decreasing values of field intensity than for increasing values, (*c*) that the iron is heated when the flux density changes.

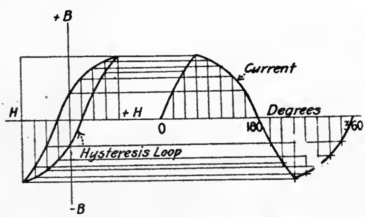


FIG. 277. Hysteresis Loop and Magnetizing Current Wave.

When the flux density B in the iron and the magnetizing force H of a magnetic circuit are plotted for a complete magnetic cycle, *i.e.*, for all possible values between a given positive maximum and an equal negative maximum density, a curve similar to Fig. 277 is obtained. Let

N = the number of turns in the exciting coil of an electromagnet,
i = the current in c.g.s. units flowing in the coil,
V = the volume of the iron core (l centimeters in length and A square centimeters in cross-sectional area),
ϕ = the flux in the iron core.

Then

$$e = N\frac{d\phi}{dt}, \tag{45}$$

$$ei\,(dt) = Ni\,(d\phi), \tag{46}$$

$$eit = Ni\int d\phi. \tag{47}$$

But $$V = lA, \tag{48}$$

$$\phi = AB \tag{49}$$

and $$Ni = \frac{Hl}{4\pi}. \tag{50}$$

Therefore, $$W_h = \frac{V}{4\pi}\int_{-B_m}^{+B_m} H\,dB \text{ c.g.s. units,} \tag{51}$$

i.e., the energy expended in carrying a volume of iron through a magnetic cycle is proportional to the area of the hysteresis loop, and the average power loss due to hysteresis in the iron is

$$P_h = \frac{fV}{4\pi}\int_{-B_m}^{+B_m} H\,dB \text{ ergs} \tag{52}$$

when f = the number of magnetic cycles per second through which the iron passes.

Dr. C. P. Steinmetz has shown that the area of a hysteresis loop is approximately proportional to the 1.6 power of the maximum flux density attained during a magnetic cycle. The loss per cycle per cubic centimeter of iron due to magnetic hysteresis is, then,

$$W_h = \eta B^{1.6} \text{ ergs,} \tag{53}$$

when η = the hysteretic (magnetic) constant and is dependent on the physical properties of the iron. Table XX.

TABLE XX

HYSTERETIC CONSTANTS (η)

Kind of iron	Constant
Best annealed sheet	0.0015
Good annealed sheet	0.003*
Ordinary annealed sheet	0.004
Soft annealed cast iron	0.008
Soft machine steel	0.01
Cast steel	0.12
Cast iron	0.16
Hardened steel	0.25

* Largely used for dynamo armature punchings.

8. Distortion of current wave due to hysteresis. — From Section 7 it is evident that the magnetizing current in a coil having an iron core in which the flux varies harmonically cannot be harmonic, but has a distorted wave shape as indicated in Fig. 277.

9. Growth and decay of current in an inductive circuit. — When an electromotive force of constant value is applied to a circuit containing both resistance and inductance, the current does not immediately rise to its final value $\frac{E}{R}$, because of the counter-electromotive force induced in the circuit by the

increasing flux; when a circuit containing both resistance and inductance is disconnected from a source of constant electromotive force, and the circuit closed, the current does not immediately fall to zero, but is maintained by the electromotive force induced in the circuit by the decreasing flux. Let

E = the applied electromotive force,
R = the resistance of the circuit,
L = the inductance of the circuit,
$\frac{di}{dt}$ = the rate at which the current flowing in the circuit changes.

For an increasing current,

$$E = Ri + L\frac{di}{dt}. \tag{54}$$

Transposing equation (54) and multiplying by $-R$,

$$\frac{-R\,(di)}{E - Ri} = \frac{-R\,(dt)}{L}, \tag{55}$$

$$\int_0^{\frac{E}{R}} \frac{-R\,(di)}{E - Ri} = -\frac{R}{L}\int_0^t dt, \tag{56}$$

$$\log_e \frac{E - Ri}{E} = -\frac{Rt}{L}, \tag{57}$$

and

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right). \tag{58}$$

For a decreasing current,

$$Ri + L\frac{di}{dt} = 0, \tag{59}$$

$$\frac{R\,(di)}{Ri} = -\frac{R\,(dt)}{L}, \tag{60}$$

$$\int_i^{\frac{E}{R}} \frac{R\,(di)}{Ri} = -\frac{R}{L}\int_t^0 dt, \tag{61}$$

$$\log_e \frac{i}{\frac{E}{R}} = -\frac{Rt}{L} \tag{62}$$

and

$$i = \frac{E}{R}\epsilon^{-\frac{Rt}{L}}. \tag{63}$$

APPENDIX C

CAPACITANCE

1. **The dielectric field.** — When the potential of a body is greater or less than zero (the surface of the earth is assumed to be at zero potential), the body is said to be electrified or charged. The energy required to produce electrification is stored in the surrounding medium, and there is set up in the medium a stressed condition termed a dielectric or electrostatic field. The dielectric flux emanating from a surface is normal to the surface as indicated in Fig. 278, and is represented by lines which terminate at another surface.

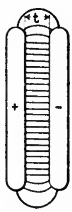


FIG. 278.

2. **Properties of electric charges.** — The following properties of electric charges have been established by experiment:

(*a*) Like charges repel; unlike charges attract.

(*b*) When a charge is produced on any body, an equal and opposite charge is produced on the same or on some other body.

(*c*) The force exerted between two charges of electricity is directly proportional to the product of the charges, and inversely proportional to the square of the distance separating them.

$$f = \frac{q_1 q_2}{r^2}. \tag{1}$$

3. **Electrostatic units.** — The units of the electrostatic system, and the equivalent values in practical units are:

(*a*) The unit of quantity (charge) is that quantity of electricity which repels with a force of one dyne a similar and equal quantity of electricity placed at a distance of one centimeter in air. To reduce electrostatic units of quantity to coulombs divide by 3×10^9.

(*b*) Unit current is that which conveys unit quantity past a given point on a conductor in one second. To reduce electrostatic units of current to amperes divide by 3×10^9.

(*c*) Unit electrostatic force or unit difference of potential exists between two points when one erg of work is expended in transferring unit quantity of electricity from one point to the other against the force of the electrostatic field. To reduce electrostatic units of potential to volts multiply by 300.

(*d*) Unit capacitance is that which produces unit difference of potential when charged with unit quantity of electricity. To reduce electrostatic units of capacitance to microfarads divide by 900,000.

(*e*) The specific inductive capacity or dielectric constant of a substance is the ratio of the capacitance of a condenser having that substance as a dielectric, to

the capacitance of the same condenser using dry air, at 0° C. and a pressure of 76 centimeters, as the dielectric. (Table XXI.)

TABLE XXI

DIELECTRIC CONSTANTS (K)

Material	Constant
Air	1.0
Oil (transformer)	2.1
Shellac	2.75
Paraffin	2.3
Rubber	2.35
Paper	2 to 4
Gutta percha	3 to 5
Glass	3 to 10
Mica	4 to 8
Varnished cambric	4 to 6
Pure water	80

(f) The intensity of a dielectric field is the ratio of the total flux, when propagated in air, to the area of the surface from which it emanates, and is equal to the force in dynes at the point.* (Symbol F.)

(g) The density of a dielectric field is the product of the field intensity and the dielectric constant.* (Symbol D.)

4. Flux due to unit charge. — The force with which a unit charge acts on another unit charge distant r centimeters in air is (from 2 c and 3 a)

$$f = \frac{1}{r^2} \text{ dynes} \tag{2}$$

and the intensity of the dielectric field is

$$F = \frac{1}{r^2} \text{ lines per square centimeter.} \tag{3}$$

But the surface of a sphere, the diameter of which is $2r$ centimeters is $4\pi r^2$ square centimeters. Therefore, the total dielectric flux emanating from unit charge is

$$\psi = \frac{1}{r^2} \times 4\pi r^2 \tag{4}$$

$$= 4\pi \text{ lines.} \tag{5}$$

5. Potential difference between points. — The potential difference between points is, by definition, equal to the work in ergs done when unit quantity of electricity is transferred from one point to the other against the force of the electrostatic field, and is, therefore, the line integral of the field intensity between the points.

$$E = \int F\,dx \tag{6}$$

* Compare with the corresponding terms used for the magnetic circuit (Chapter 2).

6. Capacitance. — Capacitance has been defined (Chapter 1, Section 12) as the ratio of the quantity of electricity displaced (the charge), to the electromotive force producing the displacement.

$$C = \frac{Q}{E}. \tag{7}$$

7. Capacitance of parallel plates. — Determine the capacitance of two parallel plates (Fig. 278) A square centimeters in area, and separated by t centimeters of dielectric, the constant of which is K. Let Q be the charge on each square centimeter of the positive plate and $-Q$ the charge on each square centimeter of the negative plate. Then

$$F = \frac{4\pi Q}{K} \text{ lines per square centimeter,} \tag{8}$$

$$E = Ft \tag{9}$$

$$= \frac{4\pi Qt}{K} \text{ electrostatic units,} \tag{10}$$

and

$$C = \frac{Q}{\frac{4\pi Qt}{K}} \tag{11}$$

$$= \frac{K}{4\pi t} \text{ electrostatic units per square centimeter} \tag{12}$$

$$= \frac{AK}{4\pi t} \text{ electrostatic units} \tag{13}$$

$$= \frac{AK}{4\pi t} \div 900{,}000 \text{ microfarads,} \tag{14}$$

i.e., the capacitance of a plate condenser is directly proportional to the product of the area of plates and the dielectric constant, and inversely proportional to the distance between the plates.

Commercial condensers are made up of a large number of sheets of tinfoil, to

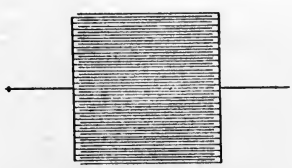


FIG. 279. The Plate Condenser.

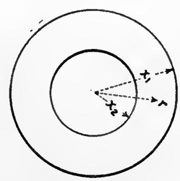


FIG. 280.

obtain the required area of the plates, connected as indicated in Fig. 279, and separated by sheets of paraffined paper.

8. Capacitance of concentric cylinders. — Determine the capacitance of two concentric cylinders (Fig. 280), the radii x_1 and x_2 of the cylinders, their length l, and the dielectric constant K being given. Let the charge per centimeter length of the positive cylinder be Q and the charge per centimeter length of the

negative cylinder be $-Q$. Then the field intensity at any point distant r centimeters from the axis of the cylinders is

$$F = \frac{4\pi Q}{2\pi r K} \tag{15}$$

$$= \frac{2Q}{rK} \text{ lines per square centimeter.} \tag{16}$$

The potential difference between the two cylinders is found by integrating equation (15) between the limits $r = x_1$ and $r = x_2$

$$E = \frac{2Q}{K}\int_{x_2}^{x_1}\frac{dr}{r} \tag{17}$$

$$= \frac{2Q}{K}\log_e\frac{x_1}{x_2} \tag{18}$$

and

$$C = \frac{Q}{\frac{2Q}{K}\log_e\frac{x_1}{x_2}} \tag{19}$$

$$= \frac{K}{2\log_e\frac{x_1}{x_2}} \text{ electrostatic units per centimeter length of cylinders.} \tag{20}$$

9. **Capacitance of parallel wires.** — Determine the capacitance of two parallel wires (Fig. 281), their radii r, the distance between their centers D, their length l and the dielectric constant K being given. Let the charge per centimeter length of the positive wire be Q and the charge per centimeter length of the negative wire be $-Q$. Then the field intensity at any point distant x centimeters from the axis of conductor A is

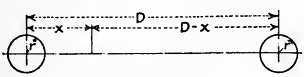


FIG. 281.

$$F = \frac{2Q}{K}\left(\frac{1}{x} + \frac{1}{D-x}\right) \tag{21}$$

and the potential difference between A and B is found by integrating equation (21) between the limits $x = D - r$ and $x = r$.

$$E = \frac{2Q}{K}\int_{r}^{D-r}\left(\frac{1}{x} + \frac{1}{D-x}\right)dx \tag{22}$$

$$= \frac{4Q}{K}\log_e\frac{D-r}{r}^{*} \tag{23}$$

* The expression $\log_e \frac{D-r}{r}$ may be replaced by $\log_e \frac{D}{r}$ without appreciable error when the ratio $\frac{D}{r}$ is large.

and $$C = \frac{Q}{\frac{4Q}{K}\log_e\frac{D-r}{r}} \tag{24}$$

$$= \frac{K}{4\log_e\frac{D-r}{r}} \text{ electrostatic units per centimeter length of circuit.} \tag{25}$$

10. Capacitance of each conductor in a three-phase circuit.—The capacitance of a conductor in a three-phase system depends on its position relatively to the other conductors in the system. There will be considered here only the two commonly used arrangements: (*a*) when the conductors are placed at the vertices of an equilateral triangle, (*b*) when the conductors are in the same plane.

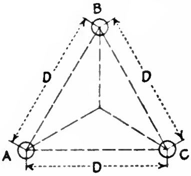


FIG. 282a.

(*a*) *When the conductors are placed at the vertices of an equilateral triangle.* — From equation (22) the potential difference between any conductor (Fig. 282a) and the neutral plane halfway between the conductor and either of the other conductors is

$$E = \frac{2Q}{K}\log_e\frac{D-r}{r} \tag{26}$$

and the capacitance of each conductor is

$$C = \frac{Q}{\frac{2Q}{K}\log_e\frac{D-r}{r}} \tag{27}$$

$$= \frac{K}{2\log_e\frac{D-r}{r}} \text{ electrostatic units per centimeter length of conductor,} \tag{28}$$

i.e., the capacitance of each conductor in a three-phase system, when the conductors are placed at the vertices of an equilateral triangle, is *twice* the capacitance of the loop formed by any two of the conductors.

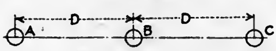


FIG. 282b.

(*b*) When the conductors are in the same plane. From equation (22) the potential difference between A or B (Fig. 282b) and their neutral plane is

$$E_1 = \frac{2Q}{K}\log_e\frac{D-r}{r}, \tag{29}$$

the potential difference between A or C and their neutral plane is

$$E_2 = \frac{2Q}{K}\log_e\frac{2D-r}{r}, \tag{30}$$

and the potential difference between B or C and their neutral plane is

$$E_3 = \frac{2Q}{K}\log_e \frac{D-r}{r}. \tag{31}$$

Under the common practice of transposing the conductors of polyphase systems, the capacitances of the conductors are sensibly equal, and may be calculated by means of the following equation:

$$C = \frac{3Q}{\frac{4Q}{K}\log_e \frac{D-r}{r} + \frac{2Q}{K}\log_e \frac{2D-r}{r}} \tag{32}$$

$$= \frac{3K}{4\log_e \frac{D-r}{r} + 2\log_e \frac{2D-r}{r}} \tag{33}$$

$$= \frac{K}{2\log_e \frac{1.26(D-r)}{r}} \text{ electrostatic units per centimeter length of conductor.*} \tag{34}$$

11. Charging current in a three-phase system. — From equation (16), Chapter 1, the current due to the capacitance of a single-phase line is

$$I_C = 2\pi fCE. \tag{35}$$

when the capacitance is concentrated. But the voltage between the terminals of a condenser star-connected to a three-phase system is $\frac{1}{\sqrt{3}}$ times the line-to-line voltage. Therefore, the charging current due to the capacitance of a three-phase line is

$$I_C = \frac{4\pi fCE}{\sqrt{3}}, \tag{36}$$

and is $\frac{2}{\sqrt{3}}$ (= 1.15) times the charging current flowing in the circuit formed by any two of the conductors when the voltage between lines is equal to the line-to-line voltage of the three-phase system.

12. Energy stored in the dielectric field. — When the intensity of a dielectric field increases or decreases, a current flows to or from the charged body.

$$i = \frac{dq}{dt} \tag{37}$$

and

$$q = Ce. \tag{38}$$

Therefore,

$$i = C\frac{de}{dt}. \tag{39}$$

* Equation (33) gives values only slightly greater than those given by equation (27).

But
$$W_C = \int p\,dt \tag{40}$$
$$= \int ei\,dt \tag{41}$$
$$= C\int e\,de \tag{42}$$
$$= \frac{Ce^2}{2} \text{ ergs} \tag{43}$$

when the applied voltage varies from e to zero.

Substituting in equation (43) the value of e obtained in equation (9), and that of C obtained in equation (13)

$$W_C = \frac{AK}{4\,\pi t} \times \frac{F^2 l^2}{2} \tag{44}$$
$$= \frac{AKF^2 l}{8\,\pi} \text{ ergs.} \tag{45}$$

But
$$F = \frac{D}{K} \tag{46}$$

and
$$Al = V \text{ cubic centimeters.} \tag{47}$$

Therefore,
$$W_C = \frac{D^2}{8\,\pi K} \times V \text{ ergs} \tag{48}$$
$$= \frac{DF}{8\,\pi} \text{ ergs per cubic centimeter} \tag{49}$$
$$= \frac{D^2}{8\,\pi K} \text{ ergs per cubic centimeter.} \tag{50}$$

13. Dielectric hysteresis. — It has been experimentally shown that the energy (heat) dissipated in the dielectric of a condenser is greater with alternating current than when a constant difference of potential exists between the condenser terminals, the effective value of the alternating electromotive force being equal to the constant electromotive force. This additional loss is due to so-called dielectric hysteresis.

When a condenser is connected to an alternating-current circuit, the changing value of the dielectric flux induces alternating electromotive forces in any conducting particles that may have become imbedded in the dielectric. Dielectric hysteresis is, therefore, more nearly analogous to eddy currents than to magnetic hysteresis. Dielectric hysteresis losses are always small, and are usually neglected.

14. Charging and discharging current in a condenser circuit. — When an electromotive force of constant value is applied to a circuit containing both resistance and capacitance, a charging current flows in the circuit; when a circuit containing both resistance and capacitance is disconnected from a

source of constant electromotive force, and the circuit closed, a discharging current flows in the circuit. Let

$E =$ the applied electromotive force,
$R =$ the resistance of the circuit,
$C =$ the capacitance of the circuit,
$\frac{dq}{dt} =$ the rate at which the capacitance charges or discharges.

For a charging current,

$$E = \frac{q}{C} + R\frac{dq}{dt}, \tag{51}$$

$$\frac{dq}{q - CE} = -\frac{dt}{RC}, \tag{52}$$

$$\int_q^0 \frac{dq}{q - CE} = -\int_t^0 \frac{dt}{RC}, \tag{53}$$

$$\log_e\left(\frac{q - CE}{-CE}\right) = -\frac{t}{RC} \tag{54}$$

and

$$q = CE\left(1 - \epsilon^{-\frac{t}{RC}}\right). \tag{55}$$

But

$$CE = Q \tag{56}$$

and

$$i = \frac{dq}{dt}. \tag{57}$$

Therefore

$$i = \frac{Q}{RC}\left(e^{-\frac{t}{RC}}\right). \tag{58}$$

For a discharging current,

$$\frac{q}{C} + R\frac{dq}{dt} = 0, \tag{59}$$

$$\frac{dq}{q} = -\frac{dt}{RC}, \tag{60}$$

$$\int_q^Q \frac{dq}{q} = -\int_t^0 \frac{dt}{RC}, \tag{61}$$

$$\log_e \frac{q}{Q} = -\frac{t}{RC}, \tag{62}$$

$$q = Qe^{-\frac{t}{RC}} \tag{63}$$

and

$$i = -\frac{Q}{RC}\epsilon^{-\frac{t}{RC}}. \tag{64}$$

15. Corona. — The phenomena known as corona * is an electrostatic (leakage) discharge between wires of different potential, and takes place when the

* See The Transactions of the American Institute of Electrical Engineers, Vol. XXX, pages 1889–1965 and Vol. XXXI, pages 1051–1092, "Law of Corona and the Dielectric Strength of Air," by F. W. Peek, Jr., Vol. XXXI, pages 1035–1049, "Corona Losses Between Wires at High Voltages," by C. Francis Harding.

potential difference between the wires exceeds a certain "critical" value, depending on the diameters of the wires and their distance apart. In an alternating-current system, the power losses due to corona are proportional to the frequency and to the square of the increase in voltage above the critical value, but are usually negligible for voltages up to 45,000. At voltages materially higher than the critical value, a visible halo-like envelope surrounds the conductor, the diameter of the envelope increasing as the voltage increases.

APPENDIX D

THE COMPLEX QUANTITY

Admittance, Conductance and Susceptance

1. **The complex quantity.** — It is evident that the equation

$$a = \sqrt{b^2 + c^2} \tag{1}$$

may be written

$$a = b \pm jc, \tag{2}$$

when it is specified that j indicates that b and c are at right angles, and are to be combined geometrically.

j may be defined as the complex operator, the effect of which is to rotate a vector to which it is applied, 90 degrees forward in relation to the reference line. The effect of $j \times j$ or j^2 is to rotate the vector 180 degrees in relation to the reference line, and the algebraic value of

$$j^2 = -1. \tag{3}$$

Therefore,

$$j = \sqrt{-1}. \tag{4}$$

Addition, subtraction, multiplication and division of equations involving the complex quantity are essentially algebraic operations, and are governed by the laws of such processes. The use of the complex quantity greatly simplifies the solution of some alternating-current problems, particularly those relating to transmission lines and distributing networks.

2. **Examples.** — (1) Find the line current when the currents in the parallel branches of a circuit are: $I_1 = 6 - j\,3$, and $I_2 = 8 - j\,2$.

$$\begin{aligned} I &= 6 + 8 - j\,3 - j\,2 \\ &= 14 - j\,5 \\ &= \sqrt{(14)^2 + 5^2} \\ &= 14.86 \text{ amperes.} \end{aligned}$$

(2) Find the applied electromotive force, the power component, the wattless component, and the power in a series circuit when: $I = 5 + j\,2$ and

$$\begin{aligned} Z &= 20 + j\,5. \\ E &= ZI \\ &= (20 + j\,5)(5 + j\,2) \\ &= 100 + j\,25 + j\,40 + j^2\,10 \\ &= 90 + j\,65 \text{ (since } j^2 = -1) \\ &= \sqrt{(90)^2 + (65)^2} \\ &= 111 \text{ volts (applied).} \\ E_1 &= 90 \text{ volts (power component).} \end{aligned}$$

$$
\begin{aligned}
E_2 &= 65 \text{ volts (wattless component).}\\
EI &= (90+j\,65)\,(5+j\,2)\\
&= 450+j\,325+j\,180+j^2\,130\\
&= 320+j\,505\\
P &= 320 \text{ watts.}
\end{aligned}
$$

(3) Find the power in an electric circuit when: $I = 10 + j\,4$ and

$$
\begin{aligned}
E &= 150 - j\,24.\\
EI &= (150 - j\,24)\,(10 + j\,4)\\
&= 1500 - j\,240 + j\,600 - j^2\,96\\
&= 1596 + j\,360,\\
P &= 1596 \text{ watts.}
\end{aligned}
$$

3. **Admittance, conductance and susceptance.** — The value of the current in any circuit is expressed by the equation

$$I = \frac{E}{Z} \tag{5}$$

$$= \frac{E}{\sqrt{R^2 + X^2}} \tag{6}$$

$$= \frac{E}{R \pm jX}. \tag{7}$$

Multiplying the right-hand member of equation (7) by $R \mp jX$

$$I = \frac{(R \mp jX)\,E}{R^2 + X^2} \tag{8}$$

$$= \left(\frac{R}{R^2 + X^2} \mp j\,\frac{X}{R^2 + X^2}\right) E. \tag{9}$$

The value of the current in the circuit may also be expressed by the equation

$$I = YE \tag{10}$$

$$= (g \mp jb)\,E, \tag{11}$$

when $Y = \frac{1}{Z}$ and is termed the admittance of the circuit,

$g = Y \cos \phi$ and is termed the conductance of the circuit,
$b = Y \sin \phi$ and is termed the susceptance of the circuit.

From equations (11) and (9)

$$g \mp jb = \frac{R}{R^2 + X^2} \mp j\,\frac{X}{R^2 + X^2}, \tag{12}$$

i.e., the conductance of a series circuit is equal to its resistance divided by the square of its impedance

$$g\,^* = \frac{R}{Z^2}, \tag{13}$$

* Compare equations (13) and (14) with equations (124) and (129), Chapter 1.

and the susceptance of a series circuit is equal to its reactance divided by the square of its impedance.

$$b\,^* = \frac{X}{Z^2}. \tag{14}$$

Admittances, like impedances, must be combined geometrically; conductances or susceptances are combined algebraically.

APPENDIX D — PROBLEMS

1–6. Solve Problems 1 to 6, Chapter 18, using Complex Quantity.

7–14. Solve Problems 22, 30, 34, 35, 36, 40, 41 and 42, Chapter 1, using admittance, conductance and susceptance values.

* Compare equations (13) and (14) with equations (120) and (125), Chapter 1.

APPENDIX E

RULES RECOMMENDED BY COMMISSION ON RESUSCITATION FROM ELECTRIC SHOCK

REPRESENTING

The American Medical Association
The National Electric Light Association
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Follow these instructions even if victim appears dead.

I. IMMEDIATELY BREAK THE CIRCUIT

With a single quick motion, free the victim from the current. Use any *dry non-conductor* (clothing) rope, board, to move either the victim or the wire. Beware of using metal or any moist material. While freeing the victim from the live conductor have every effort also made to shut off the current quickly.

II. INSTANTLY ATTEND TO THE VICTIM'S BREATHING

1. As soon as the victim is clear of the conductor, rapidly feel with your finger in his mouth and throat and remove any foreign body (tobacco, false teeth, etc.). Then *begin artificial respiration at once.* Do not stop to loosen the victim's clothing now; *every moment of delay is serious.* Proceed as follows:

(*a*) Lay the subject on his belly, with his arms extended as straight forward as possible and with face to one side, so that nose and mouth are free for breathing (see Fig. 1). Let an assistant draw forward the subject's tongue.

FIG. 1.—Inspiration; pressure off.

(*b*) Kneel, straddling the subject's thighs, and facing his head; rest the palms of your hands on the loins (on the muscles of the small of the back), with fingers spread over the lowest ribs, as in Fig. 1.

(*c*) With arms held straight, swing forward slowly so that the weight of your body is gradually, but *not violently*, brought to bear upon the subject (see Fig. 2). This act should take from two to three seconds.

(*d*) Then immediately swing backward so as to remove the pressure, thus returning to the position shown in Fig. 1.

(*e*) Repeat deliberately twelve to fifteen times a minute the swinging forward and back — a complete respiration in four or five seconds.

(*f*) As soon as this artificial respiration has been started and while it is being continued, an assistant should loosen any tight clothing about the subject's neck, chest, or waist.

2. Continue the artificial respiration (if necessary, two hours or longer), *without interruption*, until natural breathing is restored, or until a physician arrives. If natural breathing stops after being restored, use artificial respiration again.

FIG. 2.—Expiration; pressure on.

3. *Do not give any liquid by mouth until the subject is fully conscious.* Give the subject fresh air, but keep him warm.

III. SEND FOR NEAREST DOCTOR AS SOON AS ACCIDENT IS DISCOVERED

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